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The Selection of Investment Priorities using Expanded Dual Hesitant Fuzzy Sets

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Abstract— The concept of expanded dual hesitant fuzzy set is a concept that can be applied in decision-making problems. In decision-making problems, expanded dual hesitant fuzzy set can be applied to represent the opinions of multiple experts or stakeholders in a more elaborate manner. It enables decision-makers to provide more detailed information about their preferences and hesitancy.

Keywords— Decision-Making Problems, Expanded Dual Hesitant Fuzzy Set, ω-weighted score, (ω, δ)-sequence score.

I. Introduction

The concept of Fuzzy sets [7] has been developed in extraordinary variations. One of the developments of the Fuzzy Set concept is regarding hesitant fuzzy sets and their development, such as expanded hesitant fuzzy sets, dual hesitant fuzzy sets, and extended hesitant fuzzy sets ([1], [2], [3], [9], [8]). Hesitant fuzzy sets explain that decision makers can provide more than one degree of membership for each object in decision-making problems. An Expanded hesitant fuzzy set is known that the decision maker gives the degree of membership for each object in the decision-making problem [3]. Extended hesitant fuzzy sets explain that there are lots of possibilities from a membership degree that are defined by a bunch of decision-makers in the decision-making problems [8]. The Dual hesitant fuzzy set explain that the decision maker can provide the membership degree and non-membership degree for each object in the decision-making problem [9].

In 2018, Fatimah and Alcantud [5] proposed the new concept namely expanded dual hesitant fuzzy set. The concept of an expanded dual hesitant fuzzy set is a combination of the concepts of an expanded hesitant fuzzy set and a dual hesitant fuzzy set. In expanded dual hesitant fuzzy set explained that membership and nonmembership degree with decision maker that given the membership and nonmembership degree for each object in decision making problem.

This research aims to examine the application of an expanded dual hesitant fuzzy set in the selection of investment priorities in decision-making problems.

II. Hesitant Fuzzy Set, Extended Hesitant Fuzzy Set, Dual Hesitant Fuzzy Set, Expanded Hesitant Fuzzy Set, Expanded Dual Hesitant Fuzzy Set.

In this section, we will review some related definitions introduced in the previous article, which form the basis for the study in this article. Let X be a non-empty set.

Definition 2.1. ([4],[6]) *A hesitant fuzzy set in* X *is a function* $h_M: X \rightarrow \mathcal{P}^*([0,1])$. *A typical hesitant fuzzy sets in* X *are function* $\bar{h}_M: X \rightarrow \mathcal{F}^*([0,1])$, *where* $\mathcal{P}^*([0,1])$ *is collection of the sets of all HFE and* $\mathcal{F}^*([0,1])$ *is collection of the sets of all THFE*

Definition 2.2. [8] *An extended hesitant fuzzy element is a cartesian product of a hesitant fuzzy set, denoted by* $\overline{H}(x)$, *where* $\overline{H}(x) = h_1(x) \times \ldots \times h_m(x)$ *where* $h_i(x)$ *is a membership set of x, where* $i = 1, \ldots, m$.

Definition 2.3. [8] *An extended hesitant fuzzy set of X is*

$$\overline{H}_x = \{(x, \overline{H}(x)) | x \in X\}.$$

Definition 2.4. [9] *Dual hesitant fuzzy elements are a pair* $d = (H, G)$ *with* $H, G \subseteq [0,1]$ *such that* $\gamma^+ + \eta^+ \leq 1$, $\gamma^+ = \sup\{\gamma | \gamma \in H\}$, $\eta^+ = \sup\{\eta | \eta \in G\}$.

Definition 2.5. [9] *Given dual hesitant fuzzy elements* $d(x) = (h(x), g(x))$ *for each* $x \in X$, *then the dual hesitant fuzzy set of X is*

$$D = \{(x, h(x), g(x)) | x \in X\}.$$

Suppose $P_m = \{1, \ldots, m\}$ denotes a set representing m decision makers and $X = \{1, \ldots, m\}$ denotes a set representing M objects to be assessed by decision makers.

