International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

Vol. 37 No. 1 February 2023, pp. 170-177

Synthesis Of Four-Bar Linkages For Grippers With Application Of The Theory Of Stationary Curvature

Vitan Galabov[1], Roumen Roussev[2], Blagoyka Paleva-Kadiyska[3],

[1]Technical University
Sofia, Bulgaria
[2]Faculty of Technics and Technology
Trakia University
Yambol, Bulgaria
[3]Mechanical Engineering and Technologies
South-West University "Neofit Rilski"
Blagoevgrad, Bulgaria
[1]vitangalabov@abv.bg
[2]roussev_r@abv.bg
[3]paleva-kadiyska.bl@abv.bg

Abstract – A new computer-applied linear mathematical model for the synthesis of grippers with four-bar linkages is derived, applying the theory of cubic of stationary curvature and in particular the Carter-Hall circle to generate Burmester curves. The model includes and a condition for achieving a certain value of the pressure angle, whereupon is uniquely defined the kinematic diagram of the mechanism. The mathematical model is illustrated by two examples for the synthesis of the mechanisms of two grippers with practical application. The first refers to the synthesis of a mechanism with an asymmetric opening of the jaws, and the second - to the gripper for transferring plates from one position to another in various technologies in microelectronics. The proposed new model significantly eases the work of engineers in the synthesis of four-link mechanisms, which are very often found in modern devices, such as e.g. some grippers for robots in microelectronics. The new method avoids the non-linear mathematical models for synthesis, to which leads by the known methods of synthesis by infinitely close positions.

Keywords – Carter-Hall circle, Burmester curves, cubic of stationary curvature, pressure angle, synthesis of four-bar linkages

I. Introduction

Grippers are end effectors of robots, manipulators, and other technical devices. The grippers convert input motion and input force into required output motion and gripping force. They are rarely optimized so that the change in the gripping force follows a corresponding change in the parameters of the manipulated object. Different types of grippers and conditions required for their design have been studied [1-4]. Various approxsimation methods (Leasts Square, MiniMax, Maximally Contracted Evolution Method, etc.) have been used for the synthesis of grippers with higher kinematic pairs when the gripping force changes in accordance with the change in the parameters of the manipulated objects [5-8].

When changing the gripping force in the close vicinity of the design position of the gripping levers, synthesis methods for infinitely close positions are more suitable, for example with the corresponding Burmester curve for infinitely close four positions (cubic of stationary curvature) [9-11] or the *Carter-Hall circle* [12-16]. These methods lead to non-linear mathematical models for synthesis. They are difficult to implement in routine engineering practice.

With this article, the authors set themselves the task of easing the work of engineers in the synthesis of four-link mechanisms, which are very often found in modern devices, such as e.g. some grippers for robots in microelectronics.

The aim is to develop a computer-applied linear mathematical model for the synthesis of four-bar mechanisms for grippers, applying the theory of stationary curvature, in particular Burmester curves, generated by the Carter-Hall circle. Another goal is to unambiguously determine the kinematic diagram of the mechanism, fulfilling an additional condition for a certain value of the pressure angle [17].

II. Mathematical Model For Determining Burmester Curves

In this section is presented the a computer-applicable linear mathematical model for the synthesis of grippers with four-bar linkages is derived, applying the theory of cubic of stationary curvature and in particular the Carter-Hall circle to generate Burmester curves.

2.1. Mathematical Model for Determining Burmester Curves

When rotating the input and output links, a coordinate system is Oxy with an axis $x \equiv OC$ is used, which determines the position of the fixed link with length $L = l_{OC}$. The values of the geometric transfer functions up to the third sequence are stated $\psi'(\varphi)$, $\psi''(\varphi)$ и $\psi'''(\varphi)$. The kinematic invariants are determined (Fig. 1):

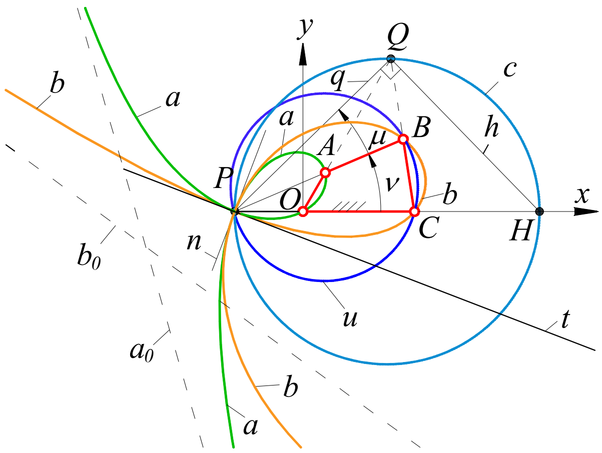

Fig. 1. Burmester curves ***a*** and ***b*** with asymptotes ***a0*** and ***b0***, respectively, obtained by means of a Carter-Hall circle ***c***. Four-bar linkage, synthesized at a given (definite) angle *ν*

