

Analysis of the Material on Direct and Inverse Proportions in Mathematics Teaching Materials for Seventh Grade Based on Praxeology

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Abstract – This research aims to analyze school mathematics teaching materials from a practical perspective, where the researcher seeks to understand how students engage with the content presented in the teaching module. The concept of praxeology is a key construct of the Anthropological Theory of Didactics (ATD). Teaching materials serve as empirical sources that can reveal the knowledge to be taught during didactic transposition. Through document study (in phenomenological design qualitative research) of the school mathematics teaching materials based on the Merdeka Curriculum, the results obtained are: (1) The task component in the material of direct and inverse proportions uses operational techniques and includes standalone tasks for students to complete; (2) The technique component is appropriate because the information presented in each session has continuity; (3) The technology component is not yet fully aligned, thus requiring improvements and enhancements in the development of information and communication technology-based learning media; and (4) The theory component aligns with constructivist theory where students can build knowledge based on their individual experiences.

Keywords – Mathematics, Praxeology Review, Textbook, Comparison Of Direct And Inverse Proportion.

I. INTRODUCTION

One of the mathematical concepts taught to seventh-grade junior high school students is direct proportion and inverse proportion. The concept of comparison can be used in everyday life, such as determining the size and design of a house, assessing how delicious coffee and sugar are, deciding the number of employees to be hired over a certain period, and so on. According to Shield and Dole (2002), many subjects in secondary school are related to the concept of comparison. Understanding the concept of comparison can help solve many problems related to that concept. This indicates that students are beginning to use comparison when solving math problems in high school. The idea of indirect comparison arises in plane geometry, particularly related to similarity. The trigonometric sine rule also implicitly contains the concept of ratios in high school. Students still need help to differentiate between direct and inverse proportions, even though the concept of proportion often appears in everyday life. (Dewi et al., 2022). This is also in line with the research conducted by Binaria Lawana (2022) based on the analysis of students' answers regarding the comparison material, which still needs to be considered low.

Many factors influence the low ability of students' math achievement on comparison material. This low achievement indicates that students have difficulty in learning. Factors that cause students' difficulties in understanding mathematics can come from within and outside the student. Internal factors include students' need for more interest in learning math, poor study habits, and difficulties they have faced in previous schools. In contrast, the things that come from external sources include teachers' teaching methods that are too fast, material that is considered too dense, and inappropriate textbooks (Nursyifa et al., 2020; Ramda, 2017; Setiawan, 2019). However, textbooks are crucial as a supporting tool in the learning process, making lessons easier for students to understand. Textbooks are systematically organized by considering the explanation and material of the field of study. At the same

time, textbooks have a vital role as a supporting tool in learning material so that the students can more easily understand and master the lesson. The use of textbooks involves a selection process that considers learning objectives, learning orientation, and student development, to assist students in understanding learning materials (Ramda, 2017; Setiawan, 2019). Textbooks have an essential role as a strategic medium to shape students' reasoning, attitudes, and interests as well as thinking, imagining, expressing, and feeling happy in participating in the teaching and learning process (Dewi, 2022; Halitopo, 2020). Thus, the textbooks used should contain good qualifications. A good mathematics textbook will also positively impact students' mathematics learning achievement. This textbook should be analyzed first before being used in the classroom. If there are discrepancies or inaccuracies in the book, follow-up steps can be taken to address them earlier. The analysis of the book is carried out based on four criteria, namely criteria related to competence, material, approach, and assessment in the 2013 curriculum student book (Tusyana & Luciana, 2019; Yenni, 2016).

In this study, the researcher uses the concept of praxeology to analyze teaching materials. The textbook being analyzed is the teaching material teachers use as a reference in implementing learning. Praxeology is a key component of Didactic Anthropology Theory (Chevallard, 2006) and consists of two blocks: praxis (the practical block) and logos (the theoretical block) (Khasanah et al., 2021; Z. H. Putra et al., 2020). The praxis block consists of task (T) and technique (τ). The logos block has two components: technology (θ) and theory (Θ). T represents the type of task being completed, τ is the technique used to complete that task, θ is the technology employed, which can also be knowledge for analyzing techniques or considering discourse, and Θ is the theory used to justify or test the technology. (Putra, 2019).

