International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

http://ijpsat.ijsht-journals.org
Vol. 13 No. 1 February 2019, pp. 21-30

Empirical Analyze of Exogenous Economic Growth

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Absctrat - In 1956, Robert Solow published a paper on economic growth and development titled "A Contribution to the Theory of Economic Growth." For this work and for his later contribution to our understanding of economic growth, Solow was awarded the Nobel Prize in Economics in 1987. This descriptive paper illustrated Solow model in a short overview. But this model has not yet been assumed various factors such as social capital, international trade, income distribution etc. which are theoretically considered important for the growth of economies. Even though there are some issues in the Solow model with regard to the way the properties of production factors and certain variables were used, it is a mathematically correct model up to date. Moreover, the final section of this Paper illustrates how the Solow model, or its simple extensions can be used to interpret economic growth overtime, highlighting the factors such as investment or capital accumulation. However, this model has the potential to go a long way if the main characteristics were considered thoroughly.

Keywords - Basic Model, Solow Diagram, Comparative Statics, Technology, Empirical Growth.

I. The Basic Solow Model

The Solow model is built around two equations; a production function and a capital accumulation equation. The production function explains how input combines to produce output. To simplify the model, it groups the input into three categories: capital K, labor L, technological process A, and generated output Y. The production function is estimated to have the Cobb-Douglas form[1] and it is given by,

$$Y = F(K(t), L(t), A(t)) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}$$

For $1 < \alpha < 0$ (1)

This production function shows constant returns of scale, which means, if all the inputs are doubled, the output will definitely be doubled.[2] $K(t)$ and $L(t)$ are the stocks of capital and labor in the economy. α and $(1-\alpha)$ are parameters or measurements of the elasticizes in K and L. $A(t)$ is an index of the technological process of the economy. It grows over time at a constant rate of λ. Moreover λ is given exogenously, and it is shown by,

$$A(t) = A(1-\lambda)^t \quad (2)$$

The marginal productivity of the capital (MPK) and labor (MPL) is partly derivative of the function (1) with respect to K and L. It is given by,

$$MPK = \frac{\delta Y}{\delta K} = \alpha A K^{(\alpha-1)} L^{(1-\alpha)} = \frac{\alpha A K^{\alpha} L^{(1-\alpha)}}{K} = \alpha \frac{Y}{K} \quad (3)$$

$$MPL = \frac{\delta Y}{\delta L} = (1-\alpha) A K^{\alpha} L^{-\alpha} = (1-\alpha)\frac{Y}{L} \quad (4)$$

Production factors are paid according to their marginal productivity, capital share, and wage share of Y. It means that the firms in this economy pay workers a wage w, for each unit of labor and pay r in order to rent a unit of capital for one period. We assume that there is a large number of firms in the economy so that a perfect competition prevails

[1] Charles Cobb and Paul Douglas (1928] this functional form in their analysis of US. Manufacturing. Interestingly, they argued that this production function,

[2] In generally, multiplying the K and L by any positive constants C causes output to change by the same quantity of the K and L.

$$F(cK, cAL) = cF(K, AL) \qquad \text{For a } c < 0$$

and that the firms are price takers. [3]Under this condition, profit maximizing firms solve the following problem;

$$max_{K,L} = F(K,L) - rK - wL \qquad (5)$$

According to the first order conditions for this problem, a firm will hire labor until the marginal production of labor equals to the wage and will rent capital until the marginal production of capital is equal to the rental price.

$$w = \frac{\partial F}{\partial L} = (1-\alpha)\frac{Y}{L} \qquad (6)$$

$$r = \frac{\partial F}{\partial K} = \alpha\frac{Y}{K} \qquad (7)$$

According to that, $wL + rK = Y$, that is, payments of inputs or payments of factors are completely based on the value of the aggregate output production. At this point there are no economic profits to be earned. Also, the share of the output paid to labor is , $\frac{wL}{Y} = 1 - \alpha$ and the share of the paid capital is , $\frac{rK}{Y} = \alpha$.