Definition 2.6. [3] *An expanded hesitant fuzzy element of degree* m *is* $H_M^m \in \mathcal{P}^*([0,1] \times \mathcal{P}(P_m)) = \{H_M^m | H_M^m \subset [0,1] \times \mathcal{P}(P_m)\}$, *where* $\mathcal{P}(P_m) = \{C | C \subset P_m\}$, *such as* $(a, C)(a', C') \in H_M^m$ *with* $a = a'$ *imply* $C = C'$. *An typical expanded hesitant fuzzy element of degree* m *is* $\xi_M^m \in \mathcal{P}^*([0,1] \times \mathcal{P}^*(P_m)) = \{\xi_M^m | \xi_M^m \subset [0,1] \times \mathcal{P}^*(P_m)\}$, *where* $P^*(P_m) = \{C | C \subset P_m \text{ and } C \neq \emptyset\}$, *such as* $(a, C)(a', C') \in \xi_M^m$ *with* $a = a'$ *imply* $C = C'$.

Definition 2.7. [3] *An expanded hesitant fuzzy set on X is* $\overline{H}_M^m = \{(x, H_M^m(x)) | x \in X\}$ *where* $H_M^m(x)$ *is expanded hesitant fuzzy element for each* $x \in X$. *A typical expanded hesitant fuzzy set on X is* $\mathcal{H}_M^m = \{(x, \xi_M^m(x)) | x \in X\}$ *where* $\xi_M^m(x)$ *is typical expanded hesitant fuzzy element for each* $x \in X$.

Definition 2.8. [3] *Let* $H_M^m = \{(a_i, C_i)\}_{i=1,\ldots,l}$ *and vector of weight* $\omega = (\omega_1, \omega_2, \ldots, \omega_m)$ *where* $\omega_i \in [0,1]$ *for each* i *and* $\omega_1 + \omega_2 + \cdots + \omega_m = 1$. *The ω-weighted score of* H_M^m *defined by*

$$\mathcal{L}_\omega(H_M^m) = \frac{a_1\, \omega^1(H_M^m)\, \lambda_1(H_M^m) + \cdots + a_l\, \omega^l(H_M^m)\, \lambda_l(H_M^m)}{\omega^1(H_M^m)\, \lambda_1(H_M^m) + \cdots + \omega^l(H_M^m)\, \lambda_l(H_M^m)}$$

where

1. $\lambda_i(H_M^m) = \begin{cases} |C_i| & , \quad C_i \neq \emptyset \\ 1 & , \quad C_i = \emptyset \end{cases};$
2. $\omega^i(H_M^m) = \begin{cases} \sum \omega_k, k \in C_i & , \quad C_i \neq \emptyset \\ min_{1=1,\ldots,m}\{\omega_i\} & , \quad C_i = \emptyset \end{cases}.$

Definition 2.9. [3] *Let* $\delta = \{\delta_1, \ldots, \delta_n, \ldots\}$ *be a monotone non-decreasing sequence of positive numbers. The* (ω, δ)*-sequence score of* H_M^m *defined by*

$$\mathcal{L}_{\omega,\delta}(H_M^m) = \frac{a_1\, \omega^1(H_M^m)\, \delta_1 + \cdots + a_l\, \omega^l(H_M^m)\, \delta_l}{\omega^1(H_M^m)\, \delta_1 + \cdots + \omega^l(H_M^m)\, \delta_l}.$$

Definition 2.10. [5] *A expanded dual hesitant fuzzy element of degree* m *is a pair* $d^m = (H^m, G^m)$, *where* H^m *and* G^m *is a expanded hesitant fuzzy element, with condition if* $(a, C) \in H^m$ *and* $(a', C') \in G^m$ *then* $a + a' \leq 1$.

Definition 2.10. [5] *An expanded dual hesitant fuzzy set of degree* m *on X is* $D^m = \{(x, h_M^m(x), g_M^m(x)) | x \in X\}$ *where* $d^m(x) = (h_M^m(x), g_M^m(x))$ *is an expanded dual hesitant fuzzy element of degree* m, *for* $x \in X$.

Definition 2.11. [5] *Let $d^m = (H_m, G_m)$ where $H_m = \{(a_i, C_i)\}_{i=1,\dots,l}$ and $G_m = \{(a_i', C_i')\}_{i=1,\dots,l}$. Let vector of weight $\omega = (\omega_1, \omega_2, \dots, \omega_m)$ where $\omega_i \epsilon [0,1]$ for each i and $\omega_1 + \omega_2 + \cdots + \omega_m = 1$. The ω-weighted score of d^m defined by*

$$\mathcal{L}_{d^m} = \mathcal{L}_\omega(H_m) - \mathcal{L}_\omega(G_m).$$

Definition 2.12. [5] *Let $\delta = \{\delta_1, \dots, \delta_n, \dots\}$ be a monotone non-decreasing sequence of positive numbers. The (ω, δ)-sequence score of d^m defined by*

$$\mathcal{L}_{\delta, d^m} = \mathcal{L}_{\omega,\delta}(H_m) - \mathcal{L}_{\omega,\delta}(G_m).$$

