

Modeling Of The 2004 Tsunami In Indonesia Between The Epicenter To The East 90 Ridge

Gabin Daniel Chan Koi Lame¹, Jacques Chrysologue Ratsimavo², Jean Eugène Randrianantenaina³

¹ Ecole doctorale Géosciences, Physique, Chimie de l'environnement et système Hôte- Pathogène –
Université de Toliara – Madagascar

^{2,3} Laboratoire de dynamique de l'atmosphère, des climats et des océans (DyACO)- Université de
Toliara- Madagascar

Corresponding author: danielgabinc@gmail.com



Abstract – Madagascar was spared the tsunami in Indonesia in december 2004. The East 90 ridge and the Mascarene plateaus have constituted a protective barrier for having considerably attenuated the energy of the tsunami on arrival on the east coast of Madagascar. But since 2018, in addition to the new geological context in the Somali and Comoros basins, producing the formation of an underwater volcanic channel in the Mayotte sea, the phenomenon of the East african rift and the phenomenon of global warming have been added, Madagascar is once again in a tsunami risk zone. This requires a new appropriate measure according to the country's possibilities.

Keywords – East 90 ridge, Somali and Comoros basins, East african rift, tsunami-risk zone, global warming.

1.- INTRODUCTION

The life of a tsunami can be summed up in three phases : the triggering phase, the propagation phase and the surge phase.

The triggering phase is still based on a probabilistic theory. We know that the phenomenon will occur but as for saying when and where, there is still a lot to do for seismologists.

The advancement of technology such as the recent discovery of gravitational waves in 2017 has contributed a lot to the reduction of the time interval between the triggering phase and the alert data.

The only phase that can be interpreted by mathematical tools is the propagation phase.

[1]

The numerical resolution of the Navier and Stokes equations is current. Added to this are simulation approaches and the use of catalogs of events that have occurred in the past for regions on red alert.

However, given the problem of the accuracy of bathymetry, it is still difficult to accurately determine the propagation time of the tsunami. [2]

The surge phase is summarized by the impact of the tsunami on exposed coasts and shores. This impact depends on the scale of the tsunami and the vulnerability of the coasts.

The only effective defense is to prepare in advance according to the possibilities of each country.

In this article, we propose to model the tsunami propagation phase between the epicenter and the East 90 ridge in order to know the phenomena that occurred during the propagation on this portion of tsunami at december 2004.

This is an area for which the bathymetric map reveals a constant depth. Therefore the curves that give the propagation duration should be circular curves.

But satellite measurements and the results of calculations in laboratories give another result. We propose to give an explanation of this difference.

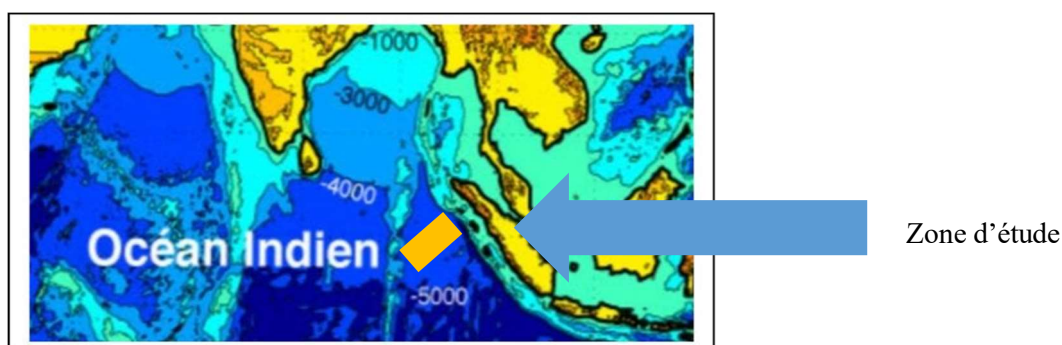
According to satellite monitoring and numerical calculations, Madagascar was reported to have been hit by the 2004 tsunami, but testimonies from the population and daily newspapers showed that this impact was without human loss or significant material damage. The East 90 ridge and the Mascarene plateaus constituted a protective barrier for Madagascar.