Marginal productivity of the capital and labor is positive but concave, which means that, with an increase in input of a single factor, the output rises less than proportionately (α *and* $(1-\alpha) < 1$). It is shown by,

$$\frac{dY}{dK} > 0 , \frac{dY}{dL} > 0 \ but \ \frac{d^2Y}{d^2K} < 0 \ ,$$

$$\frac{d^2Y}{d^2L} < 0 \qquad (8)$$

According to the (1) equation, we can derive the production function of the output per labor, $y = \frac{Y}{L}$, and capital per labor, $k = \frac{K}{L}$. At this point, Solow model is ignoring the contribution of the technological process A, and driving everything according to L. We call this the intensive or the compact form of Solow Model.

$$y = k^{\alpha} \qquad (9)$$

This production function shows a decreasing returnee of the capital per worker. It is graphed in Figure 1. This function simply explains that more capital per worker effected to generate more output per worker. However, there are diminishing returns to capital per worker. This means that the output of a worker becomes less and less per unit of capital we provide him.

[3] Simple microeconomics assume that constant retunes of scale and the number of firms are indeterminate

The other main equation of the Solow model is an equation that describes how capital accumulates; this equation is shown by,

$$\dot{K} = sY - dK \qquad (10)$$

According to this equation, the changes in the capital stock, $\dot{K}$, is equal to the amount of gross investment, sY, and less than the amount of depreciation that concentrates during the production process, dK. A detailed explanation of these three terms is as follows;

$\dot{K}$ is the continuous time version[4] of , $K_{t+1} - K_t$, that is, the change in the capital stock per period. This special notation $\dot{K}$ illustrates a derevative with respect to time:

$$\dot{K} = \frac{dK}{dt} \qquad (11)$$

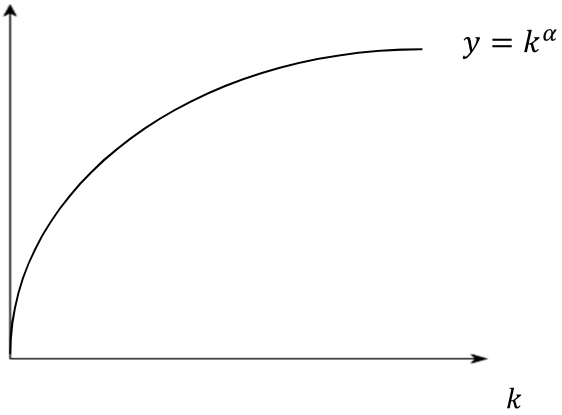
On the one hand, the term of the right-hand side of the equation (10) represents a gross investment. According to the Solow assumptions, we estimate that labor / consumers save a constant fractions of s. Since the economy is closed, Savings equals Investment, $S = I$. Thus, the economy accumulates capital only through the use of investment.

On the other hand, 10 equation illustrates the depreciation of the capital stock. The standard functional form used here implies a constant faction of d.[5]

Rewriting the capital accumulation equation in terms of capital per person, $y = k^{\alpha}$ illustrates that the amount of output per person produced for whatever capital stock per person is present in the economy. This can be demonstrated by using the simple mathematical trick, which is to "take

[4] The Solow model can be evaluated in two separate conditions, which are either in discrete or in continuous time estimations. Both conditions are used in the macroeconomics applications, but most applications formulated in continuous time.

