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# Using The Simplex Method For Optimizing Some Economic Functions Of The Enterprise "MSA Kompani" Dooel-Skopje 

Bukurie Imeri-Jusufi ${ }^{1}$, Bujamin Bela ${ }^{2}$, Teuta Jusufi-Zenku ${ }^{3}$, Azir Jusufi ${ }^{4}$<br>${ }^{1}$ University" Mother Teresa"-Skopje, bukurie.imeri@unt.edu.mk;<br>${ }^{2}$ University" Mother Teresa"-Skopje, bujamin.bela @unt.edu.mk;<br>${ }^{3}$ University" Mother Teresa"-Skopje, teuta.zenku@unt.edu.mk;<br>${ }^{4}$ University of Tetova. Tetovo, azir.jusufi@unite.edu.mk

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#### Abstract

In science, in economics, in practice, problems are often encountered in which it is required to find the largest (smallest) value of a numerical function with real values of the type $f: E \rightarrow \mathbb{R}$. More precisely, the elements of the set $E$ are required, where the function f reaches the optimal value. For this reason, such problems are called optimization problems. Optimization problems are also solved by so-called linear programming methods. Linear programming is a mathematical programming that deals with the problem of system optimization within given constraints, it is a method for solving production planning problems. The manufacturer wants to determine how to use the limited quantities of raw materials with maximum profit, the manager how to distribute the assigned work among his employees so that it is done in the shortest possible time, that is, to be as effective as possible. The goal of these problems is optimization, profit maximization or cost minimization [1]. Making profit is the main goal of the company, but many companies still need to learn the maximum profit that can be achieved by optimizing their resources, one of which is the company "MSA KOMPANI". This research focuses on the products of "MSA KOMPANI" DOOEL-SKOPJE different according to the price. The purpose of this paper is to optimize the company's profits and costs. In this paper we will use the simplex linear programming method to solve our problem in "MSA KOMPANI". Modeling a real-life problem as a linear programming problem requires teamwork of experts from several fields.


Keywords - Optimation, Profit, Income, Enterprise, Simplex Method, Linear Programming.

## I. Introduction

The focus of this research is on the products of the company "MSA KOMPANI" DOOEL-SKOPJE. This study aims to optimize profits, revenues, by determining the composition of the number of products produced. There are many ways to solve this problem, one of which is using linear programming, specifically the simplex method.

A linear program is a mathematical model for determining the optimal combination of products to maximize benefits or minimize costs. A linear program has three essentials: the objective function, the decision variable, and the constraints.

In the beginning, we build the mathematical model from the data of the real problem and then from these data we form the first simplex table. There is often more than one way to describe a particular algorithm. The simplex method is one of the techniques for determining the optimal solution used in linear programming. The simplex method is a general method, which means that it solves any linear programming problem.

The simplex method is an iterative method with a finite number of steps starting from an initial solution and in the last step of the iterations, the optimal solution is obtained. This is the reason why the simplex method is easily applied to the computer as well.

## II. Formulation of the linear programming problem. Mathematical model

In science, in economics, in practice, problems are often encountered in which it is required to find the largest (smallest) value of a numerical function with real values of the type $f: E \rightarrow \mathbb{R}$. More precisely, the elements of the set $E$ are required, where the function $f$ reaches the optimal value. For this reason, such problems are called optimization problems. Such a function is called a goal function. In general it is a function of $n$ variables $x_{1}, x_{2}, \ldots x_{n}$, t.e. its starting set is a subset of Euclidean space $\mathbb{R}^{n}$ of vectors $x=\left(x_{1}\right.$, $x_{2}, \ldots x_{n}$ ) with $n$ dimensions, t.e. $E \subset \mathbb{R}^{n}$. Wide practical use represents the case when the objective function $f$ is a linear function of the variables $x_{1}, x_{2}, \ldots, x_{n}$ :

$$
\begin{equation*}
f(x)=c_{1} x_{1}+c_{2} x_{2}+\ldots+c_{n} x_{n} \tag{1}
\end{equation*}
$$

for $x=\left(x_{1}, x_{2}, \ldots, x_{n}\right) \in E$. The starting set $E$ is determined by imposing various conditions on the variables $x_{j}$ for $j=1,2, \ldots, n$. In particular, $E$ is the set of solutions of the following $m$ system of linear equations or inequations of variables $x_{j}$ :

$$
\left\{\begin{array}{l}
\alpha_{11} x_{1}+\alpha_{12} x_{2}+\ldots+\alpha_{1 n} x_{n}\{\leq,=, \geq\} \beta_{1}  \tag{2}\\
\alpha_{21} x_{1}+\alpha_{22} x_{2}+\ldots+\alpha_{2 n} x_{n}\{\leq,=, \geq\} \beta_{2} \\
\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
\alpha_{m 1} x_{1}+\alpha_{m 2} x_{2}+\ldots+\alpha_{m n} x_{n}\{\leq,=, \geq\} \beta_{m}
\end{array}\right.
$$

