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Selection of Exemplary Students Using The Concept of Hesitant Multi-Fuzzy Soft Set

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Abstract – The concept of hesitant multi-fuzzy soft set is a concept that can be applied in decision-making problems. In this paper, we use an algorithm with respect to hesitant multi-fuzzy soft set to make the selection of exemplary students. The decision-making process in this case is carried out by following the steps in a constructed algorithm in order to obtain the best possible outcome from the given case study.

Keywords – ***Decision-Making Algorithm, Hesitant Multi-Fuzzy Soft Set, Scoring Method.***

I. Introduction

To solve complex real-life problems in various disciplines such as engineering, economics, and social sciences, the techniques available in classical mathematics often fail to produce satisfactory results due to various types of uncertainty and hesitancy. There are several mathematical tools and approaches to deal with uncertainty and hesitancy, but each proposed approach has its specific limitations. The theory of soft sets is a mathematical tool for handling uncertainty and vagueness. The idea of soft sets was first introduced by Molodtsov [4]. Recently, research on the theory of soft sets has been rapidly growing.

In 2002, Maji et al. [3] proposed the first development of soft sets in decision-making problems by introducing and studying the theory of fuzzy soft sets, which combines the theories of soft sets and fuzzy sets. In the theory of fuzzy soft sets, membership degrees are assigned to facilitate decision-making in a problem. However, assigning membership degrees to elements is not easy, as each element in a set may have several possible values. To address this issue, Torra [7] proposed a new concept of fuzzy sets called hesitant fuzzy sets (HFS). Later, a new type of fuzzy set called multi-fuzzy set was introduced by Sebastian and Ramakrishnan [6] as an extension of the fuzzy set concept. Meanwhile, Yang et al. [8] introduced the concept of multi-fuzzy soft sets (MFSS) and discussed their applications in decision-making problems. Furthermore, in 2019, the concept of hesitant multi-fuzzy sets and hesitant multi-fuzzy soft sets, proposed by Dey et al. [1].

This research aims to examine the application of hesitant multi-fuzzy soft sets in the selection of exemplary students and develop an algorithm for decision-making in this selection process.

II. Fuzzy Set, Soft Set, Fuzzy Soft Set, Multi-Fuzzy Set, Multi-Fuzzy Soft Set, Hesitant Fuzzy Set, Scoring Method in Fuzzy Soft Set, and Hesitant Multi Fuzzy Soft Set

In this section, we will review some related definitions introduced in the previous article, which form the basis for the study in this article.

Definition 2.1. [9] *Let U be a set of objects. A fuzzy set X over U can be defined as*

$$X = \{(\mu_x(u)/u) | u \in U, \mu_x(u) \in [0,1]\}$$

where $\mu_x : U \rightarrow [0, 1]$ is called the membership function of X over U, and $\mu_x(u)$ represents the degree of membership of $u \in U$ in the fuzzy set X.

Definition 2.2. [4] *Let U be a set of objects, P(U) be the power set of U, E be a set of parameters, and $A \subseteq E$. A soft set over U is a pair (F, A), where F is a mapping given by $F: A \rightarrow P(U)$. A soft set over U can be expressed as a set of ordered pairs:*

$$(F, A) = \{(e, F(e)) | e \in A, F(e) \in P(U)\}.$$

Definition 2.3. [2] *Let U be a set of objects, E be a set of parameters, $A \subseteq E$, and $\tilde{P}(U)$ be the set of all fuzzy sets over U. A fuzzy soft set over U is a pair $(\hat{F}, A)$, where $\hat{F}$ is a mapping given by $\hat{F} : A \rightarrow \tilde{P}(U)$. A fuzzy soft set over U can be expressed as a set of ordered pairs:*

$$(\hat{F}, A) = \{(e, \hat{F}(e)) | e \in A, \hat{F}(e) \in \tilde{P}(U)\}.$$

Definition 2.4. [6] *Let U be a set of objects, and k be a positive integer. A multi-fuzzy set $\tilde{Y}$ over U can be defined as*