But since 2018, in addition to the geological context in the Comoros basin, which has changed a lot, there is the context of the East African rift [3] and global warming [4]. These are elements that make Madagascar once again exposed to a new tsunami risk.

2.- METHODOLOGY

2.1.- Modeling of the propagation of the 2004 tsunami in Indonesia

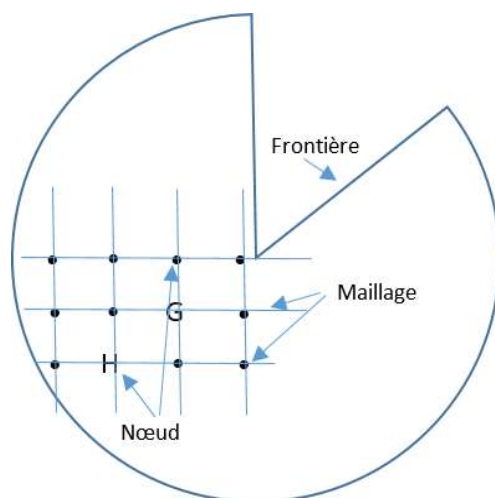
The study is carried out between the epicenter and the East 90 ridge.



2.2.- Stages of the finite difference method

This principle is broken down into several stages:

- a- The domain studied is meshed, we speak of discretization of the domain Meshing of a two-dimensional domain



The finite difference method searches for a solution at the nodes of the mesh

b – We also discretize the partial differential equation, that is to say that we will write at each node an algebraic approximation of the original equation.

c – We write as many algebraic equations as there are nodes where we are looking for a solution. Which leads to writing a system of equations

d- We solve this system of equations and the resolution will be done with MATHLAB 803 [5]

2.3.- Velocity potential by finite difference method

In the study, we will admit the following approximations:

- The fluid is perfect
- The fluid is irrotational in the Earth's gravity field
- The variation of atmospheric pressure on the ocean surface is negligible
- The variations of the gravity field due to the altitude and the constitution of the oceanic crust are also negligible
- The stress forces due to the wind on the ocean during the tsunami propagation period are considered negligible.[6]

Which gives for the continuity equation of the Navier and Stokes equations

$\text{div}(\vec{u}) = 0$ where \vec{u} is the velocity vector

We can consider a velocity potential \mathcal{V} defined by $\vec{u} = \text{grad} \mathcal{V}$ [7]

Which gives $\text{Laplacien de } \mathcal{V} = 0$

Equation 1

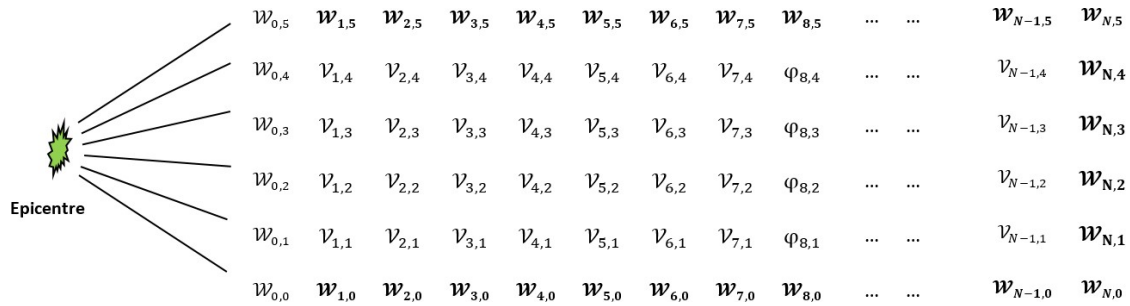
In two Cartesian dimensions, the Laplacian is written: $\frac{\partial^2 \mathcal{V}}{\partial x^2} + \frac{\partial^2 \mathcal{V}}{\partial y^2} = 0$

Equation 2

We must therefore discretize the second derivative in the form of a finite difference, which is done as always using limited developments.