Usually variables $x_{j}$ in practice they take non-negative values, therefore the condition is added to the system (2) of conditions:
$x_{1} \geq 0, x_{2} \geq 0, \ldots, x_{n} \geq 0$.
Any solution of system (2) that satisfies condition (3) is called an allowed solution of system (2), while the its allowed solution $\left(x_{1}^{0}, x_{2}^{0}, \cdots x_{n}^{0}\right)$, where the function $f$ reaches the largest or smallest value is called the optimal solution of the problem (1), (2), (3). The set of allowed solutions of system (2) is denoted $\Omega$ and is called the allowed set (area) of problem (1), (2), (3).

In this case the problem of optimizing the function, using the symbol $\Sigma$ of an adder, takes the form of the following mathematical model:

- To find the largest (or smallest) value of a function
$f(x)=\sum_{j=1}^{n} c_{j} x_{j}$, where $x=\left(x_{1}, x_{2}, \ldots, x_{n}\right)$ and variables $x_{1}, x_{2}, \ldots, x_{n}$ meet the conditions
$\sum_{j=1}^{n} a_{i j} x_{j}(\leq,=, \geq) \beta_{i}, \quad i=1,2, \ldots, m$ and $x_{j} \geq 0, j=1,2, \ldots, n$.
The theory that studies problems of the type (1), (2), (3) is called Linear Programming, while the problem presented according to the model (1), (2), (3) is called a linear programming problem (LPP for short) in general form, in particular a minimization problem when the smallest value of the function $f$ is required and a maximization problem when its largest value is required.


## III. The simplex method

Definition 3.1. [2]. The standard form of LPP is called the form:
Minimize the function
$f(x)=c_{1} x_{1}+c_{2} x_{2}+\ldots+c_{n} x_{n}$, for $x=\left(x_{1}, x_{2}, \ldots, x_{n}\right) \in \mathbb{R}^{n}$
in the conditions when $\left\{\begin{array}{l}a_{11} x_{1}+a_{12} x_{2}+\ldots+a_{1 n} x_{n}=b_{1}, \\ a_{21} x_{1}+a_{22} x_{2}+\ldots+a_{2 n} x_{n}=b_{2}, \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \\ a_{m 1} x_{1}+a_{m 2} x_{2}+\ldots+a_{m n} x_{n}=b_{m} .\end{array}\right.$
and

$$
\begin{equation*}
x_{j} \geq 0, j=1,2, \ldots, n . \tag{6}
\end{equation*}
$$

We consider the system of conditions (5) of a LPP in standard form. It is known that it has no solution, if the rank of the matrix $A$ its is different from the rank of the expanded matrix $A^{*}$; there is only one solution, if $\operatorname{rg}(A)=\operatorname{rg}\left(A^{*}\right)=n$; and there is an infinity of solutions with $\quad n-r$ degrees of freedom, if $r g(A)=r g\left(A^{*}\right.$ $)=r<n$. Of interest for the LPP solution is the third case. In this case the system contains $r$ basic variables and $n-r$ free variables. To find them, with the Gauss-Jordan method, we bring the system to the system below, equivalent to it. From here we find that the basic variables are $x_{1}, x_{2}, \ldots, x_{r}$ and free the other variables $x_{r+1}, x_{r+2}, \ldots, x_{n}$

If the free terms $\beta_{i}$ in system (11) are $\beta_{i} \geq 0$, then the system (7) is called the canonical form of the system of conditions (5).
We substitute the basic variables found according to (7) in the form (4) of the objective function.
Reception:
$f(x)=c_{1}\left[\beta_{1}-\left(\alpha_{1,+1} x_{r+1}+\ldots+\alpha_{1 n_{n}} x_{n}\right)\right]+\ldots+c_{r}\left[\beta_{r}-\left(\begin{array}{ll}\left.\left.\alpha_{r+1} x_{r+1}+\ldots .+\alpha_{r}{ }_{n} x_{n}\right)\right]+\quad+c_{r+1} x_{r+1}+\ldots+c_{n} x_{n}, ~\end{array}\right.\right.$ $\Rightarrow f(x)=d_{0}-\left(d_{r+1} x_{r+1}+\ldots+d_{n} x_{n}\right)$.

