International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

Vol. 40 No. 2 September 2023, pp. 146-150

Determining the Amount of Car Production to Obtain Maximum Profit Using the Concept of Dual Hesitant Fuzzy Set

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Abstract— In the context of decision-making problems related to car production, the dual-hesitant fuzzy set can be a tool to deal with uncertainty and hesitation in determining the optimal number of cars to be produced for each type of car. The score values associated with the dual-hesitant fuzzy set can represent the level of preference or confidence in producing the number of cars necessary to obtain maximum profit.

Keywords— Decision-Making Problems, Dual Hesitant Fuzzy Set, Score Values.

I. Introduction

The basic concept of fuzzy sets was initially introduced by L.A. Zadeh in 1965 [3]. Fuzzy sets are sets in which the elements have membership values ranging from [0, 1]. Based on the concept of fuzzy sets, several developments have been made, such as Intuitionistic Fuzzy Sets (IFS) [1], Fuzzy Multisets (FMS), and Hesitant Fuzzy Sets (HFS) [2].

Intuitionistic fuzzy sets were introduced by Attanasov in 1986 [1] as a generalization of fuzzy sets. Hesitant fuzzy sets were introduced by Torra in 2010 [2] and defined as the values of the membership of an element represented as a set of possible values. Based on the concepts of Intuitionistic fuzzy sets and Hesitant fuzzy sets, the concept of a dual hesitant fuzzy set was developed by Zhu Bin et al. [4]. The Dual Hesitant Fuzzy Set is defined by two functions: the hesitant membership function and the hesitant non-membership function. This concept distinguishes the Dual Hesitant Fuzzy Set from fuzzy sets, Intuitionistic fuzzy sets, and Hesitant fuzzy sets, especially concerning the membership and non-membership values of the elements and objects considered in decision-making problems. Consequently, fuzzy sets, Intuitionistic fuzzy sets, and Hesitant fuzzy sets can be considered special cases of the Dual Hesitant fuzzy set.

In this article, we will determine the car production amount to obtain maximum profit using the dual hesitant fuzzy sets and the scoring method. In forecasting problems, event probabilities are often used to derive expectations. However, in reality, probabilities often cannot represent the wishes of a decision-maker, and they also cannot describe the level of certainty of an event. So, DHFS is used to replace probability in decision-making problems.

II. Fuzzy Set, Intuitionistic Fuzzy Set, Hesitant Fuzzy Set, Dual Hesitant Fuzzy Set, and Scoring VALUES in Dual Hesitant Fuzzy Set

In this section, we will review some related definitions introduced in the previous article, which form the basis for the study in this article.

Definition 2.1. [3] *Let X be a non-empty universe set. A fuzzy set A over X can be defined as*

$$A = \{\langle x, \mu(x)\rangle | x \in X\}$$

where $\mu: X \rightarrow [0, 1]$ *is called the membership function of A over X, and* $\mu(x)$ *represents the degree of membership of* $x \in X$ *in the fuzzy set A.*

Definition 2.2. [1] *Let X be a non-empty universe set, A intuitionistic fuzzy set I over X can be defined as*

$$I = \{\langle x, \mu(x), \nu(x)\rangle | x \in X\}$$

where $\mu: X \rightarrow [0, 1]$ *and* $\nu: X \rightarrow [0, 1]$ *respectively, is a membership function and a non-membership function, with the condition* $0 \leq \mu(x) + \nu(x) \leq 1$, *for every* $x \in X$. *It is also defined that the hesitant degree of x in I is* $\pi(x) = 1 - \mu(x) - \nu(x)$.

Definition 2.3. [2] *Let X be non-empty universe set, A hesitant fuzzy set H over X can be defined as*

$$H = \{\langle x, h(x)\rangle | x \in X\}$$

where h: X → P[0, 1] is called the membership function of I over X where P*[0, 1] is a collection of sets whose members are subsets [0, 1] , h(x) is called a set whose members are in the interval [0, 1], and h(x) is also called a Hesitant fuzzy element (HFE), which is denoted as h with* $\gamma \in h$.

