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Modeling Of A Magnet-Spring System With Damping

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Abstract – This research discusses the modeling of a magnet-spring system by taking into account the damping factor. This magnet-spring model represents the movement of a magnet suspended from the ceiling at the bottom of a spring, and directly below it, there is another identical magnet permanently fixed on the floor. The modeling of this system is a continuation of the study by Fay and Med (2005), which discussed the same system but ignored the damping factor.

Keywords – magnet-spring, damping, Newton's Second Law

I. Introduction

One of the common topics discussed in mathematical modeling is the oscillatory motion of a mass hanging on a spring, known as a mass-spring system [4]. The oscillatory motion produced by the system is modeled in the form of a second-order ordinary differential equation by applying Newton's Second Law. The forces acting on the mass-spring system include the mass, the spring restoring force following Hooke's Law, the damping force, and the external forces [1].

The mass-spring system can be applied, e.g., in suspension systems of motorcycles, cars, trains, and other vehicles [4]. The suspension can reduce shocks or vibrations in the vehicle due to uneven road surfaces. As a result, it can increase driving comfort. Several studies related to the spring-mass system have been conducted. For example, Gilberto Flores [3] discussed the dynamical instability of the spring-mass system in MEMS (Micro Electro Mechanical Systems) devices. In addition, Mingzhao Zhuoa et al. [7] examined the analytical solution of the mass-spring system containing the effects of nonlinearity and hysteresis.

In its development, the study of the mass-spring system is modified by replacing the mass with a magnet, known as the magnet-spring system. Some researchers have studied this magnet-spring system. Fay and Mead [2] investigated the effect of a magnet, as a mass attached to the bottom of a spring, on another identical magnet that is permanently mounted on the floor. Furthermore, Fay and Mead explained the stability of the system without considering damping factors. The study by Fay and Mead was then continued by Jannah et al. [5] to determine the interval value of the spring constant that has physical meaning in the case of attracting magnetic force. This paper will extend the modeling of the magnet-spring system by adding a damping factor, making it more realistic to implement.

II. Newton's Second Law And Magnetic Force

Newton's Second Law states that the acceleration of an object is directly proportional to the total force acting on it and inversely proportional to its mass. The direction of acceleration is the same as the direction of the total force acting on it. Newton's Second Law is formulated as follows [4]:

$$\sum F = ma,$$

where $\sum F$ represents the net force (N), m indicates the mass (kg), and a means the acceleration (m/s^2). The object will increase its speed if the direction of motion of the object is the same as the direction of the total force. However, the object will slow down or even stop if the direction of motion of the object is opposite to the direction of the total force. The change in velocity experienced by the object results in acceleration. In other words, acceleration is caused by the total force given to the object.

The main object of the spring-magnet system in this study is the magnet itself, either as a mass or permanently mounted on the floor. A magnet is an object that can attract a certain object, such as iron or steel, that is near it. Each magnet consists of two parts that have the strongest attraction force. In a bar magnet, the strongest attraction force is located at the ends of the magnet. The part of the magnet with the strongest attraction force is called the magnetic pole. Therefore, each magnet has two poles: the north pole and the south pole. If the north pole and south pole are brought close together, there will be an attraction between the magnets, whereas if the north pole is brought close to the north pole, there will be a repulsion. Similarly, if the south pole is brought close to the south pole, there will also be a repulsion. In other words, if two magnets with the same pole are brought close together, they will repel each other, whereas two magnets with opposite poles will attract each other.

A magnet, for example a bar magnet, will produce a magnetic field around it. The direction of the magnetic lines (B) is from the north pole (N) to the south pole (S). The further an object is from the magnet, the faster the magnetic force disappears. The magnetic field always depends on the lines of force. The denser the lines of force, the greater the magnetic field [4]. The interaction between magnetic poles produces a force called the magnetic force. The magnetic force between two magnets can be obtained using a vector derivative approach. Let z denote the distance between the magnetic poles. According to [6], the magnetic force between two magnetic poles is proportional to $\pm\frac{1}{z^4}$, where the positive sign indicates the case of attraction, while the negative sign indicates the case of repulsion.