By transposing Taylor's formula, $f(a + h) = f(a) + hf'(a) + \frac{h^2}{2}f''(a) + O(h^3)$,

we can write a development of order 3 along the x direction:



$$\mathcal{V}(x_i + \delta; y_j) = \mathcal{V}(x_i; y_j) + \delta \frac{\partial \mathcal{V}}{\partial x}(x_i; y_j) + \frac{\delta^2}{2} \frac{\partial^2 \mathcal{V}}{\partial x^2}(x_i; y_j) + \frac{\delta^3}{3!} \frac{\partial^3 \mathcal{V}}{\partial x^3} + O(h^3)$$

And

$$\mathcal{V}(x_i - \delta; y_j) = \mathcal{V}(x_i; y_j) - \delta \frac{\partial \mathcal{V}}{\partial x}(x_i; y_j) + \frac{\delta^2}{2} \frac{\partial^2 \mathcal{V}}{\partial x^2}(x_i; y_j) - \frac{\delta^3}{3!} \frac{\partial^3 \mathcal{V}}{\partial x^3} + O(h^3)$$

we also write a development of order 3 along the y direction: And Laplace's equation imposes

$$\Delta \mathcal{V} = 0$$

Which gives for all $(i;j)$ in the domain and to order 2 in δ :

$$\mathcal{V}[i+1;j] + \mathcal{V}[i-1;j] + \mathcal{V}[i;j+1] + \mathcal{V}[i;j-1] - 4\mathcal{V}[i;j] = 0 \quad [8] \quad \text{Equation 3}$$

Let us call $\mathcal{V}_{i,j}$ the values of the velocity potentials to be determined on each node of the meshes and $\mathcal{W}_{i,j}$ the values of the velocity potentials at the boundaries of the domain. They are deduced from the formula obtained by the formula $u = \overrightarrow{\text{grad}}\mathcal{W}$ (therefore known)

After linearization, Equation 3 is written in matrix form: $[A] * [\mathcal{V}] = [B]$ so that
 $[\mathcal{V}] = [A]^{-1} * [B]$ Equation 4

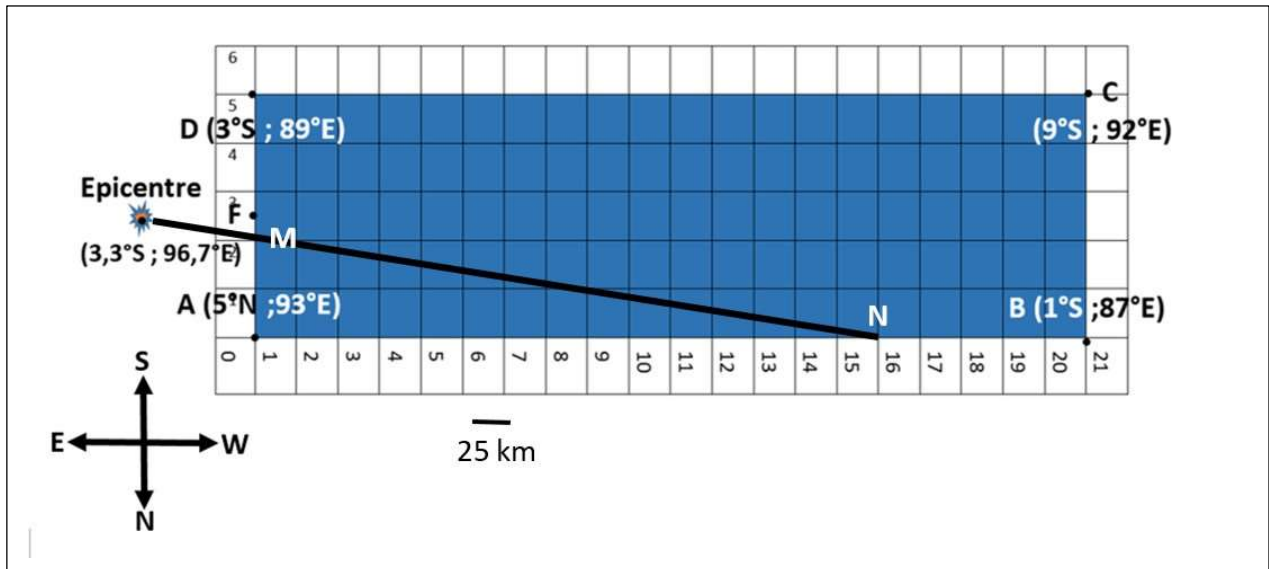
2.4.- Meshing of the domain

The length of the study domain is 500 km The depth of the sea in the area is 5000m Rectangular domain with a pitch of 25 km