Definition 3.2 [2]. The canonical form of LPP is called the form:
Minimize the function $f(x)=d_{0}-\left(d_{r+1} x_{r+1}+\ldots+d_{n} x_{n}\right)$

and $\quad x_{j} \geq 0, j=1,2, \ldots, n$.

Taking $x_{r+1}=x_{r+2}=\ldots=x_{n}=0$ in system (9) we find its solution $\left(\beta_{1}, \beta_{2}, \ldots, \beta_{r}, 0, \ldots, 0\right)$, which is an allowed solution of system (4) of the standard LPP (since $x_{j} \geq 0, j=1,2, \ldots, n$ ). The ranked system ( $x_{1}, x_{2}, \ldots, x_{r}$ ) of the basic variables of the system (9) is called the basis of LPP, while the allowed solution ( $\beta_{1}, \beta_{2}, \ldots, \beta_{r}, 0, \ldots$, 0 ) is called its basic solution. It also seems easy that, $d_{0}$ is the value of the objective function $f$ in the basic solution of LPP in canonical form.

We emphasize that the condition that the free terms of the system (9) are $\geq 0$ is essential for the canonical form of the LPP, according to the Gauss-Jordan method, system (5), when there is a solution, always behaves in the form (7), even in many ways. Therefore, the question arises: For what conditions does the standard LPP have at least one canonical form and is it the only one?

For now we can say, convinced and from the examples, that in general the canonical form of the standard LPP is not unique. However, such LPP cannot have more than $C_{n}^{r}$ canonical form, because one of its bases is a combination of variable $r$ from $n$ that are total.

Note: We will explain the simplex algorithm in detail in the solution of our problem.

## IV. The USE OF THE SIMPLEX METHOD FOR THE OPTIMIZATION OF SOME ECONOMIC FUNCTIONS OF THE ENTERPRISE MSA KOMPANI DOOEL-SKOPJE

The company MSA KOMPANI DOOEL SKOPJE is a medium-sized manufacturing company, which deals with the production of equipment and toys for kindergartens. This company has 23 employees, of which 15 are in production. From the company's products, we have singled out 5 types of its products, which are: SCS 01 playground; Pencil fence; Swing; Soft blocks; seesaw, which we will refer to as production A;B;C;D and E respectively. The official person of the company A.S. was very kind and willing to provide us with the following information about the company:

| PRODUCE | PRODUCTI ON TIME FOR UNIT | $$ | NUMBE <br> R OF <br> WORKE <br> RS PER <br> UNIT | REQUES <br> T/ <br> DAY | $\begin{aligned} & \hline \text { OFFER } \\ & / \\ & \text { DAY } \end{aligned}$ | PROFIT/ <br> UNIT | SELLING PRICE/ <br> UNIT | $\begin{aligned} & \hline \text { COST/ } \\ & \text { UNIT } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A) $\operatorname{SCS} 01$ <br> playground | 8 hour | 4 hour | 4 | 1 | 5 | 30000 den | 80600.00 den. | 50600 den |
| B)Pencil fence | 10 minutes | 5 minutes | 3 | 30 | 10 | 800 den | 2800.00 den | 2000 den |
| C) Swing | 3 hour | 1 hour | 4 | 1 | 5 | 12000 den | 45000.den | 33000 den |
| D) Soft <br> blocks  | 24 hour | 0 | 1 | 0.5 | 2 | 35000 den | 150000 den | 115000 den |
| E) seesaw | 3 hour | 1 hour | 3 | 1 | 5 | 5000 den | 18000 den | 13000 den |

The company works in one shift. From the above data we calculated that the company has available 120 hours of work per day, of which Production and Assembly are in the ratio P:M=38,25:6,083. We solve the system $\left\{\begin{array}{c}P+M=120 \\ P: M=38,25: 6,083\end{array}\right.$ and we get that the company has 103,5 hours of daily capacity for production and 16,083 daily capacity hours for assembly.