Definition 2.4.[4] *Let U be a non-empty universe set, A dual hesitant fuzzy set D over X can be defined as*

$$D = \{\langle x, h(x), g(x)\rangle | x \in X\}$$

where h(x) and g(x) are two sets whose members are in the interval [0, 1], which respectively denote the set of membership values and the set of non-membership values of $x \in X$ *for the set D. The pair* $d(x) = (h(x), g(x))$ *is said to be a DHFE denoted* $d = (h, g)$, *where for* $\gamma \in h$ *and* $\eta \in g$ *is provided that*

$$0 \leq \gamma \leq 1, 0 \leq \eta \leq 1, 0 \leq \gamma^+ + \eta^+ \leq 1, \text{ with } \gamma^+ = \sup\{\gamma: \gamma \in h\}, \text{and } \eta^+ = \sup\{\eta: \eta \in g\}.$$

Definition 2.5.[4] *Let* $d_i = (h_{di}, g_{di})$ *where (i = 1, 2) are two DHFE,* $S_{d_i} = \left(\frac{1}{\#h}\right)\sum_{\gamma \in h}\gamma - \left(\frac{1}{\#g}\right)\sum_{\eta \in g}\eta$ *is the score value of* d_i, *and* $P_{d_i} = \left(\frac{1}{\#h}\right)\sum_{\gamma \in h}\gamma + \left(\frac{1}{\#g}\right)\sum_{\eta \in g}\eta$ *is the accuracy value of* d_i, *where #h and #g, respectively, are the number of elements in h and g. Hereinafter it is defined*

1. *If* $s_{d_1} > s_{d_2}$, d_1 *is said to be higher than* d_2 *, which is denoted by* $d_1 > d_2$*;*
2. *If* $s_{d_1} = s_{d_2}$,
   (a) *If* $p_{d_1} = p_{d_2}$, d_1 *is said to be equivalent to* d_2, *which is denoted by* $d_1 \sim d_2$*;*
   (b) *If* $p_{d_1} > p_{d_2}$, d_1 *is said to be higher than* d_2, *which is denoted by* $d_1 > d_2$.

In the following, an algorithm for decision-making problems will be presented as an application of the Dual Hesitant Fuzzy Set concept [4].

Algorithm:

1. Define DHFS D over X;
2. Calculating score values based on Definition 2.5;
3. Transform the score using the formula $f_i = \sum_{j=1}^{n}\frac{s_{d_{ij}}+1}{2}$;
4. Calculating $b_{ij} = \sum_{j=1}^{n}\frac{s_{d_{ij}}+1}{2f_i}$;
5. Calculate the expected value using the formula $e_i = (\overline{c_{ij}})(\overline{b_{ij}})$.

III. RESULT AND DISCUSSION

In this section, a case of a DHFS application will be given for decision-making problems.

Case. Suppose it is denoted that the i-th type of car from a car company is y_i $(i = 1,2,3,4)$ and c_{ij} $(j = 1,2,3)$ is the number of i-th cars sold for the j-th assumption. For each y_i and c_{ij}, a decision maker will determine the level of confidence (membership value / γ) that y_i will be sold as much as c_{ij} and the level of uncertainty (non-membership value / η) that y_i will sell as much c_{ij} as can be written in DHFE $d_{ij} = \left(\bigcup_{\gamma\in h}\{\gamma\}, \bigcup_{\eta\in g}\{\eta\}\right)$, where $\gamma \in [0,1]$, $\eta \in [0,1]$, $\gamma + \eta \leq 1$. This car company wants to determine the car production amount to obtain maximum profit for each type of car from the company based on the assumption of the number of sales. Types of cars and the assumption data on the number of cars sold and the DHFE can be seen in the following table.