We will have 20 grids along the length and 5 grids along the width

The domain will be rectangular consisting of 20 grids along the length and 5 grids along the width

The pitch of each grid is set at 25 km.



The values of the velocity potentials at the boundaries of the domain are the boundary conditions.

Usually, these values are given by the Dirichlet conditions but here, they are calculated from the property of the propagation velocity for $h \ll \lambda$ such that $c = \sqrt{gh}$ [9]

The propagation velocity is constant so the velocity potential can be written between two points (A) and (B): $\mathcal{W}_A - \mathcal{W}_B = c(r_A - r_B)$ Equation 5



3.- RESULTS

3.1.- The velocity potentials at the boundaries

6800	33377	45800	66200	78200	106400	118256	176200	184000	201600	221600	241600	259000	276700	296600	316700	336700	361700	381900	401700	421700	6800
5121																					5121
3770																					3770
3200																					3200
3770																					3770
5121																					5121
6800	33377	45800	66200	78200	106400	118256	176200	184000	201600	221600	241600	259000	276700	296600	316700	336700	361700	381900	401700	421700	6800

We will take the potential at the epicenter as zero $W_0=0$

At any point (M) on the boundary, we have: $W_M=c.(OM)$ where $(OM)=\sqrt{x^2+y^2}$

3.2.- Determination of the values of the velocity potentials in the domain

3.2.1.- Linearization

The linearization of Equation 3 results in the relation : $[A]*[V]=[B]$

Linearization block matrix $[A]$ is

E	I	O	O	O
O	E	I	O	O
O	I	E	I	O
O	O	O	E	I
O	O	O	O	E

$[O]$ and $[I]$ are respectively the zero matrix and the unit matrix, both of dimension 20×20

Matrix E is of dimension 20×20

-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	-4	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	-4	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	-4	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	-4	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	-4	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	-4	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	-4	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-4	1

[B] is the following column matrix of dimension 1 X 100 :

$$[B] = \begin{Bmatrix} [B1] \\ [B2] \\ [B3] \\ [B4] \\ [B5] \end{Bmatrix}$$

Where [B1], [B2], [B3], [B4] and [B5] are respectively following matrix column

$$[B1] = \begin{Bmatrix} -93\,600 \\ -79\,600 \\ -99\,600 \\ -119\,600 \\ -139\,600 \\ -159\,600 \\ -179\,600 \\ -199\,600 \\ -219\,600 \\ -239\,600 \\ -259\,600 \\ -279\,600 \\ -299\,600 \\ -319\,600 \\ -339\,600 \\ -359\,600 \\ -379\,600 \\ -399\,600 \\ -419\,600 \\ -893\,600 \end{Bmatrix} \quad [B2] = \begin{Bmatrix} -28\,800 \\ -448\,800 \end{Bmatrix} \quad [B3] = \begin{Bmatrix} -30\,200 \\ -50\,200 \end{Bmatrix} \quad [B4] = \begin{Bmatrix} -28\,800 \\ -448\,800 \end{Bmatrix} \quad [B5] = \begin{Bmatrix} -93\,600 \\ -79\,600 \\ -99\,600 \\ -119\,600 \\ -139\,600 \\ -159\,600 \\ -179\,600 \\ -199\,600 \\ -219\,600 \\ -239\,600 \\ -259\,600 \\ -279\,600 \\ -299\,600 \\ -319\,600 \\ -339\,600 \\ -359\,600 \\ -379\,600 \\ -399\,600 \\ -419\,600 \\ -893\,600 \end{Bmatrix}$$