- We compile the mathematical model for maximizing the company's daily profit according to the offer presented:
Decision variables:
$\mathrm{x}_{1}=$ Quantity to be produced of the type A
$\mathrm{x}_{2}=$ Quantity to be produced of the type B
$\mathrm{x}_{3}=$ Quantity to be produced of the type B
$\mathrm{X}_{4}=$ Quantity to be produced of the type B
$\mathrm{x}_{5}=$ Quantity to be produced of the type B
Mathematical modeling of our case
Determine the quantities $x_{1}, x_{2}, x_{3}, x_{4}$ and $x_{5}$, with restrictions:
Total units of production: $x_{1}+x_{2}+x_{3}+x_{4}+x_{5} \leq 27$
Total cost: $50600 x_{1}+2000 x_{2}+33000 x_{3}+115000 x_{4}+13000 x_{5} \leq 733000$
Total time: $12 x_{1}+0.25 x_{2}+4 x_{3}+24 x_{4}+4 x_{5} \leq 120$
$x_{1}, x_{2}, x_{3}, x_{4}, x_{5} \geq 0$
To maximize the profit function $\mathrm{F}=30000 x_{1}+800 x_{2}+12000 x_{3}+35000 x_{4}+5000 x_{5}$

So we have:
To maximize the objective function

$$
F(x)=30000 x_{1}+800 x_{2}+12000 x_{3}+35000 x_{4}+5000 x_{5}
$$

Under the conditions:

$$
\left\{\begin{array}{c}
x_{1}+x_{2}+x_{3}+x_{4}+x_{5} \leq 27 \\
50600 x_{1}+2000 x_{2}+33000 x_{3}+115000 x_{4}+13000 x_{5} \leq 733000 \\
12 x_{1}+0.25 x_{2}+4 x_{3}+24 x_{4}+4 x_{5} \leq 120
\end{array}\right.
$$

Where, $x_{1}, x_{2}, x_{3}, x_{4}, x_{5} \geq 0$.
We take the additional variables and build the system:

$$
\left\{\begin{array}{c}
x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+s_{1}=27 \\
50600 x_{1}+2000 x_{2}+33000 x_{3}+115000 x_{4}+13000 x_{5}+s_{2}=733000 \\
12 x_{1}+0.25 x_{2}+4 x_{3}+24 x_{4}+4 x_{5}+s_{3}=120 \\
-30000 x_{1}-800 x_{2}-12000 x_{3}-35000 x_{4}-5000 x_{5}+F(x)=0
\end{array}\right.
$$

We build the first simplex table:

| Base | $\mathbf{x}_{\mathbf{1}}$ | $\mathbf{x}_{\mathbf{2}}$ | $\mathbf{x}_{\mathbf{3}}$ | $\mathbf{x}_{\mathbf{4}}$ | $\mathbf{x}_{\mathbf{5}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{3}$ | $\mathbf{F}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{s}_{\mathbf{1}}$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 27 |
| $\mathbf{s}_{\mathbf{2}}$ | 50600 | 2000 | 33000 | 115000 | 13000 | 0 | 1 | 0 | 0 | 733000 |
| $\mathbf{s}_{3}$ | 12 | 0.25 | 4 | 24 | 4 | 0 | 0 | 1 | 0 | 120 |
| $\mathbf{F}$ | -30000 | -800 | -12000 | -35000 | -5000 | 0 | 0 | 0 | 1 | 0 |

## Step 1:

In the fourth row and the columns of $x$, we look for the largest negative number in absolute value, which is $\mid-$ $35000 \mid=35000$ and the $x_{4}$ column is the key column. We divide the free terms by the corresponding coefficients of the $x_{4}$ column and get:

$$
\frac{120}{24}=5 ; \frac{733000}{115000}=6.374, \frac{27}{1}=27
$$

We look for the smallest factor which is 5 and fix the row of $s_{3}$, we see that the fixed element is 24 . Since the key must always be 1 , then we divide the third row (the $s_{3}$ ) with 24 and we get:

| Base | $\mathbf{x}_{\mathbf{1}}$ | $\mathbf{x}_{\mathbf{2}}$ | $\mathbf{x}_{\mathbf{3}}$ | $\downarrow_{\mathbf{x}}$ | $\mathbf{x}_{\mathbf{5}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{\mathbf{3}}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{\mathbf { s } _ { 1 }}$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 27 |
| $\mathbf{s}_{\mathbf{2}}$ | 50600 | 2000 | 33000 | 115000 | 13000 | 0 | 1 | 0 | 733000 |
| $\leftarrow \mathbf{s}_{\mathbf{3}}$ | 0.5 | $1 / 96$ | $1 / 6$ | $[1]$ | $1 / 6$ | 0 | 0 | $1 / 24$ | 5 |
| $\mathbf{F}$ | -30000 | -800 | -12000 | -35000 | -5000 | 0 | 0 | 0 | 0 |