$$\tilde{Y} = \{u/(\mu_1(u), \mu_2(u), \cdots, \mu_k(u)) | u \in U\}$$

where $\mu_i: U \rightarrow [0, 1]$, i = 1, 2, · · · , k is called the membership function of $\tilde{Y}$ over U for the i-th dimension, and $\mu_i(u)$ represents the degree of membership of $u \in U$ in the multi-fuzzy set $\tilde{Y}$ for the i-th dimension. The function $\mu_{\tilde{Y}}(u) = (\mu_1(u), \mu_2(u), \cdots, \mu_k(u))$ is a mapping given by $\mu_{\tilde{Y}}: U \rightarrow [0,1] \times [0,1] \times \ldots \times [0,1]$ and is called the multi-membership function of the multi-fuzzy set $\tilde{Y}$, and k is referred to as the dimension of $\tilde{Y}$. The set of all k-dimensional multi-fuzzy sets over U is denoted as $M^kFS(U)$.

Definition 2.5. [8] *Let U be a set of objects, E be a set of parameters, and $A \subseteq E$. A k-dimensional multi-fuzzy soft set, abbreviated as MFSS, over U is a pair $(\breve{F}, A)$ where $\breve{F}$ is a mapping given by $\breve{F}: A \rightarrow M^kFS(U)$. A k-dimensional multi-fuzzy soft set over U can be expressed as a set of ordered pairs:*

$$(\breve{F}, A) = \{(e, \breve{F}(e)) | e \in A, \breve{F}(e) \in M^kFS(U)\}.$$

Definition 2.6. [7] *Let U be a set of objects. A hesitant fuzzy set over U is denoted by $R = \{(u, h_R(u)) | u \in U\}$, where $h_R(u)$ is a set of multiple values in the interval [0,1], indicating the possible degree of membership of the element $u \in U$ to the set R. For simplicity, $h_R(u)$ is referred to as a hesitant fuzzy element (HFE).*

Definition 2.7.[5] *Let's consider a fuzzy soft set $(\hat{F}, A)$. A comparison table of $(\hat{F}, A)$ is a table with an equal number of rows and columns, where the rows and columns are labeled with the names of objects. The entry at position $\{ij\}$, c_{ij}, represents the number of parameters for which the membership value of u_i is greater than or equal to the membership value of u_j for i, j = 1, 2, ..., n.*

The comparison table of the fuzzy soft set $(\hat{F}, A)$ can be represented in the following table.

The comparison table of the fuzzy soft set $(\hat{F}, A)$

	u_1	u_2	…	u_n
u_1	c_{11}	c_{12}	…	c_{1n}
u_2	c_{21}	c_{22}	…	c_{2n}
⋮	⋮	⋮	⋱	⋮
u_n	c_{n1}	c_{n2}	…	c_{nn}

Next, a score table will be formed by determining the sum of c_{ij} for the i-th row, the sum of of c_{ij} for the j-th column, and the score values, as explained below. For each i, the sum of c_{ij} for the object u_i, denoted as r_i, is calculated using the formula

$$r_i = \sum_{j=1}^{n} c_{ij},$$

where i = 1, 2,…, n. For each j, the sum of c_{ij} for the object u_j, denoted as t_j is calculated using the formula

$$t_j = \sum_{i=1}^{n} c_{ij},$$

where j = 1, 2,…, n. For each i, the score of an object u_i, denoted as S_i, is calculated using the formula $S_i = r_i - t_i$. All the values of r_i, t_i, and S_i for each u_i are presented in a fuzzy soft set score table $(\hat{F}, A)$, as shown in the following table.

Score Table of Fuzzy Soft Set $(\hat{F}, A)$

	r_i	t_i	Skor (S_i)
u_1	r_1	t_1	S_1
u_2	r_2	t_2	S_2
⋮	⋮	⋮	⋮
u_n	r_n	t_n	S_n

Definition 2.8.[1] *Let U be a set of objects and k be a positive integer. A hesitant multi-fuzzy set $\tilde{A}$ of dimension k over U can be expressed as*

$$\tilde{A} = \left\{\left(u, \tilde{h}_{\tilde{A}}(u)\right) | u \in U\right\}$$

where $\tilde{h}_{\tilde{A}}(u) = \left(h^1_{\tilde{A}}(u), h^2_{\tilde{A}}(u), \dots, h^k_{\tilde{A}}(u)\right)$and $h^i_{\tilde{A}}(u)$ represents a hesitant fuzzy element for each $u \in U$, with i=1, 2, … , k. The set of all k-dimensional hesitant multi-fuzzy sets over U is denoted as $HM^kFS(U)$.