Definition 2.12. [5] *Let $d^m = (H_m, G_m)$ and $\bar{d}^m = (\bar{H}_m, \bar{G}_m)$ be typical expanded dual hesitant fuzzy element. Select score $\mathcal{L}$ of typical expanded hesitant fuzzy element. Define:*

$$s_{d^m} = \mathcal{L}(H_m) - \mathcal{L}(G_m),$$

$$\bar{s}_{d^m} = \mathcal{L}(\bar{H}_m) - \mathcal{L}(\bar{G}_m),$$

$$p_{d^m} = \mathcal{L}(H_m) + \mathcal{L}(G_m),$$

$$\bar{p}_{d^m} = \mathcal{L}(\bar{H}_m) + \mathcal{L}(\bar{G}_m).$$

1. *When $s_{d^m} > \bar{s}_{d^m}$ so it can be said that d^m is superior than $\bar{d}^m$, denoted $d^m \succ \bar{d}^m$.*
2. *When $s_{d^m} = \bar{s}_{d^m}$, then*
    a. *d^m superior than $\bar{d}^m$ when $p_{d^m} > \bar{p}_{d^m}$, denoted by $d^m \succ \bar{d}^m$; and*
    b. *d^m equivalent than $\bar{d}^m$ when $p_{d^m} > \bar{p}_{d^m}$, denoted by $d^m \succ \bar{d}^m$.*

Based on Definition 2.8 and 2.9 then:

1. select score $\mathcal{L} = \mathcal{L}_\omega$ then $s_{d^m} = \mathcal{L}_{d^m}$,
2. select score $\mathcal{L} = \mathcal{L}_{\omega,\delta}$ then $s_{d^m} = \mathcal{L}_{\delta, d^m}$.

III. Result and Discussion

This section will give a case that can be represented as a form of Expanded Dual Hesitant Fuzzy Set.

Case. A husband and a wife want to invest considering the growth in wealth from each investment. There are three types of investments considered by the husband and wife, namely gold, property and stocks. Type of investment denoted by y_i where $i = 1,2,3$ with gold denoted as y_1, property denoted as y_2, and stocks denoted as y_3. Assumption of wealth growth denoted by c_{ij} where $j = 1,2,3$ with C_{ij} indicating the type of investment i assuming wealth growth to j. Let the opinions of the husband and wife are collected in the form of a typical expanded dual hesitant fuzzy element where for each y_i and c_{ij} the husband and wife will provide an assessment based on how much they believe and do not believe that y_i will have a wealth growth of the size of c_{ij} as follows:

$$d_{11} = (\{(0.6, \{1,2\})\}, \{(0.1, \{2\}), (0.3, \{1\})\}),$$

$$d_{12} = (\{(0.5, \{1,2\}), (0.6, [1\})\}, \{(0.3, \{1,2\})\}),$$

$$d_{13} = (\{(0.3, \{2\}), (0.4, \{1,2\}), (0.5, \{2\})\}, \{(0.3, [1,2\})\}),$$

$$d_{21} = (\{(0.8, \{2\})\}, \{(0.1, \{1\})\}),$$

$$d_{22} = (\{(0.3, \{1,2\}), (0.4, \{1,2\})\}, \{(0.4, \{1\})\}),$$

$$d_{23} = (\{(0.8, \{1\})\}, \{(\emptyset, \{1,2\})\}),$$

$$d_{31} = (\{(0.1, \{1\}), (0.4, \{2\})\}, \{(0.5, \{1\})\}),$$

$$d_{32} = (\{(0.7, \{1,2\}), (0.8, \{1\})\}, \{(0.1, \{1,2\})\}),$$

$d_{33} = (\{(0.5, \{2\})\}, \{(\emptyset, \{1,2\})\})$.