3.2.2.- Result of the matrices [V] of the velocity potentials

We start from the matrix relation $[A] * [V] = [B]$

If $[A]^{-1}$ is the inverse matrix of $[A]$ then, $[V] = [A]^{-1} * [B]$

$$[V]_1 = \begin{Bmatrix} 27331.4104 \\ 45715.0896 \\ 64723.5899 \\ 82562.1362 \\ 103726.839 \\ 122334.98 \\ 143915.211 \\ 167580.679 \\ 184602.275 \\ 202703.666 \\ 221741.159 \\ 240869.799 \\ 259384.337 \\ 278060.404 \\ 297547.494 \\ 317420.029 \\ 337699.594 \\ 358986.138 \\ 378836.139 \\ 397616.32 \end{Bmatrix} \quad [V]_2 = \begin{Bmatrix} 25160.5522 \\ 45005.3581 \\ 64417.1337 \\ 83598.1165 \\ 103610.238 \\ 123441.87 \\ 144149.186 \\ 165105.229 \\ 184124.755 \\ 202871.231 \\ 221791.17 \\ 240753.7 \\ 259607.144 \\ 278609.784 \\ 298009.543 \\ 317733.027 \\ 337692.208 \\ 357708.818 \\ 376842.097 \\ 394609.14 \end{Bmatrix} \quad [V]_3 = \begin{Bmatrix} 24562.4403 \\ 44728.6567 \\ 64341.4705 \\ 83802.9577 \\ 103674.128 \\ 123673.076 \\ 144134.436 \\ 164566.294 \\ 183920.285 \\ 202865.334 \\ 221798.59 \\ 240746.686 \\ 259680.755 \\ 278762.047 \\ 298147.865 \\ 317810.328 \\ 337627.394 \\ 357314.831 \\ 376214.292 \\ 393858.143 \end{Bmatrix}$$

$$[V]_4 = \begin{Bmatrix} 25160.5522 \\ 45005.3581 \\ 64417.1337 \\ 83598.1165 \\ 103610.238 \\ 123441.87 \\ 144149.186 \\ 165105.229 \\ 184124.755 \\ 202871.231 \\ 221791.17 \\ 2408753.7 \\ 259607.144 \\ 278609.784 \\ 298009.543 \\ 317733.027 \\ 337692.208 \\ 357708.818 \\ 376842.097 \\ 394609.14 \end{Bmatrix} \quad [V]_5 = \begin{Bmatrix} 27331.4104 \\ 45715.0896 \\ 64723.5899 \\ 82562.1362 \\ 103726.839 \\ 122334.98 \\ 143915.211 \\ 167580.679 \\ 184602.275 \\ 202703.666 \\ 221741.159 \\ 240869.799 \\ 259384.337 \\ 278060.404 \\ 297547.494 \\ 317420.029 \\ 337699.594 \\ 358986.138 \\ 378836.139 \\ 397616.32 \end{Bmatrix}$$

The resolution of this equation is done by MATLAB and gives as result the following column matrix of velocity potentials $[V]$:

$$[V] = \begin{Bmatrix} [v1] \\ [v2] \\ [v3] \\ [v4] \\ [v5] \end{Bmatrix}$$

We obtain the tables of values of the speed potentials between columns 1 to column 22

1	2	3	4	5	6	7	8	9	10
6800	33377	45800	66200	78200	106400	118256	141596	176200	184000
5120	27 331	45 715	64 723	82 562	103 727	122 335	143 915	167 581	184 602
3770	25 160	45 005	64 417	83 598	103 610	123 442	144 143	165 105	184 125
3200	24 562	44 729	64 341	83 803	103 674	123 673	144 134	164 566	183 920
3770	25 161	45 005	64 417	83 598	103 610	123 442	144 143	165 105	184 125
5120	27 331	45 715	64 724	82 562	103 727	122 335	143 915	167 581	184 602
6800	33377	45800	66200	78200	106400	118256	141596	176200	184000