Get out of the base $\mathbf{s}_{3}$, enter on the base $\mathrm{x}_{4}$. We perform the transformations: $R_{1}-R_{3}, R_{2}-115000 R_{3}$ and $R_{4}+35000 R_{3}$ and we get:

| Base | $\mathbf{x}_{\mathbf{1}}$ | $\mathbf{x}_{\mathbf{2}}$ | $\mathbf{x}_{\mathbf{3}}$ | $\mathbf{x}_{\mathbf{4}}$ | $\mathbf{x}_{\mathbf{5}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{\mathbf{3}}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{s}_{\mathbf{1}}$ | 0.5 | 0.99 | 0.83 | 0 | 0.83 | 1 | 0 | -0.04 | 22 |
| $\mathbf{s}_{\mathbf{2}}$ | -6900 | 802.08 | 13833.33 | 0 | -6166.67 | 0 | 1 | -4791.67 | 158000 |
| $\mathrm{X}_{4}$ | 0.5 | 0.01 | 0.17 | 1 | 0.17 | 0 | 0 | 0.04 | 5 |
| $\mathbf{F}$ | -12500 | -435.42 | -6166.67 | 0 | 833.33 | 0 | 0 | 1458.33 | 175000 |

Thus the first step of the simplex algorithm is completed.
Step 2: We act the same as in the step 1 and fix the element 0.5 . To make it 1, we multiply the third row ( the $\mathrm{x}_{4}$ ) with 2 .

| Base | $\downarrow_{\mathbf{x}}$ | $\mathbf{x}_{\mathbf{2}}$ | $\mathbf{x}_{\mathbf{3}}$ | $\mathbf{x}_{\mathbf{4}}$ | $\mathbf{x} \mathbf{5}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{\mathbf{3}}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{s}_{\mathbf{1}}$ | 0.5 | 0.99 | 0.83 | 0 | 0.83 | 1 | 0 | -0.04 | 22 |
| $\mathbf{s}_{\mathbf{2}}$ | -6900 | 802.08 | 13833.33 | 0 | -6166.67 | 0 | 1 | -4791.67 | 158000 |
| $\leftarrow \mathbf{x}_{4}$ | $[1]$ | 0.02 | 0.34 | 2 | 0.34 | 0 | 0 | 0.08 | 10 |
| $\mathbf{F}$ | -12500 | -435.42 | -6166.67 | 0 | 833.33 | 0 | 0 | 1458.33 | 175000 |

Get out of the base $\mathrm{x}_{4}$, enter on the base $\mathrm{x}_{1}$. We perform the transformations: $R_{1}-1 / 2 R_{3}, R_{2}+6900 R_{3}$ and $R_{4}-12500 R_{3}$ and we get:

| Base | $\mathbf{x}_{\mathbf{1}}$ | $\mathbf{x}_{\mathbf{2}}$ | $\mathbf{x}_{\mathbf{3}}$ | $\mathbf{x}_{\mathbf{4}}$ | $\mathbf{x}_{\mathbf{5}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{\mathbf{3}}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{\mathbf { s } _ { 1 }}$ | 0 | 0.98 | 0.67 | -1 | 0.67 | 1 | 0 | -0.08 | 17 |
| $\mathbf{s}_{\mathbf{2}}$ | 0 | 945.83 | 16133.33 | 13800 | -3866.67 | 0 | 1 | -4216.67 | 227000 |
| $\mathrm{X}_{1}$ | 1 | 0.02 | 0.33 | 2 | 0.33 | 0 | 0 | 0.08 | 10 |
| $\mathbf{F}$ | 0 | -175 | -2000 | 25000 | 5000 | 0 | 0 | 2500 | 300000 |