[5] Which means, the capital stock depreciates every period ,for the example we estimate $d = .04$, so that 4 percent of the capital depreciation rate in our model economy were out each year



$\boldsymbol{Figure\ (1)}$: $Cobb - Douglas\ Productions\ function$

logs of the equation and then derivatives", given below (without tenological progess[6]) as,

$$y = k^{\alpha} \rightarrow \log Y = \alpha \log L \tag{12}$$

$$\rightarrow \frac{\dot{y}}{y} = \alpha \frac{\dot{k}}{k}$$

And,

$$k \equiv \frac{K}{L} \rightarrow \log K - \log L, \tag{13}$$

$$\rightarrow \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \quad .$$

By applying (13) and equation (10) , we can rewrite the capital accumulation equation per labor, but this time we consider the growth rate of the labor force,$\frac{\dot{L}}{L}$. At that point, labor force participation rate is constant and the population growth rate is given by,n, (every member of the population is considered as a worker). This implies that the labor force growth rate $\frac{\dot{L}}{L}$ is also given by n. For the example , if $n = 0.02$, then population and the labor force are growing at the rate of two percent per year. This growth can be represented by,

$$L(t) = L_0 e^{nt} \tag{14}$$

Next, the log is taken to differentiate this equation (14) and combined with the (13) equation and (10),

$$\frac{\dot{k}}{k} = \frac{sY}{K} - n - d \tag{15}$$

$$= \frac{sY}{k} - n - d$$

According to this ,we can derive the capital accumalation equation per worker;

$$\dot{k} = sy - (n + d)k \tag{16}$$

This equation explains that the change in capital per labor in each period is determined by three terms: investment rate ,sy , depreciation for the capital, d, and

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[6] We can rewrite this with the technologycal process $A(t)$.it is represented as,

$$\log Y = \log A + \alpha \log K + (1 - \alpha)L$$

And then totally deffrentiate (2.14) in relation to the time, is given by,

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \alpha \frac{\Delta K}{K} + (1 - \alpha)\frac{\Delta L}{L}$$

$$\rightarrow \quad g = \lambda + \alpha k \quad (1 - \alpha)l$$

Where, g is the growth rate over time of GDP, technical growth λ, capital stock growth k, and labor force growth rate l.Summarizing this production function illustrates production growth (GDP) coming from an increased in capital, labor and technological progress.

population growth n. If there were no new investments and no depreciation, capital per labor, $\dot{k}$, would decline because of the increase of the labor force (population growth). In this equation the relationship between technological progress and changes in capital per labor is not clearly specified. The next section illustrates these relationships in diagrams.

II. The Solow Diagram

Solow diagram is constant to two curves and they are combined as a function of the capital-labour ratio, k. The first curve is the amount of investment per person, $sy = sk^{\propto}$. This curve has the same shape as the productions function in Figure (1). The second curve is the line $(n + d)k$. This illustrates the amount of new investment per person required to keep the amount of capital per worker constant. Thus, both depreciation and the growing workforce tend to reduce the amount of the capital per person in the economy. The difference between these two curves is the change in the amount of the capital per worker. When this change is positive, and the economy is growing its capital per worker, the capital deepening occurs.

Let's consider a specific example: suppose an economy has capital equals to the amount $k_{0\,(today)}$ and the capital per worker is constant, so that the capital deepening occurs. That is, k increases over time. This capital deepening will continue until $k = k^*$, and at this point $sy = (n + d)k. Therefore,\ \dot{k} = 0$. At this point, the amount of capital per worker remains constant, and we call such a point a steady state. It is illustrated in Figure 2