Based on the background description above, this research implements the interpretive paradigm in didactical design research (DDR) aimed at analyzing the presentation of material on direct and inverse proportions in teaching materials that focus on the praxis block and the logos block. From this analysis, it seeks to identify the learning obstacles faced by students. This research is also expected to yield findings that will contribute to the development of future teaching materials.

II. METHODOLOGY

2.1 Research Design

This research aims to answer the research question about how praxeology theory helps students learn. Qualitative research was chosen as an alternative method. Data is analyzed inductively from specific to general themes; data is usually collected in participant settings, and the researcher interprets its meaning. (Creswell, 2014). The design used in this research is phenomenology. Alase (2017) explains that phenomenology is a qualitative research approach that allows researchers to utilize and apply their experiences and their relationship with subjectivity during the research process. Therefore, the phenomenon observed in this research is the phenomenon underlying the process of designing learning designs, particularly concerning the reflection and assessment of the design of learning materials in direct and inverse proportions.

2.2 Instrument

Qualitative research has a flexible and multiparadigmatic nature, where researchers operate from various worldviews. (seperti post-positivisme, interpretivisme, dan orientasi kritis). The diversity of paradigms makes qualitative research a broad field of inquiry, allowing researchers to engage in various research projects. (The Oxford Handbook of Qualitative Research, diedit oleh Patricia Leavy: 2014). Therefore, the researcher acts as the primary instrument that fully controls the research process. In addition, the researchers also utilized supplementary instruments such as documents to support the study. The document used is a teaching material for teachers for Grade VII of the Merdeka Curriculum in junior high school/ Mts. The material analyzed is the proportion, which is the third topic in the teaching module for the odd semester, consisting of three sub-chapters of discussion, including functions, direct proportion, and inverse proportion. In this study, these sections are limited to analyzing the presentation of the sub-chapters on direct and inverse proportion.

2.3 Data Collection Technique

In accordance with the research objectives, data were collected through document analysis. This is an investigative process that focuses on written materials, records, or documents, and is commonly used in educational research when textbooks or curricula are the main source of data being analyzed. The documents in this study included mathematics teaching materials for class VII SMP/ MTs Merdeka Curriculum and the design tasks in these documents are analyzed accompanied by relevant conceptual frameworks.

2.4 Data Analysis Technique

Data analysis in qualitative research was conducted concurrently with other stages in the development of qualitative research, such as data collection and writing up research results (Creswell, 2015). The data analysis procedure in this study consisted of three main stages. The first stage involved selecting any task design unit in the selected teaching module and grouping related topics into a praxeology table. In the second stage, the researchers coded the test design independently. The third stage involved validating the reliability of the coding by an expert in measurement and evaluation. At this stage, the researchers reconverged and identified inconsistencies in the taxonomy table. This process then continued in a cyclical manner, where task designs that were inconsistent in coding by the researchers would be discussed until a mutual agreement was reached.

III. RESULTS AND DISCUSSION

There are two parts of analysis presented, namely the praxis block and the logos block, which are elements in the application of praxeology theory. Before describing the difference between the praxis block and the logos block, the researcher first outlines the introduction of the teaching module. In this teaching material, the Problem-Based Learning learning method was applied. The learning process was done for five lesson hours with a total time allocation of 200 minutes, and required prerequisite skills in understanding proportion in the domain or topic of direct and inverse proportions and its application in daily life. At the initial stage, students were given information related to the competencies to be achieved, material coverage, learning objectives, benefits, stages in learning, and assessment methods used. Visualization of the introduction to the teaching materials is expected to make students comprehend many terms for 'proportion' in their daily life. Generally, the teaching module does not mention that before learning about direct and inverse proportions, students were reminded of the definition of proportion and how to write it. This can trigger didactical obstacles in the use of mathematical terms that are not yet appropriate.