Thus the second step of the simplex algorithm is completed.
Step 3. We act the same as in the step 1 and fix the element 16133.33. To make the key 1, we divide the second row (the $\mathrm{s}_{2}$ ) with 16133.33 and we get:

| Base | $\mathbf{x}_{\mathbf{1}}$ | $\mathbf{x}_{\mathbf{2}}$ | $\downarrow_{\mathbf{x}}$ | $\mathbf{x}_{\mathbf{4}}$ | $\mathbf{x}_{\mathbf{5}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{\mathbf{3}}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{s}_{1}$ | 0 | 0.98 | 0.67 | -1 | 0.67 | 1 | 0 | -0.08 | 17 |
| $\leftarrow \mathbf{s}_{\mathbf{2}}$ | 0 | 0.06 | $[1]$ | 0.86 | -0.24 | 0 | 0 | -0.26 | 14.07 |
| $\mathrm{x}_{1}$ | 1 | 0.02 | 0.33 | 2 | 0.33 | 0 | 0 | 0.08 | 10 |
| $\mathbf{F}$ | 0 | -175 | -2000 | 25000 | 5000 | 0 | 0 | 2500 | 300000 |

Get out of the base $\mathrm{s}_{2}$, enter on the base $\mathrm{x}_{3}$. We perform the transformations: $R_{1}-0.67 R_{2}, R_{3}-0.33 R_{2}$ and $R_{4}+2000 R_{2}$ and we get:

| Base | $\mathbf{x}_{\mathbf{1}}$ | $\mathbf{x}_{\mathbf{2}}$ | $\mathbf{x}_{\mathbf{3}}$ | $\mathbf{x}_{\mathbf{4}}$ | $\mathbf{x}_{\mathbf{5}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{\mathbf{3}}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{s}_{\mathbf{1}}$ | 0 | $\mathbf{0 . 9 4}$ | 0 | -1.57 | 0.83 | 1 | 0 | 0.09 | 7.62 |
| $\mathrm{x}_{3}$ | 0 | 0.06 | 1 | 0.86 | -0.24 | 0 | 0 | -0.26 | 14.07 |
| $\mathrm{x}_{1}$ | 1 | 0 | 0 | 1.71 | 0.41 | 0 | 0 | 0.17 | 5.31 |
| $\mathbf{F}$ | 0 | -57.75 | 0 | 26710.74 | 4520.66 | 0 | 0.1 <br> 2 | 1977.27 | 328140.5 |

Thus the third step of the simplex algorithm is completed.
Step 4. We act the same as in the step 1 dhe fiksojmë elementin $\mathbf{0 . 9 4}$. To make the key 1 , we divide the first row (the $\mathrm{s}_{1}$ ) with 0.94 and we get:

| Base | $\mathbf{x}_{\mathbf{1}}$ | $\downarrow_{\mathbf{x} \mathbf{2}}$ | $\mathbf{x}_{\mathbf{3}}$ | $\mathbf{x}_{4}$ | $\mathbf{x}_{\mathbf{5}}$ | $\mathbf{s}_{\mathbf{1}}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{\mathbf{3}}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\leftarrow \mathbf{s}_{\mathbf{1}}$ | 0 | $[1]$ | 0 | -1.67 | 0.88 | 1.06 | 0 | 0.1 | 8.11 |
| $\mathrm{x}_{3}$ | 0 | 0.06 | 1 | 0.86 | -0.24 | 0 | 0 | -0.26 | 14.07 |
| $\mathrm{x}_{1}$ | 1 | 0 | 0 | 1.71 | 0.41 | 0 | 0 | 0.17 | 5.31 |
| $\mathbf{F}$ | 0 | -57.75 | 0 | 26710.74 | 4520.66 | 0 | 0.12 | 1977.27 | 328140.5 |

Get out of the base $\mathrm{s}_{1}$, enter on the base $\mathrm{x}_{2}$. We perform the transformations: $R_{2}-0.06 R_{1}, R_{4}+57.75 R_{1}$ and we get:

| Base | $\mathbf{x}_{1}$ | $\mathbf{x}_{\mathbf{2}}$ | $\mathbf{x}_{3}$ | $\mathbf{x}_{4}$ | $\mathbf{x}_{5}$ | $\mathbf{s}_{1}$ | $\mathbf{s}_{\mathbf{2}}$ | $\mathbf{s}_{3}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{x}_{2}$ | 0 | 1 | 0 | -1.67 | 0.88 | 1.06 | 0 | 0.1 | $\mathbf{8 . 1 1}$ |
| $\mathbf{x}_{3}$ | 0 | 0 | 1 | 0.95 | -0.29 | -0.06 | 0 | -0.27 | $\mathbf{1 3 . 6}$ |
| $\mathbf{x}_{1}$ | 1 | 0 | 0 | 1.72 | 0.41 | 0 | 0 | 0.17 | $\mathbf{5 . 3}$ |
| $\mathbf{F}$ | 0 | 0 | 0 | 26614.29 | 4571.43 | 61.43 | 0.12 | 1982.86 | $\mathbf{3 2 8 6 0 8 . 5 7}$ |