11	12	13	14	15	16	17	18	19	20	21	22
201600	221600	241600	259000	276700	296700	316700	336700	361700	381900	401700	421700
202 704	221 741	240 870	259 384	278 060	297 547	317 420	337 670	358 986	378 836	397 616	415300
202 871	221 791	240 753	259 607	278 610	298 009	317 733	337 692	357 709	376 824	394 609	410120
202 865	221 799	240 747	259681	278 762	298 148	317 810	337 627	357 315	376 214	393 858	410000
202 871	221 791	240 753	259 607	278 610	298 009	317 733	337 723	357 709	376 842	394 609	410120
202 704	221 741	240 870	259 384	278 060	297 547	317 420	337 670	358 986	378 836	397 616	415300
201600	221600	241600	259000	276700	296700	316700	336700	361700	381900	401700	421700

4- DISCUSSION OF RESULTS

4.1.- Validity of the results of the speed potentials

In general, as a precautionary measure on the imprecision of the bathymetry of the area, we take a fairly small mesh (of the order of 15 m). Here, I took a mesh of 25 km.

Let's see if this can stick to reality.

Let's consider six (6) arbitrary portions L_1 ; L_2 ; L_3 ; L_4 ; L_5 ; L_6 and calculate the speed for this one.

The results are given in the following table:

Portion	V_2	V_2	$r_2 - r_1$	Vitesse (km/h)
L_1	103 601	44 729	$\sqrt{75^2 + 25^2}$	745
L_2	144 149	103 727	$\sqrt{50^2 + 25^2}$	723
L_3	165 105	103 675	$\sqrt{75^2 + 25^2}$	777,6
L_4	259 607	202 871	$\sqrt{75^2 + 25^2}$	745
L_5	337 733	278 762	$\sqrt{75^2 + 25^2}$	745,93
L_6	394 609	337627	$\sqrt{75^2 + 25^2}$	721,3

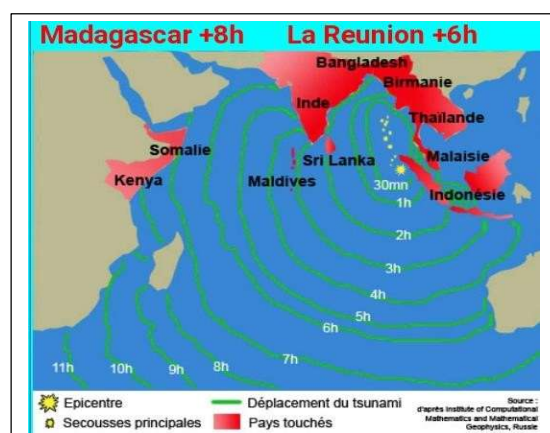
For each portion, we find an average speed value of around 740 km/h

A result that is justified by the propagation time between the epicenter and the East 90 ridge.

4.2.- The phenomena that occur between the epicenter and the East 90 ridge

The East 90 ridge is an obstacle to propagation. The ridge reflects the majority of the wave energy at its level.

The inclination of the study area corresponds to the gap on the ridge. From there, the wave is then diffracted to go up towards the Maldives islands. And it is these refracted waves that arrive in the Maldives three hours after the tsunami is triggered.



Source : Institute of Computational Mathematics and MAThematical Geophysics, Russia

Without the East 90 ridge, the wave propagation speed will be the same in all horizontal directions. We can then have an explanation why the curves of the same duration are not circles between the epicenter and the East 90 ridge.

4.3.- Did the tsunami hit Madagascar ?

Yes, the tsunami did hit Madagascar 8 hours after it started and alerts were given to the population of the east coast of Madagascar.

However, when it reached Madagascar, the tsunami energy was greatly attenuated and no human losses were recorded.

In a way, the Mascarene Plateau and the East 90 ridge were in a way like a protection for the countries to the west of the East 90 ridge and even more so for the countries to the west of the Mascarene Plateau

5.- CONCLUSION

Since 2004, Madagascar has been considered a tsunami-protected area.

This is why, during a survey with the BNGRC, the tsunami risk is considered moderate. The result of the internet search gives the same observation.