Definition 2.9.[1] *Let $U = \{u_1, u_2, \dots, u_n\}$ be a set of objects, $E = \{e_1, e_2, \dots, e_m\}$ be a set of parameters, and $A \subseteq E$. Also, assume that $HM^kFS(U)$ denote the set of all k-dimensional hesitant multi-fuzzy sets over U. A pair $(\tilde{F}, A)$ is called a k-dimensional hesitant MFSS over U, where $\tilde{F}$ is a mapping given by $\tilde{F}: A \to HM^kFS(U)$. A k-dimensional hesitant multi-fuzzy soft set over U can be expressed as a set of ordered pairs:*

$$(\tilde{F}, A) = \left\{\left(e, \tilde{F}(e)\right) | e \in A, \tilde{F}(e) \in HM^kFS(U)\right\}.$$

Definition 2.10.[1] *Let $U = \{u_1, u_2, \dots, u_n\}$ be a set of objects, $E = \{e_1, e_2, \dots, e_m\}$ be a set of parameters, and $A \subseteq E$. Also, assume $(\tilde{F}, A)$ is a 3-dimensional hesitant MFSS over U, where $\tilde{F}$ is a mapping given by $\tilde{F}: A \to HM^3FS(U)$. Let's assume*

$$\tilde{F}(e) = \{u_1/(\{a^1_1, a^1_2, \dots, a^1_{\alpha_1}\}, \{b^1_1, b^1_2, \dots, b^1_{\beta_1}\}, \{c^1_1, c^1_2, \dots, c^1_{\gamma_1}\}), u_2/(\{a^2_1, a^2_2, \dots, a^2_{\alpha_2}\}, \{b^2_1, b^2_2, \dots, b^2_{\beta_2}\}, \{c^2_1, c^2_2, \dots, c^2_{\gamma_2}\}), \dots, u_n/ (\{a^n_1, a^n_2, \dots, a^n_{\alpha_n}\}, \{b^n_1, b^n_2, \dots, b^n_{\beta_n}\}, \{c^n_1, c^n_2, \dots, c^n_{\gamma_n}\})\}.$$

RMSS-level soft set for HMFSS is defined as a function $rmss\left(\tilde{F}(e)\right)$, with $\tilde{F}: \tilde{A} \to \tilde{P}(U)$ for $(\tilde{F}, A)$, $\tilde{A} = \{\tilde{F}(e) | e \in A\}$ and $\tilde{P}(U)$ is the collection of all fuzzy sets over U,

$$rmss\left(\tilde{F}(e)\right) = \left\{u_1/\tfrac{1}{3}\left(\sqrt{\tfrac{1}{\alpha_1}\textstyle\sum_{i=1}^{\alpha_1}(a^1_i)^2} + \sqrt{\tfrac{1}{\beta_1}\sum_{i=1}^{\beta_1}(b^1_i)^2} + \sqrt{\tfrac{1}{\gamma_1}\sum_{i=1}^{\gamma_1}(c^1_i)^2}\right), u_2/\tfrac{1}{3}\left(\sqrt{\tfrac{1}{\alpha_2}\sum_{i=1}^{\alpha_2}(a^2_i)^2} + \sqrt{\tfrac{1}{\beta_2}\sum_{i=1}^{\beta_2}(b^2_i)^2} + \sqrt{\tfrac{1}{\gamma_2}\sum_{i=1}^{\gamma_2}(c^2_i)^2}\right), \dots, u_n/\tfrac{1}{3}\left(\sqrt{\tfrac{1}{\alpha_n}\sum_{i=1}^{\alpha_n}(a^n_i)^2} + \sqrt{\tfrac{1}{\beta_n}\sum_{i=1}^{\beta_n}(b^n_i)^2} + \sqrt{\tfrac{1}{\gamma_n}\sum_{i=1}^{\gamma_n}(c^n_i)^2}\right)\right\}.$$

From the above definition, it can be seen that RMSS-level soft set transforms a hesitant multi-fuzzy soft set into the form of a fuzzy soft set, which can then be represented in a fuzzy soft set table.