Since the row of $F$ has no negative values, then the algorithm ends.
We conclude that the optimal solution is reached for $x_{1}=5.3, x_{2}=8.11 . x_{3}=13.6, x_{4}=0$ and $x_{5}=0$ as well as the maximum profit per day that can be achieved is max $F=328608.57$ den.

## V. Conclusions:

Based on the calculation using the simplex linear programming method and verifying the results also with the help of the online software https://linprog.com/, the following conclusions can be drawn:

1. To maximize profits, the company should produce 5.3 units of type A) SCS01 playground, 8.11 units of type B) Pencil fence and 13.6 units of type C) Swing, while from two other products from 0 units
2. The maximum profit received in one day in this case will be 328608.57 den. from 178000 den that it was at the beginning according to the offer made, representing an enormous increase in profits of $185 \%$.
3. Under optimal conditions, the total production cost increased to 733200 den (initially 733000 den) and the required production time was the same as before the study, which represents a negligible cost increase of $0.03 \%$.

- We compile the mathematical model for minimizing the cost of production, so that the daily profit according to the company's request is preserved:

Mathematical modeling:
Determine the quantities $x_{1}, x_{2}, x_{3}, x_{4}$ and $x_{5}$, with restrictions:
Total units of production: $x_{1}+x_{2}+x_{3}+x_{4}+x_{5} \leq 27$

Profit: $30000 x_{1}+800 x_{2}+12000 x_{3}+35000 x_{4}+5000 x_{5}=88500$
Total time: $12 x_{1}+0.25 x_{2}+4 x_{3}+24 x_{4}+4 x_{5} \leq 120$
$x_{1}, x_{2}, x_{3}, x_{4}, x_{5} \geq 0$
So we have:
To minimize the cost function

$$
C=50600 x_{1}+2000 x_{2}+33000 x_{3}+115000 x_{4}+13000 x_{5}
$$

Under the conditions:

$$
\left\{\begin{array}{c}
x_{1}+x_{2}+x_{3}+x_{4}+x_{5} \leq 27 \\
30000 x_{1}+800 x_{2}+12000 x_{3}+35000 x_{4}+5000 x_{5}=88500 \\
12 x_{1}+0.25 x_{2}+4 x_{3}+24 x_{4}+4 x_{5} \leq 120
\end{array}\right.
$$

Where, $x_{1}, x_{2}, x_{3}, x_{4}, x_{5} \geq 0$.

We also verified the results of this linear programming minimization problem in the online software https://linprog.com/. Take:

We add the additional variables and get:

$$
C=50600 x_{1}+2000 x_{2}+33000 x_{3}+115000 x_{4}+13000 x_{5}+0 s_{1}+0 s_{2}+\mathrm{M} s_{3}
$$

Under the conditions:

$$
\left\{\begin{array}{c}
x_{1}+x_{2}+x_{3}+x_{4}+x_{5}+s_{1}=27 \\
30000 x_{1}+800 x_{2}+12000 x_{3}+35000 x_{4}+5000 x_{5}+s_{3}=88500 \\
12 x_{1}+0.25 x_{2}+4 x_{3}+24 x_{4}+4 x_{5}+s_{2}=120
\end{array}\right.
$$

The initial table is:

| Basis | $\mathbf{x} 1$ | $\mathbf{x} 2$ | $\mathbf{x} 3$ | $\mathbf{x} 4$ | $\mathbf{x} 5$ | $\mathbf{S}_{1}$ | $\mathbf{s}_{2}$ | $\mathbf{s} 3$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{S} 1$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 27 |
| $\mathbf{S 3}$ | 30000 | 800 | 12000 | 35000 | 5000 | 0 | 0 | 1 | 88500 |
| $\mathbf{S} 2$ | 12 | 0.25 | 4 | 24 | 4 | 0 | 1 | 0 | 120 |
| Min C | 30000 M <br> -50600 | $800 \mathrm{M}-$ <br> 2000 | $12000 \mathrm{M}-$ <br> 33000 | $35000 \mathrm{M}-$ <br> 115000 | $5000 \mathrm{M}-$ <br> 13000 | 0 | 0 | 0 | 88500 M |