Application of Definition 2.11 produces the evaluation below. Let $\omega = (0.5,0.5)$ and $\mathcal{L} = \mathcal{L}_\omega$. Then,

$$\mathcal{L}_{d_{ij}} = \frac{a_1\ \omega^1(H_{ij})\,\lambda_1(H_{ij})+\cdots+a_l\,\omega^l(H_{ij})\,\lambda_l(H_{ij})}{\omega^1(H_{ij})\,\lambda_1(H_{ij})+\cdots+\omega^l(H_{ij})\,\lambda_l(H_{ij})} - \frac{a_1\ \omega^1(G_{ij})\,\lambda_1(G_{ij})+\cdots+a_l\,\omega^l(G_{ij})\,\lambda_l(G_{ij})}{\omega^1(G_{ij})\,\lambda_1(G_{ij})+\cdots+\omega^l(G_{ij})\,\lambda_l(G_{ij})}$$

obtained

$$\mathcal{L}_{d_{11}} = \frac{0.6\cdot(0.5+0.5)\cdot 2}{(0.5+0.5)\cdot 2} - \frac{0.1\cdot 0.5\cdot 1+0.3\cdot 0.5\cdot 1}{0.5\cdot 1+0.5\cdot 1} = 0.4$$

$$\mathcal{L}_{d_{12}} = \frac{0.5\cdot(0.5+0.5)\cdot 2+0.6\cdot 0.5\cdot 1}{(0.5+0.5)\cdot 2+0.5\cdot 1} - \frac{0.3\cdot(0.5+0.5)\cdot 2}{(0.5+0.5)\cdot 2} = 0.22$$

$$\mathcal{L}_{d_{13}} = \frac{0.3\cdot 0.51+0.4\cdot(0.5+0.5)\cdot 2+0.5\cdot 0.5\cdot 1}{0.5\cdot 1+(0.5+0.5)\cdot 2+0.5\cdot 1} - \frac{0.3\cdot 0.5\cdot 1}{0.5\cdot 1} = 0.1$$

$$\mathcal{L}_{d_{21}} = \frac{0.8\cdot 0.5\cdot 1}{0.5\cdot 1} - \frac{0.1\cdot 0.5\cdot 1}{0.5\cdot 1} = 0.7$$

$$\mathcal{L}_{d_{22}} = \frac{0.3\cdot(0.5+0.5)\cdot 2+0.4\cdot(0.5+0.5)\cdot 2}{(0.5+0.5)\cdot 2+(0.5+0.5)\cdot 2} - \frac{0.4\cdot 0.5\cdot 1}{0.5\cdot 1} = -0.05$$

$$\mathcal{L}_{d_{23}} = \frac{0.8\cdot 0.5\cdot 1}{0.5\cdot 1} - 0 = 0.8$$

$$\mathcal{L}_{d_{31}} = \frac{0.1\cdot 0.5\cdot 1+0.4\cdot 0.5\cdot 1}{0.5\cdot 1+0.5\cdot 1} - \frac{0.5\cdot 0.5\cdot 1}{0.5\cdot 1} = -0.25$$

$$\mathcal{L}_{d_{32}} = \frac{0.7\cdot(0.5+0.5)\cdot 2+0.8\cdot 0.5\cdot 1}{(0.5+0.5)\cdot 2+0.5\cdot 1} - \frac{0.1\cdot(0.5+0.5)\cdot 2}{(0.5+0.5)\cdot 2} = 0.62$$

$$\mathcal{L}_{d_{33}} = \frac{0.5\cdot 0.5\cdot 1}{0.5\cdot 1} - 0 = 0.5.$$

Next, expectations for gold, property, and stocks are determined using predictive values and scores. Then the expectation obtained from y_1, y_2, dan y_3 as follows.

Table 1. The Example of XDHFE

Investment Type	c_{ij}	Predictive Value (million)	d_{ij}
y_1	c_{11}	50	d_{11}
	c_{12}	70	d_{12}
	c_{13}	60	d_{13}
y_2	c_{21}	100	d_{21}
	c_{22}	60	d_{22}
	c_{23}	70	d_{23}
y_3	c_{31}	80	d_{31}
	c_{32}	70	d_{32}
	c_{33}	90	d_{33}

$$E_1 = 50 \times 0.4 + 70 \times 0.22 + 60 \times 0.1 = 41.4$$

$$E_2 = 100 \times 0.7 + 60 \times (-0.05) + 70 \times 0.8 = 123$$

$$E_3 = 80 \times (-0.25) + 70 \times 0.62 + 90 \times 0.5 = 68.4$$

Based on the evaluation of gold, property, and stock expectations, $E_2 > E_3 > E_1$ is obtained, so it is concluded that the optimal choice for investment is y_2 that is property based on wealth growth.

IV. Conclusion

Decision-making problems using expanded dual hesitant fuzzy sets involves determining the best choice or alternative based on the given criteria and preferences. Expanded dual hesitant fuzzy sets are an extension of hesitant fuzzy sets, which allow decision-makers to express their hesitancy more explicitly by considering both positive and negative evaluations for each alternative with respect to each criterion.

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