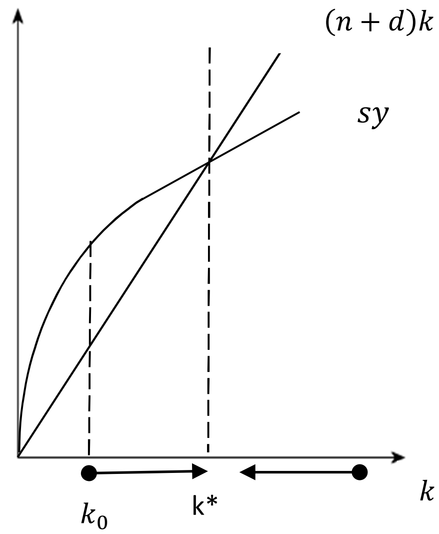
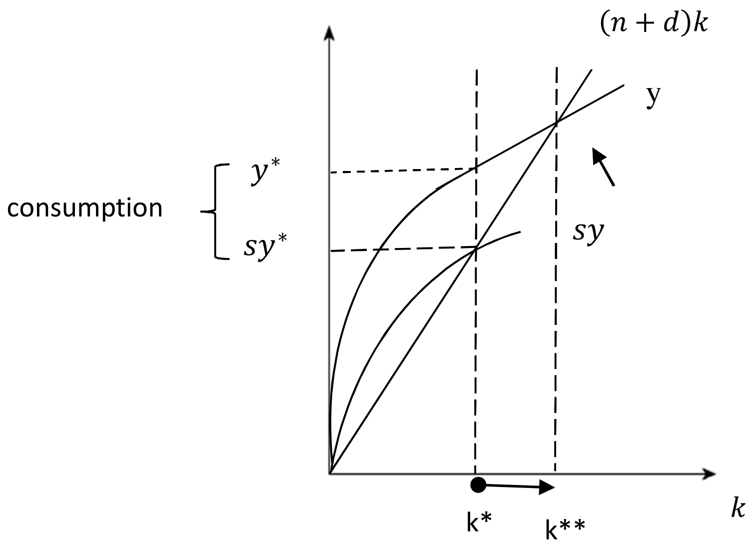


***Figure* (2)**: The Basic Solow Diagram

Notice that Solow diagram determines the steady state value of capital per worker. The production function of equation $y = k^{\alpha}$ then determines the steady state value of output per worker, y^*,as a function of k^*. It is convenient to include the production function in the Solow diagram itself to make this point clearer. This is presented in Figure 3. Note that the steady state consumption per worker is then given by the difference between the steady state output per worker, y^*, and the steady state investment per worker, sy^*.



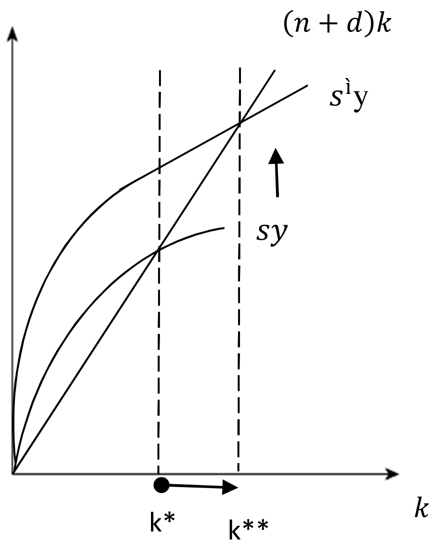
$\boldsymbol{Figure}$ (**3**): The Basic Solow Diagram and the Production Funtion

III. Comparative Statics

Comparative statics are used to examine the response of the model to changes in the value of various parameters. In this section, we will consider an increase in the investment rate, s, and an increase in the population growth rate, n.

A. An Increase in the Investment Rate

Consider an economy in the steady state condition. Now consumers in the economy decide to increase the investment rate sy to $s^{ì}y$. What happens to the economy? Figure 4 illustrates the answer;



$\boldsymbol{Figure}$ (**4**): An Increase in The Investment Rate

The increase in the investment rate shifts the sy curve to $s^{\grave{}}y$. At the current value of the capital stock, k*, investment per worker now exceeds the amount required to keep capital per worker constant, and therefore the economy begins the capital deepening again. This capital deepening continues until $s^{\grave{}}y = (n+d)k$ and capital stock per worker reaches a higher value, represented by the point k**.

B. An Increase in The Population Growth Rate

Suppose that the economy has reached its steady state. But this time population growth rate rises from n to $n^{\grave{}}$. Now, what happens to the economy? The answer is found in Figure 5.