Based on the analysis of the journal from Shinno and Takeuchi (2018), the following guidelines were used as a reference in analyzing the presentation of materials and tasks:

Table 1. Definition of Praxiology Elements

<i>The Notion</i>	<i>Meaning</i>	<i>Indicator</i>
<i>Type of Task (T)</i>	<i>A type of task T that is a collection of tasks which can be solved by some technique τ</i>	<i>Point praxeology (containing a single type of task)</i>
		<i>Local praxeology (containing a set of types of tasks organised around a common technological discourse)</i>
		<i>Regional praxeology (containing all point and local praxeologies that share a common theory)</i>
<i>Technique (τ)</i>	<i>Technique is refers to a way of</i>	<i>Perceptual technique (a perceptua; technique mainly relies on visual judgement based on the appearance of given shapes)</i>

	<i>performing this of type of task</i>	<i>Physical technique (a physical technique is performed using physical tools for drawing and measuring, such as a mirror, ruler, or compass;)</i>
		<i>Operational technique (an operational technique is performed with shapes on a grid sheet (squared paper) for drawing by using coordinates)</i>
		<i>Algebraic technique (technique is based on expressions by column vectors)</i>
<i>Technology (θ)</i>	<i>Technology is about a way of explaining and justifying (or designing) the technique</i>	
<i>Theory (Θ)</i>	<i>To explain, justify, or generate whatever part of the technology that may sound unclear or missing</i>	

Praxis Block Analysis

The tasks in the praxis block were classified into task types in the teaching materials. The teaching materials have different T numbers, starting from T_1 , T_2 , ..., to T_n . The concept of direct and inverse proportions in the praxis block in the teaching materials is presented through six types of tasks (T_1 , T_2 , ..., T_5). The five types of tasks in the Merdeka Curriculum teaching materials were generally divided into three categories, namely understanding the definition of valued proportion using a proportion table (T_1), in-depth understanding of arithmetic operations on direct and inverse proportions (T_2 , T_3), and applying arithmetic operations on direct and inverse proportions in solving everyday problems (T_4 , T_5). [Table 2]

Techniques (τ) in the praxis block refer to the methods used to complete a particular type of task (T). In order to identify the techniques for each task type, one begins by examining what solutions and approaches are required to solve the questions in each task type and then created several categories that described these solutions and approaches. In this study, the types of techniques used were based on the findings of Takeuchi and Shinno (2020), which consist of four types of techniques, namely perceptual (τ_1), physical (τ_2), operational (τ_3), and algebraic (τ_4). The τ_1 technique involved completing a task by using visual judgment based on the appearance of the form presented. Then, the τ_2 technique involved solving the task by using physical aids, such as a ruler, compass, or other tools. The τ_3 technique involved completing the task through investigation or discovery by students, aiming to develop their understanding. While the τ_4 technique involved solving the task by using mathematical expressions.

Table 2: Praxis block of teacher teaching materials

Task Type (T)	Engineering (τ)	Description of each technique
T_1 = Mr. Hendra is paid IDR 800.000.00 for 5 hours to provide training at a course. The time Mr. Hendra uses for training if he gets a salary of IDR 10.800.000,00 is ...	τ_3 , τ_4	Performing mathematical activities and expressing mathematical ideas, including identifying comparisons, aims to infer the time spent through the process of perception and pre-existing (a priori) memories.

T_2 = With IDR 15.000, Fatimah can buy 6 notebooks. If she wants to buy 4 more notebooks, then the amount of additional money needed is...

τ_3, τ_4

Performing mathematical activities and expressing mathematical ideas, including identifying types of proportions, aims to deduce the additional money required through the process of perception and a priori memory.