Step 1. We set the key which is 35000 , we divide the corresponding row by 35000 .

| Basis | $\mathbf{x} 1$ | $\mathbf{x} 2$ | $\mathbf{x} 3$ | $\downarrow \mathbf{x} 4$ | $\mathbf{x} 5$ | $\mathbf{s} 1$ | $\mathbf{S} 2$ | $\mathbf{S 3}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{S} 1$ | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 27 |
| $\leftarrow \mathbf{S 3}_{3}$ | 0.86 | 0.02 | 0.34 | $[1]$ | 0.14 | 0 | 0 | 0 | 2.53 |
| $\mathbf{S} \mathbf{2}$ | 12 | 0.25 | 4 | 24 | 4 | 0 | 1 | 0 | 120 |
| Min C | 30000 M <br> -50600 | $800 \mathrm{M}-$ <br> 2000 | $12000 \mathrm{M}-$ <br> 33000 | $35000 \mathrm{M}-$ <br> 115000 | $5000 \mathrm{M}-$ <br> 13000 | 0 | 0 | 0 | 88500 M |

We perform the necessary transformations. At the base enters $\mathrm{x}_{4}$, while $\mathrm{s}_{3}$ exits.
After the appropriate calculations are done, we move on
Step 2. We set the key which is $\mathbf{0 . 8 6}$, we divide the corresponding row by 0.86 .

| Basis | $\downarrow \mathbf{x} 1$ | $\mathbf{x} 2$ | $\mathbf{x} 3$ | $\mathbf{x} 4$ | $\mathbf{x}$ | $\mathbf{S}_{1}$ | $\mathbf{s} 2$ | $\mathbf{S 3}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{S} 1$ | 0.14 | 0.98 | 0.66 | 0 | 0.86 | 1 | 0 | 0 | 24.47 |
| $\leftarrow \mathbf{x} 4$ | $[1]$ | 0.03 | 0.4 | 1.17 | 0.17 | 0 | 0 | 0 | 2.95 |
| $\mathbf{S} \mathbf{2}$ | -8.57 | -0.3 | -4.23 | 0 | 0.57 | 0 | 1 | 0 | 59.31 |
| Min C | 47971.43 | 628.57 | 6428.57 | 0 | 3428.57 | 0 | 0 | $-\mathrm{M}+3.29$ | 290785.71 |

We perform the necessary transformations. At the base enters $\mathrm{x}_{1}$, while $\mathrm{x}_{4}$ exits.
After the appropriate calculations are done, we move on

## Step 3:

| Basis | $\mathbf{x} 1$ | $\mathbf{x} 2$ | $\mathbf{x} 3$ | $\mathbf{x} 4$ | $\mathbf{x} 5$ | $\mathbf{S} 1$ | $\mathbf{s} 2$ | $\mathbf{s 3}$ | T.L |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| S1 | 0 | 0.97 | 0.6 | -0.17 | 0.83 | 1 | 0 | 0 | 24.05 |
| $\mathbf{x} 1$ | 1 | 0.03 | 0.4 | 1.17 | 0.17 | 0 | 0 | 0 | $\mathbf{2 . 9 5}$ |
| S2 | 0 | -0.07 | -0.8 | 10 | 2 | 0 | 1 | 0 | 84.6 |
| Min C | 0 | -650.67 | -12760 | -55966.67 | -4566.67 | 0 | 0 | $-\mathrm{M}+1.69$ | $\mathbf{1 4 9 2 7 0}$ |

The algorithm is completed and the results obtained are: $\operatorname{Min} \mathrm{C}=149270$ which is achieved for $\mathrm{x}_{1}=2.95$ and others 0 .

From which we get that the cost can be reduced to 149270 den for the same profit of 88500 den from 214100 den that was at the beginning, if we manage to sell 2.95 units of production SCS01 playground. So,
in this case, we will have a cost reduction of 64830 den, which would be transferred to the coffers of the enterprise as profit, and the total amount of profit will be 153330 den.

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