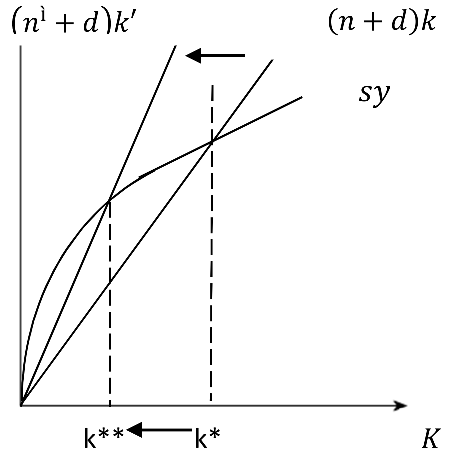


***Figure* (5)**: An Increase in The Population Growth Rate

The $(n+d)k$ curve rotates up to the new curve $\left(n^{\grave{}}+d\right)k'$. At the current value of the capital stock, (steady state value), k*, investment per worker is now no longer higher enough to keep the capital labor ratio constant in the face of the rising population. Therefore, the capital-labor ratio begins to fall. It continues to fall until the point where $sy = \left(n^{\grave{}}+d\right)k'$.

IV. TECHNOLOGY AND THE SOLOW DIAGRAM

We analyzed Solow model with technological progress in section 1.1, then set up the differential equations, and analyze it in a Solow diagram to find the steady state. In this situation, the only important difference is that the variable k is no longer constant in the long - run. Therefore, we use the differential equation in terms of another variable. At this point, the new state variable will be $\tilde{k} \equiv \frac{K}{AL}$. The variable $\tilde{k}$ represents the ratio of the capital per worker to technology or "capital technology" ratio.

According to this rewriting, the production functions in terms of the $\tilde{k}$,

$$\tilde{y} = \widetilde{k^{\alpha}} \tag{17}$$

where $\tilde{y} = \frac{Y}{AL} = y/A$. Following the terminology above, we will refer to $\tilde{y}$ as the output technology ratio. According to this rewriting the capital accumulation equation in terms of $\tilde{y}$ is,

$$\frac{\widetilde{k^{\grave{}}}}{\tilde{k}} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} - \frac{\dot{L}}{L}. \tag{18}$$

We already know that A is said to be labor augmenting. Technological progress occurs when A increases over time. A unit of labor is more productive when the level of technology is higher. Moreover, an important assumption of the Solow model is that the technological progress is exogenous, and it is growing at a constant rate. We can rewrite this as follows where g is a parameter representing the growth rate of technology,

$$\frac{\dot{A}}{A} = g \tag{19}$$

The combination of this and the capital accumulation equation (2.16) is presented in Figure 6.

$$\widetilde{k^{\grave{}}} = s\tilde{y} - (n+g+d)\tilde{k} \tag{20}$$

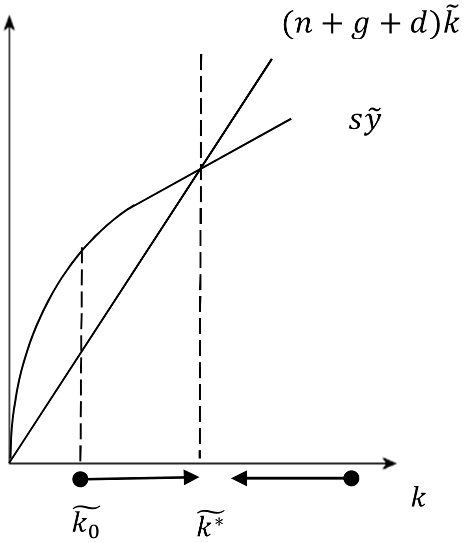


***Figure* (6)**: The Basic Solow Diagram with technological progress

The analysis of this diagram is similar to a situation when there is no technological progress. But the interpretations are slightly different. If the economy begins with capital technological ratio below its steady state level, at a point $\widetilde{k_0}$ will increase, because the amount of investment being undertaken exceeds the amount needed to keep the capital technological ratio constant. This increment will happen until $\widetilde{k^*}, or\ s\tilde{y} = (n+g+d)\tilde{k}$.