Type of Car	c_{ij}	Assuming the car is sold (units)	d_{ij}
y_1	c_{11}	400	$(\{0.5, 0.6\}, \{0.2\})$
	c_{12}	450	$(\{0.3, 0.4\}, \{0.4\})$
	c_{13}	500	$(\{0.2, 0.3\}, \{0.5, 0.6\})$
y_2	c_{21}	350	$(\{0.4, 0.5\}, \{0.4\})$
	c_{22}	200	$(\{0.8\}, \{0.1\})$
	c_{23}	400	$(\{0.2, 0.3, 0.4\}, \{0.5\})$
y_3	c_{31}	250	$(\{0.6, 0.7\}, \{0.1\})$
	c_{32}	300	$(\{0.3\}, \{0.5,0.6\})$
	c_{33}	450	$(\{0.2, 0.3\}, \{0.3, 0.4\})$
y_4	c_{41}	300	$(\{0.7, 0.8\}, \{0.1, 0.2\})$
	c_{42}	400	$(\{0.1, 0.2\}, \{0.6, 0.7\})$
	c_{43}	350	$(\{0.2, 0.3, 0.4\}, \{0.4, 0.5\})$

To solve the problem above, a logical procedure is given that can determine the car production amount to obtain maximum profit based on the assumption that many cars are sold. Completion steps:

1. Calculating score values, $S_{d_{ij}} = \left(\frac{1}{\#h}\right)\sum_{\gamma\in h}\gamma - \left(\frac{1}{\#g}\right)\sum_{\eta\in g}\eta$;

Table score values $S_{d_{ij}}$

$S_{d_{11}}$	$S_{d_{12}}$	$S_{d_{13}}$	$S_{d_{21}}$	$S_{d_{22}}$	$S_{d_{23}}$	$S_{d_{31}}$	$S_{d_{32}}$	$S_{d_{33}}$	$S_{d_{41}}$	$S_{d_{42}}$	$S_{d_{43}}$

2. Transform the score using the formula $f_i = \sum_{j=1}^{n} \frac{s_{d_{ij}}+1}{2}$;

$$f_1 = \frac{(0.35+1)+(-0.05+1)+(-0.3+1)}{2} = \frac{3}{2} = 1.5$$

$$f_2 = \frac{(0.05+1)+(0.7+1)+(-0.2+1)}{2} = \frac{3.55}{2} = 1.775$$

$$f_3 = \frac{(0.55+1)+(-0.25+1)+(-0.1+1)}{2} = \frac{3.2}{2} = 1.6$$

$$f_4 = \frac{(0.6+1)+(-0.5+1)+(-0.15+1)}{2} = \frac{2.95}{2} = 1.475$$

3. Calculating $b_{ij} = \sum_{j=1}^{n} \frac{s_{d_{ij}}+1}{2f_i}$;

Table b_{ij} percentage weight d_{ij} to f_i.

$S_{d_{11}}$	$S_{d_{12}}$	$S_{d_{13}}$	$S_{d_{21}}$	$S_{d_{22}}$	$S_{d_{23}}$	$S_{d_{31}}$	$S_{d_{32}}$	$S_{d_{33}}$	$S_{d_{41}}$	$S_{d_{42}}$	$S_{d_{43}}$
	0.3167	0.2333	[illegible]		[illegible]						

4. Calculate the expected value using the formula $e_i = (\overline{c_{ij}})(\overline{b_{ij}})$.

Table e_i

e_1	e_2	e_3	e_4
438.65	289.25	315	331.5

Based on the expected value of the request, it can be concluded that each type of y_i car as follows:
(a) The optimal number of productions for car type y_1 is 439 units.
(b) The optimal number of productions for car type y_2 is 290 units.
(c) The optimal number of productions for car type y_3 is 315 units.
(d) The optimal number of productions for car type y_4 is 331 units.

IV. Conclusion

This research solves the problem of making decisions about determine the car production amount to obtain maximum profit based on the assumption that the number of cars sold uses the score in DHFS. There are four types of cars y_i $(i = 1,2,3,4)$, and the assumption of the number of cars sold is c_{ij} $(j = 1,2,3)$. Based on the calculation of the expected value of e_i, the car production amount to obtain maximum profit for each type of car is obtained.

V. Acknowledgment

This research is supported by research fund from Andalas University.

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