III. Result And Discussion

In this section, a spring-magnet model will be constructed, taking into account damping factors. First, consider the following spring-mass model. Suppose a spring with length l is vertically suspended in a static position [see Figure 1(a)]. Next, a mass m is applied to the bottom of the spring. As a result, the spring elongates by L in the downward vertical direction (having a positive value) [see Figure 1(b)]. In this condition, two forces are acting on the system. The first force is the weight force given by $W = mg$, where g represents the gravitational acceleration acting vertically downward. Secondly, the restoring force of the spring F_s acts in the upward vertical direction. According to Hooke's Law, the spring's restoring force is proportional to L [4]. Therefore, this force can be denoted as $F_s = -kl$, where $k > 0$ is the spring constant and the negative sign indicates that the force acts in the upward vertical direction. In equilibrium, these two forces are equal, which can be expressed as $mg - kL = 0$.

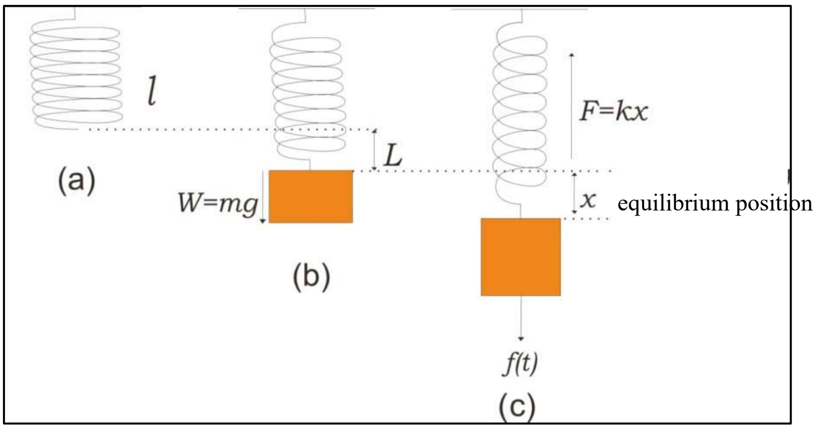

Figure 1. Illustration of a spring-mass system [1].

If the mass is subjected to an external force, it will move from equilibrium [see Figure 1(c)]. Let x(t) denotes the displacement of the mass from the equilibrium position at time t. In this case, a positive value of x(t) indicates the displacement in the downward vertical direction. In contrast, a negative value indicates the displacement in the upward vertical direction. Applying Newton's Second Law to this system yields [1]:

$$m\ddot{x} = f(t), \tag{1}$$

where $\ddot{x}$ represents the acceleration experienced by the mass, and $f(t)$ is the net force acting on the mass.

In determining $f(t)$, four forces need to be considered, they are[1]:

1. The weight W

   The weight of the mass hanging on the spring is equal to $W = mg$, acting in the downward vertical direction.

2. The restoring force of the spring $F_s(t)$

   The restoring force of the spring $F_s(t)$ is assumed to be proportional to the total elongation of the spring $L + x$ and acts to restore the spring to its original position. If $L + x > 0$, then the spring is stretched, and the restoring force of the spring acts in the vertical upward direction. In this case, the restoring force of the spring is expressed as follows :

$$F_s(t) = -k(L + x).$$

3. Damping force $F_d(t)$

   Damping force $F_d(t)$ acts in the opposite direction of the motion of the mass. This force can be caused by various sources, such as air resistance or other media in which the mass is moving, or friction between the mass and the surface (if any). It is assumed that the damping force is proportional to the velocity $|\dot{x}|$. If $\dot{x} > 0$ increases, the mass will move vertically downward. As a result, $F_d(t)$ has a vertically upward direction, and its value is given by

   $F_d(t) = -\gamma\dot{x}$,

   where $\gamma > 0$ is the damping constant.