Until this point, we explained the basic Solow model, Solow diagrams (under the increasing investment and population growth), and technological progress with Solow model. The next section illustrates how the Solow model, or its simple extensions can be used to interpret economic growth overtime, highlighting the factors such as investment or capital accumulation. Let us consider some empirical evidence of these relationships in the next section.

V. Empirical Economics Growth

Focusing on the economic growth carried out during the period of 2015-2017 in world countries, this study mentioned below show that the growth was positively depended on investment, especially in the counting period. The two studies (Table 01) show that, especially in the counting period, growth was positively depended to investment growth. Here, the DGP increase was mainly due to the investment growth, specially fixed assets including land improvement plans, machinery, and equipment purchases, the construction of roads, railways, and the like, as well as schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings. Inventories, stocks of goods held by firms to meet temporary or unexpected fluctuations in production, can also be taken as investments.

Table 1: Investment growth and Total GDP, Annual investment growth rate (%), 2005 – 2017 and Total GDP, Million US dollars, 2015 – 2017

COUNTRY	2015	2016	2017
ITALY (INVESTMENT)	2.08	3.05	4.28
JAPAN (INVESTMENT)	1.70	1.10	2.47
USA (INVESTMENT)	3.26	1.65	4.02
GERMANY (INVESTMENT)	1.61	3.53	2.88
EROPE UNION (INVESTMENT)	4.89	3.12	3.07

ITALY	**(GDP)**	2237096	2367104	2477387
JAPAN	**(GDP)**	5142456	5245730	5333444
USA	**(GDP)**	18219297	18707189	19485394
GERMANY	**(GDP)**	3919280	4110953	4345630
EROPE UNION	**(GDP)**	19734878	20560615	21786840

Source: Aggregate National Accounts, SNA 2008

Moreover, both output per worker and capital per worker grow at the rate of exogenous technological changes. Therefore, technological growth is an important factor in long-run economic growth. We have seen several examples of how transition dynamics can allow countries to grow at different rates from their long-run economic growth. For example, an economy with a capital technology ratio $\tilde{k}$, reaches its steady states level in the long-run. This reasoning may help explain why countries such as Japan and Germany, which had their capital stock wiped out by World War II, have grown more rapidly than the USA over the last fifty years. It may also explain why an economy that increases its investment rate will grow rapidly as it makes the transition to a higher output technology ratio. This explanation may work well for countries such as South Korea, Singapore and Taiwan. Their investment rates have increased dramatically since 1950. Therefore, technological changes have positively affected the capital and economic growth in the history of economics.

Solow model answers to the following question: why some of us are rich and the others are poor? Countries that have a higher investment rate will tend to be richer. Such countries accumulate more capital per worker. Countries with more capital per worker have more output per worker, and countries that have higher population growth rates will tend to be poorer. According to the Solow model a higher function of the savings in these countries must go simply to keep the capital-labor ratio constant in the face of the growing population. But the problem is that these countries do not have strong savings within the economy. Therefore, this capital widening requirement makes capital deepening more difficult and these economies tend to accumulate less capital per worker. For example, most of the Asian, Latin American and African countries are facing these capital deepening conditions.