But since 2018, the start of the seismo-volcanic crisis in Mayotte following the formation of underwater volcanic chains, the context is no longer the same in the Somali basin and in the Comoros basin because of the East African rift phenomenon and global warming.[10], [12]

Scientific research on the tsunami risk in Mayotte has persuaded the French government, which decided in 2023 to implement measures to prevent the tsunami risk that has become increasingly likely. [11]

In the event that a tsunami occurs in the Mayotte Sea, the countries of the same Somali basin, including Madagascar, in particular the northwest coasts, will not be an exception to disasters.

At the international level, the Intergovernmental Oceanographic Commission (IOC) of UNESCO has established a Tsunami Programme in which the Coordination Unit assists IOC member states in assessing tsunami risk and implementing tsunami early warning (TEW) systems.

Under the aegis of IOC/UNESCO, regional tsunami monitoring and information centres (Tsunami Service Providers, TSPs) are now operational worldwide and are able to disseminate an alert to national authorities.

At the national level, coastal states of an entire ocean basin are encouraged to take appropriate safeguards and prepare their populations. (IOC/UNESCO, 2008 and 2013).

The objectives of evacuation plans are:

- Identify and list refuge sites (high altitude locations and places where people could take refuge).
- Secure coastal populations in a minimum of time by guiding them to these refuge sites.

- Disseminate in order to support acculturation to tsunami risk.
- Encourage authorities to organize evacuation simulation exercises (IOC/UNESCO, 2013 and 2023) [11]

As this work has already been started by the Malagasy BNGRC, the GFDRR (Global Facility for Disaster Reduction and Recovery) in the context of earthquakes, floods and tropical cyclones, we can make another effort to extend these measures for tsunami hazards.

REFERENCES :

- [1] - 0Ferdinand Cap, Innsbuck, Austria- Tsunami and Hurricanes- A Mathématique Approach- p 101 ; Springer Wien New York. 2006.
- [2] - NICOLAS JARRY, Université du Sud Toulon Var, - Etudes Expérimentales et Numériques de la Propagation des vagues au-dessus de bathymétries complexes en milieu côtier ; pp 27- 29 ; 2009
- [3] - <https://books.openedition.org/irdedition>. « Le rift Est Africain- Chapitre 1. Géophysique du rift.
- [4] - <https://www.ncei.noaa.gov>access>. –what is the climate change report for 2024. Novembre 2024
- [5] - RISSIER LAURENT- Différences finies pour la résolution numérique des équations de la mécanique des fluides pp 174-182,, 4 février 2006.
- [6] - BRANLARD EMMANUEL, <http://emmanuel.branlard.free.fr>- Rapport TIPE, juin 2005.
- [7] - H. BENHISSEN et A. KHECHEKHINCHÉ, Université de Québec à Trois Rivières. Modélisation et Résolution numérique de l'équation de Poisson à 2D par la méthode de différence finies, cas du transfert de chaleur, pp 14-17 ; 72012.
- [8] - BRIAN STOUT, brian.stout@fresnel.fr Université de Provence, Institut Fresnel, Case 161- Faculté de St Jérôme Marseille- France- Méthodes numériques de résolution d'équations différentielles, Février 2007.
- [9] - KEVIN PONS, Université de Toulon- Modélisation des Tsunami- Propagation et Impact, 14 Décembre 2018.
- [10] - MAUD H. DEVES et al. Université de Paris et Université de Sorbonne. La « crise » sismo-volcanique de Mayotte dans les récits d'actualité de la presse quotidienne. Compte rendus GEOSCIENCE pp 4-5 ; (2022).
- [11] - FREDERIC Léone et al. Hal- 04316142, Mayotte se prépare au risque tsunami : modélisation, alerte, évacuation, sensibilisation. Echogéo. 25078, 2023
- [12] - ISABELLE THINON. « Volcanisme et Tectonique découverts le long de l'Archipel des Comores entre l'Afrique et Madagascar » COMPTES RENDUS Géosciences- Sciences de la Terre- Décembre 2023.