4. The external force $F(t)$

   The external force $F(t)$ is applied to the mass in the upward ($F(t)$ negative) or downward ($F(t)$ positive) vertical direction. This external force can be caused by the movement of the spring attachment point, the influence of a magnet, electricity, and others.

The equation of Newton's Second Law is then rewritten by considering all the forces described above, resulting in the following:

$$\begin{aligned} m\ddot{x} &= W + Fs(t) + Fd(t) + F(t) \\ &= mg - k(L + x) - \gamma\dot{x} + F(t) \\ &= (mg - kL) - kx - \gamma\dot{x} + F(t). \end{aligned} \tag{2}$$

In the equilibrium state, it holds $mg - kL = 0$, so equation (2) becomes

$$m\ddot{x} + \gamma\dot{x} + kx = F(t). \tag{3}$$

This paper examines the condition where the mass hanging on the spring is a magnet. Then, another identical magnet is permanently attached to the floor beneath the mass (see Figure 2). It is assumed that the external force acting on this system only comes from the interaction between these two identical magnets.

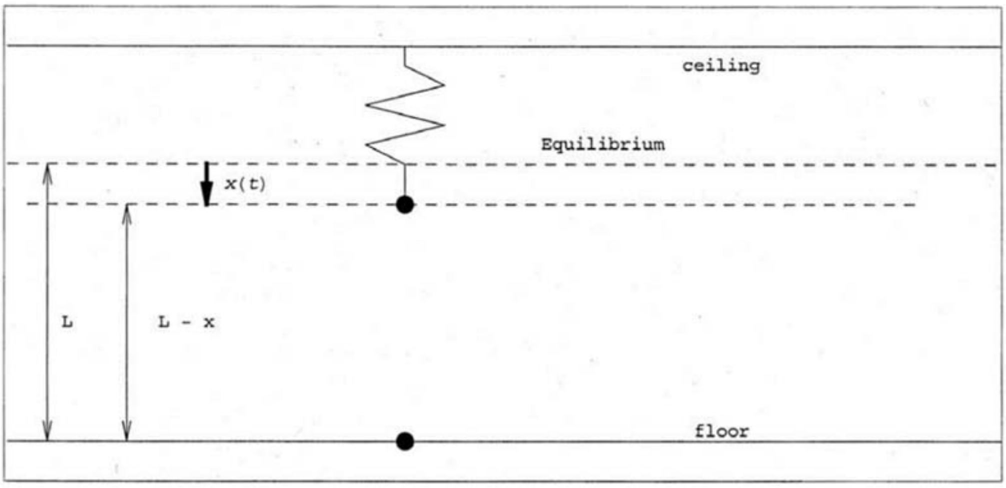

Figure 2. Illustration of a spring-magnet system [2].

From Figure 2, L represents the distance between the equilibrium position and the magnet position on the floor, and x(t) represents the displacement of the magnet from the equilibrium state. It is known that the magnetic force between two magnets is proportional to $\pm\frac{1}{z^4}$, where z is the distance between the two magnets [6]. In this case, the positive sign indicates the case of attractive force on the magnet due to the opposite pole of the magnet as the mass facing the different pole of the magnet on the floor. Conversely, the negative sign indicates the case of repulsive force on the magnet due to the same pole of the magnet as the mass facing the same pole of the magnet on the floor. For the case of the spring-magnet system, as shown in Figure 2, the distance between the two magnets is $\pm\frac{1}{z^4}$. Therefore, the external force F(t) is proportional to $\pm\frac{1}{(L-x)^4}$, so the spring-magnet system can be modeled as follows:

$$m\ddot{x} + \gamma\dot{x} + kx = \pm\frac{1}{(L - x)^4}.$$

IV. CONCLUSION

In this paper, a magnet-spring model has been constructed by taking into account the damping factorThe model describes the motion of a magnet, which is treated as a mass attached to the bottom of a spring, and is influenced by another identical magnet that is permanently mounted on the floor. The modeling is based on the mass-spring system by applying Newton's second law and considering the forces acting on the system. The external force in this system is the magnetic force that is inversely proportional to the distance between two magnets.

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