In the following, an algorithm for decision-making problems will be presented as an application of the Hesitant Multi-Fuzzy Soft Set concept [1].

Algorithm:

1. Define the set of selected parameters A, $A \subseteq E$;
2. Define a hesitant MFSS $(\tilde{F}, A)$ and represent it in a table form;
3. Calculate the RMS-level soft set for $(\tilde{F}, A)$ and display it in a table form;
4. Compute the comparison table for the RMS-level soft set;
5. Determine the maximum score, if it occurs in row i, then the optimal decision is u_i.

III. Result and Discussion

This section will give a case that can be represented as a form of Hesitant MFSS. Furthermore, referring to the construction of decision-making algorithms, a hesitant multi-fuzzy soft set will be simplified using the RMSS-level soft set method in this case.

In this case, the authors do not explain in detail the technicalities and accuracy of obtaining data because this requires separate and in-depth research involving experts in the field under study. Therefore, the data, in this case, is only illustrative. Furthermore, the case to be examined is the case of selecting exemplary students.

Case. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a set of students under consideration, and $E = \{e_1, e_2, e_3, e_4\}$ be a set of parameters, where e_1 represents "friendliness" with options of less friendly, friendly, and very friendly; e_2 represents "politeness" with options of less polite, polite, and very polite; e_3 represents "intelligence" with options of less intelligent, intelligent, and very intelligent; e_4 represents "attendance" with options of frequent absence, rare absence, and always present.

Assume a teacher is selecting a student based on a desired set of parameters, which is $A = \{e_1, e_3, e_4\}$. Here, the teacher is looking for an exemplary student. Let the hesitant MFSS $(\tilde{F}, A)$ be as follows:

$\tilde{F}(e_1) = \{u_1/(\{0.2,0.4,0.7\}, \{0.5, 0.6\}, \{0.4\}), u_2/(\{0.6, 0.7\}, \{0.3, 0.4, 0.5\}, \{0.2, 0.7\}), u_3/$ $(\{0.1, 0.4, 0.5\}, \{0.3, 0.6\}, \{0.1, 0.5, 0.6\}), u_4/(\{0.2, 0.3\}, \{0.8\}, \{0.3, 0.9\}), u_5/(\{0.4, 0.5\}, \{0.1, 0.2, 0.3\}, \{0.9\})\}$.

$\tilde{F}(e_3) = \{u_1/(\{0.8\}, \{0.2, 0.4, 0.8\}, \{0.1, 0.4, 0.5, 0.7\}), u_2/(\{0.4\}, \{0.3, 0.4, 0.5\}, \{0.3, 0.7, 0.8\}), u_3/$ $(\{0.7, 0.8\}, \{0.3, 0.4, 0.6\}, \{0.2\}), u_4/(\{0.9\}, \{0.2, 0.6, 0.8\}, \{0.3, 0.9\}), u_5/(\{0.3, 0.4, 0.5\}, \{0.1, 0.3, 0.5, 0.8\}, \{0.6, 0.7\})\}$.

$\tilde{F}(e_4) = \{u_1/(\{0.6, 0.7\}, \{0.2\}, \{0.1, 0.3, 0.6, 0.7\}), u_2/(\{0.1, 0.3, 0.4\}, \{0.3, 0.4, 0.5\}, \{0.7\}), u_3/$ $(\{0.2, 0.4, 0.6, 0.8\}, \{0.3, 0.5, 0.6\}, \{0.1, 0.6\}), u_4/(\{0.2, 0.3, 0.6\}, \{0.4, 0.6, 0.7\}, \{0.3, 0.5\}), u_5/$ $(\{0.4, 0.6, 0.7, 0.8\}, \{0.1, 0.2, 0.3\}, \{0.9\})\}$.

Furthermore, the hesitant MFSS can also be displayed in tabular form as follows.