T_3 = A project if done by 16 workers will be completed in 15 days. If the project is only done by 10 people, then the project will be completed in ...

τ_3, τ_4

Performing mathematical activities and expressing mathematical ideas, including identifying types of proportions, aims to infer the time required through the process of perception and pre-existing (a priori) memories.

T_4 = A chicken slaughtering company posts a table that people are not used to making, that is, the data is incomplete. This is to attract customers and to trigger communication with customers.

τ_2, τ_3, τ_4

Perform actions with aids to infer & find conclusions from arithmetic operations on proportion tables based on previous perceptual processes (memorial).

Many chickens	1	2	5	10	15	25	50
Cost (IDR)	4k	8k	20k	...	60k

Mrs. Sinta cut up 25 chickens for her food stall. Try to help Mrs. Sinta, how much should she pay...

T_5 = A project was undertaken by 18 workers and planned to be completed in 35 days. After 11 days of work, the project stopped for eight days. In order for the project to be completed on time the required workers are ... people.

τ_3, τ_4

Performing mathematical activities and expressing mathematical ideas, including identifying types of proportions, aims to infer the time required through the process of perception and pre-existing (a priori) memories.

Overall, in the praxis block of Merdeka Curriculum teaching materials, tasks from T_1 to T_5 are dominated by the use of τ_3 and τ_4 techniques. This indicates that the design of the tasks in the teaching materials introduces the concepts of value comparison and inverse value by involving a lot of verification, that is, observation and development of students' previously acquired knowledge, to form new knowledge. These tasks were designed by emphasizing observation and verification of knowledge through contextual examples. However, due to the over-focus on the observation aspect, the opportunity to develop knowledge through the observation process was limited, ultimately reducing student learning opportunities. Hidayah and Forgasz (2020) mentioned that textbooks often give students tasks without clear guidance or instructions to complete them. In addition, although the subject matter and tasks in textbooks are quickly explained, new ways to explore the material are largely unavailable.

T_1, T_2, T_3 , and T_5 in the teaching materials have the equation τ , namely τ_3 to formulate the theory (definition of direct and inverse proportions), which was then refined by using T_4 , namely τ_2 . The expected formulation is θ_1 , meaning 'well-defined'.

However, the design of tasks T_1 , T_2 , T_3 , and T_5 only focuses on understanding the correct results of arithmetic operations. On the other hand, not all students can achieve the expected results through the understanding formed by T_1 , T_2 , T_3 , and T_5 . Given the different levels of intelligence (Guez *et al.*, 2018), some students might not be able to deduce the correct result. This suggests that the tasks T_1 to T_5 did not give space for students to use and develop their skills of perception, memory, and introspection as they form new knowledge in the form of θ_1 . Thus, overall, the techniques applied have not created a structured learning path. The justification characteristics τ_3 in T_1 , T_2 , T_3 , and T_5 did not consider the diversity of students' knowledge, learning experiences, thinking styles, and learning potential. In addition, there is no valid verification of the new knowledge gained by students as a strong basis to support their findings.

The expectation of T_4 and T_5 in the teaching materials is the ability of students to identify a comparison so that students can independently recognize the type of comparison contained in the story, and understand the properties of equal and inverse value comparisons in the story. This aims to present for the students in the form of a mathematical model that is built based on previous experience and knowledge. τ_3 was involved in T_4 in the form of questions based on previously acquired knowledge then continued with τ_3 in T_5 by determining the correct result through students' ability to understand the correlation between the terms given with mathematical operations and being able to model the story problem properly and correctly. The design of tasks τ_4 and τ_5 is relevant to the expected formulation, where τ_3 was used appropriately and interrelatedly to build students' understanding of the theory.

Logos Block Analysis

The main components in the logos block include technology (θ) and theory (Θ). Technology (θ) serves as a tool or method used to validate a (τ), while theory (Θ) acts as a conclusion in the form of theoretical knowledge that aims to generalize the entire process of T , θ , and Θ .