However, first, while the growth rate of the output per worker in the East Asian countries are clearly remarkable, their rates of growth in total factors productivity (TFP) are less so. Several other countries such as Italy, Brazil, and Chile have also experienced a rapid TFP. Total factor productivity growth, while typically higher than the United State, was not exceptional in the East Asian countries. Second, the output per worker in the East Asian countries is much higher than the TFP growth. One of the best examples of this is Singapore. Its rapid growth of the output per worker is entirely attributable to the growth in capital, technology, and education. Most generally, a key source of the rapid growth performance of these countries' factor accumulation, especially depended on investment, technological growth, and population management. Consider the productivity of firms and industries that produce technological assets in the USA. These sectors are experiencing fundamental technical progress the ability to produce more output in the form of faster processors expanded storage capacity and increased capabilities in the economy. This affects both industry and aggregate productivity. Information technology and capital accumulation appears entirely consistent with the neoclassical model. Massive input substitution and rapid capital accumulation during the 1990s have led to an aggregate growth contribution that is now quite large. Prior to that, the contribution was modest because the stock of Information technology industry was small. Only in the late 1990s, after a major information technology investment boom, were there enough information technology inputs to have a substantial impact on growth and labor productivity at the aggregate level in the USA.

The next case is population growth and development. The population of Latin America increases annually at a rate of 2.9 percent in middle of the past century, which is higher than that of Africa (2.6 per cent), Asia (2.3 per cent), Oceania (2.0 per cent), North America (1.3 per cent) or Europe (.8 per cent). Table 2 illustrates this clearly.

Table 2: Annual Population Growth Rate in the Selected Latin American Countries

Country	Period				
	1930-39	1940-49	1950-59	1958-63	1963-70
Central America					
Guatemala	1.40	1.97	2.87	3.20	2.90
Mexico	1.88	2.45	3.13	3.20	3.50
Nicaragua	2.37	2.72	3.04	2.90	3.70
Panama	1.70	2.51	3.03	3.20	3.30
South America					
Brazil	2.09	2.31	2.97	3.10	3.20
Colombia	2.23	2.24	3.13	2.20	3.20
Ecuador	2.24	2.30	2.98	3.10	3.40
Venezuela	1.87	2.40	3.32	3.40	3.60

Source: Natural increase for 1952 computed from United Nations, Demographic Year book, 1957 (New York, 1957), pp. 164-68, 188-92; figures for 1964 computed from United Nations, Demographic Yearbook, 1975 (New York, 1975), pp. 284-88, 733-35.

The population growth of these countries negatively affected the economic development and the distribution of the income because according to the economics growth theory, the growth rate of any given country's production is a function of the growth of capital and the labor force. Increasing the supply of the capital depended essentially on the savings. Household savings constitute only a part of the total savings and according to the Solow model, population growth results in the decrease of the total savings and the investments in an economy.

VI. Advantages and Limitations

The advantages of this model include the indigenization of the capital-labor ratio. Such a ratio varies in fact according to the level of capital per worker and parameter α of the production function. Thus, this model introduces the technological concept correlation with the economic growth. Moreover, it explains a modest part of the variance of the growth. For example, it explains the growth of the output of labor in economies due to the decreasing retunes of the capital per labor while this concept is not often estimated in the real world or when it is estimated, is mainly explained by the other variables.

Limitations of the model concern the assumptions of the technological progress assumed exogenously. However, this depends on the decisions of the investment in education, research and development, innovations, etc. These are not exogenous factors in the economy. Thus, in the analysis of the convergence, the model assumes that technology is similar for all the countries. It does not consider the domestic factors (different educational capabilities and abilities to absorb imported technologies etc.) that affect the different levels of progress in countries. Moreover, this model ignores some important factors which are discussed in other recent growth models. For example, it ignores fundamental growth factors such as human capital, international trade, social capital etc. However, the neoclassical models that came after the endogenous theories, took these important factors in to consideration.

VII. Conclusion

In conclusion, this chapter illustrated Solow model in a short overview. But this model has not yet been assumed various factors such as social capital, international trade, income distribution etc. which are theoretically considered important for the growth of economies. Even though there are some issues in the Solow model with regard to the way the properties of production factors and certain variables were used, it is a mathematically correct model up to date. In my opinion, this model has the potential to go a long way if the main characteristics were considered thoroughly. In the next chapters we will try to explain this situation in detail.

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