U	Friendliness
u_1	$(\{0.2,0.4,0.7\}, \{0.5, 0.6\}, \{0.4\})$
u_2	$(\{0.6, 0.7\}, \{0.3, 0.4, 0.5\}, \{0.2, 0.7\})$
u_3	$(\{0.1, 0.4, 0.5\}, \{0.3, 0.6\}, \{0.1, 0.5, 0.6\})$
u_4	$(\{0.2, 0.3\}, \{0.8\}, \{0.3, 0.9\})$
u_5	$(\{0.4, 0.5\}, \{0.1, 0.2, 0.3\}, \{0.9\})$

U	Intelligence
u_1	$(\{0.8\}, \{0.2, 0.4, 0.8\}, \{0.1, 0.4, 0.5, 0.7\})$
u_2	$(\{0.4\}, \{0.3, 0.4, 0.5\}, \{0.3, 0.7, 0.8\})$
u_3	$(\{0.7, 0.8\}, \{0.3, 0.4, 0.6\}, \{0.2\})$
u_4	$(\{0.9\}, \{0.2, 0.6, 0.8\}, \{0.3, 0.9\})$
u_5	$(\{0.3, 0.4, 0.5\}, \{0.1, 0.3, 0.5, 0.8\}, \{0.6, 0.7\})$

U	Attendance
u_1	$(\{0.6, 0.7\}, \{0.2\}, \{0.1, 0.3, 0.6, 0.7\})$
u_2	$(\{0.1, 0.3, 0.4\}, \{0.3, 0.4, 0.5\}, \{0.7\})$
u_3	$(\{0.2, 0.4, 0.6, 0.8\}, \{0.3, 0.5, 0.6\}, \{0.1, 0.6\})$
u_4	$(\{0.2, 0.3, 0.6\}, \{0.4, 0.6, 0.7\}, \{0.3, 0.5\})$
u_5	$(\{0.4, 0.6, 0.7, 0.8\}, \{0.1, 0.2, 0.3\}, \{0.9\})$

Next, let's calculate the RMS-level soft set for $(\tilde{F}, A)$.

$rmss\left(\tilde{F}(e_1)\right) = \{u_1/0.477, u_2/0.525, u_3/0.434, u_4/0.575, u_3/0.523\}$.

$rmss\left(\tilde{F}(e_3)\right) = \{u_1/0.602, u_2/0.482, u_3/0.468, u_4/0.720, u_3/0.456\}$.

$rmss\left(\tilde{F}(e_4)\right) = \{u_1/0.446, u_2/0.468, u_3/0.445, u_4/0.466, u_3/0.586\}$.

The result of the RMS-level soft set will be displayed in tabular form as follows.

Table Representation of RMSS-Level Soft Set

U	$\tilde{F}(e_1)$	$\tilde{F}(e_3)$	$\tilde{F}(e_4)$
u_1	0.477	0.602	0.446
u_2	0.525	0.482	0.468
u_3	0.434	0.468	0.445
u_4	0.575	0.720	0.466
u_5	0.523	0.456	0.586

Then, we will calculate the comparison table for the RMS-level soft set based on the table above and display it in the following tabular form.

Comparison Table of RMS-Level Soft Set

U	u_1	u_2	u_3	u_4	u_5
u_1	3	1	3	0	1
u_2	2	3	3	1	2
u_3	0	0	3	0	1
u_4	3	2	3	3	2
u_5	2	1	2	1	3

Finally, we will calculate the number of rows (a), the number of columns (b), and the score = {number of rows(a) - number of columns(b)}, and display the results in the following table.

Score Table of RMS-Level Soft Set

U	Number of rows(a)	Number of columns (b)	Score (a-b)
u_1	8	10	-2
u_2	11	7	4
u_3	4	14	-10
u_4	13	5	8
u_5	9	9	0

From the table above, the maximum score is obtained for u_4. Therefore, a teacher will choose u_4 as the exemplary student.

IV. CONCLUSION

The decision-making process was carried out by following the steps outlined in the algorithm. To demonstrate these steps, a case of selecting an exemplary student based on the chosen parameters was provided, resulting in the decision to choose student u_4 as the best choice for an exemplary student.

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