Table 3. Logos Blocks in the Teaching Module

Theme	T number	Engineering (τ)	Technology (θ)	Theory (Θ)
Recognize types of proportion	T_1	τ_3 = operational τ_4 = algebra	θ_1 = Justify the relationship between time and salary by constructing an equation and justify the relationship between extra money and the number of notebooks.	Θ_1 = Applying the concept of direct proportion. This theory states that two quantities are directly proportional if the ratio between them remains constant.
	T_2	τ_3 = operational τ_4 = algebra		
Recognize types of proportion	T_3	τ_3 = operational	θ_2 = Justify the relationship between time and number of people by constructing an equation.	Θ_2 = Apply the concept of inverse proportion. This theory states that two quantities are inversely related if an increase in one of the quantities causes a decrease in the other by the same proportion.
	T_5	τ_4 = algebra		

Properties of proportion	T_4	τ_2 = physical τ_3 = operational	θ_3 = Use a direct proportion chart to make understanding value comparison easier.	Θ_3 = Some commonly encountered properties of proportionality are: the proportion is fixed, the graph is a straight line through the origin, and a change in one quantity will cause a change in the other quantity in the same direction.
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The first two types of T (T_1 and T_2) in the teaching material serve to improve the understanding of the concept of the type of proportion worth governed by θ_1 (justification of the relationship between time and salary by constructing the equation and justification of the relationship between extra money and the number of notebooks). θ_1 justifies τ_3 because the completion of T was based on mental actions to connect perceptual processes and develop acquired knowledge. T_4 is a form of validation to infer T_1 , T_2 , T_3 , and T_5 through τ_2 justified by θ_1 and θ_2 . Overall, the task types (T_1 , T_2 , T_3 , and T_5) in the teaching module produced θ_1 and θ_2 (proportions describe how one quantity relates or compares to another, either in the form of a ratio of direct or inverse ratio). One last type of T is T_4 , but the θ obtained is different. T_4 produced θ_3 (the proportion arithmetic operation involves a like-for-like proportion graph).

The approach used in the teaching materials to solve the tasks relies more on operational and algebraic techniques integrated with contextual problems. This approach aims to create a connection between the material being taught and students' daily lives, which can lead to a sense of immediacy (Hong & Choi, 2018). However, textbooks often only provide a simple context, which, although useful as a foundation, tends to limit students' cognitive development to higher levels (Sianturi *et al.*, 2021). This can lead to epistemological barriers when students face mathematical problems requiring more complex cognitive abilities (Fuadiah *et al.*, 2019). In the development of the concepts of Θ_1 and Θ_2 , teaching materials tend to rely on perceptual abilities by repeating pre-existing displays, thus not encouraging the construction of new knowledge. In the framework of didactic situation theory, the learning process should begin with situations that provide opportunities for students to apply their experiences and knowledge so that students can develop their perceptions of the environment and their actions within it, while facilitating the processing of information to form new understandings.

The limited learning opportunities provided by teaching materials make it difficult for students to complete tasks that are more complex or different from the examples provided. This is in accordance with Tumay's (2016) statement that learning difficulties are often caused by inappropriate delivery of material, which can result in misconceptions in knowledge acquisition. Therefore, the development of more varied and challenging tasks is highly recommended to enable students to use their various cognitive skills and build knowledge more comprehensively.

IV. CONCLUSIONS

The teaching materials on ratio and inverse value comparisons, designed by the teacher, include a task component (T) that heavily incorporates operational and algebraic techniques and uses everyday terminology closely related to these operations. The technique component (τ) is appropriately structured, ensuring continuity of information across sessions. However, there is a need to integrate approaches that help students recall prerequisite material effectively. The technology component (θ) in the learning media does not yet fully align with the praxiology criteria, indicating the necessity for improvements, particularly through integrating information and communication technology. On the other hand, the theory component (Θ) aligns well with constructivist principles, where students construct knowledge based on their own experiences.

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