- the abscissa of the relative instantaneous center of velocity (ICV) P (according to the Aronhold-Kennedy and of Willis theorems

$$x_P = L\psi' / (\psi' - 1)\ ; \quad (y_P = 0)\,, \tag{1}$$

located on the Ox axis;

- the angle μ, locked between the direction of the connecting rod (straight BP) and the collineation axis $q \equiv PQ$ according to the well-known of Bobillier (1970) and of Freudenstein theorems [18, 14] (Q is the absolute ICV)

$$tg\,\mu = \psi'(1-\psi')/\psi''\,; \tag{2}$$

- the abscissa of the intersection point H of the of the Carter-Hall circle c (with a diameter $d_c \equiv l_{PH} = |x_H - x_P|$) with an Ox axis [15, 16]

$$x_H = x_P + \frac{3L[\psi'^2(1-\psi')^2 + \psi''^2]}{[\psi''' + \psi'(1-\psi'^2)](1-\psi')^2 + 3\psi''^2(1-\psi')}. \quad (3)$$

The mathematical model includes only the abscissa x_H of (3) and the equation of lines. The model does not include the equation of the Carter-Hall circle c itself. A suitable variable parameter is the angular coefficient k_v:

$$k_\nu = tg\nu , \quad (4)$$

of the line determining the position of the connecting rod.

The angular coefficient of the collinearity axis q is

$$k_q = tg(\nu + \mu) , \quad (5)$$

where the angle μ is determined by (2).

Burmester curves are obtained entirely in the range $v \in [0, 180]$. At a definite value of v, the coordinates are determined of the centers A and B of the movable joints.

$$x_A = k_\nu x_P / (k_\nu - k_{OQ}); y_A = k_{OQ} x_A , \quad (6)$$

$$x_B = (k_{CQ} L - k_\nu x_P) / (k_{CQ} - k_\nu); y_B = k_{CQ}(x_B - L), \quad (7)$$

The centers A and B are the intersections of a line drawn through the center P at an angle v, respectively, with the line OQ and CQ. In the (6) and (7) are included:

- the angular coefficient of the straight OQ

$$k_{OQ} = y_Q / x_Q , \quad (8)$$

where: $x_Q = \left(k_q x_P + k_q^{-1} x_H\right) / \left(k_q + k_q^{-1}\right)$,

$y_Q = k_q \left(x_Q - x_P\right)$.

- the angular coefficient of the straight CQ

$$k_{CQ} = y_Q / (x_Q - L) . \quad (9)$$

A kinematic diagram of a four-bar linkage $OABC$ is obtained after determining the center B as the intersection point of the curve b with the circle u. The circle $\boldsymbol{u}$ is the geometric location of points from which the segment $\overline{PB}$ is visible at the transmission angle $\gamma = 90^o + |\theta|$, where θ is the pressure angle. The line PB intersects the curve a in the center A and makes an angle v with the axis x. Then (4) determines the project parameter $k_v = tgv$ required for the synthesis.

2.2. Special Case

A special case in which $\psi' = const$, $\psi'' = \psi''' = 0$. Then from (1), (2) and (3) is obtained

$$x_P = \psi' = const, \ \mu = 90°, \quad x_H = x_P + 3L\psi' / (1 - \psi'^2) . \quad (10)$$

The Burmester curves disintegrated to (until) the polar normal $n \equiv Ox$ and two circles a and b, symmetrically located with respect to the Ox axis. The circles have diameters, respectively

$$d_a = \left| \frac{3L\psi'}{(1-\psi')(2-\psi')} \right|, \ d_b = \left| \frac{3L\psi'}{(1-\psi')(2\psi'-1)} \right|. \quad (11)$$

III. Synthesis Of A Mechanism For Asymmetric Opening Of The Gripper's Jaws

In this section, through two examples, an exemplary application of the developed mathematical model is shown - synthesis of a mechanism for asymmetric opening of the gripper's jaws and synthesis of a gripper for transferring plates from one position to another with different technologies in microelectronics.

For synthesis, the programs MathCad (for the mathematical model) and AutoCAD (for visualizing the obtained results) were used.

3.1. Synthesis of a mechanism for asymmetric opening of the gripper's jaws

Asymmetrical opening of the jaws can be achieved by connecting the two gripping's levers with a connecting rod. A four-bar mechanism OABC (Fig. 2) is formed, which is synthesized under the condition $\psi' = const$, $\psi'' = \psi''' = 0$. When $\psi' = -0.5$, L = 45mm, from (10) is obtained xP = 15, μ = 90º, xH = -75. Burmester curves disintegrate at the polar normal $n \equiv Ox$ and into symmetrically Ox axis located circles a and b with diameters da = 18mm, db = 22.5mm. The circles touch the center P (relative instantaneous center of velocity) and intersect the Ox axis at points with ordinates xa = - 3 and xb = 37.5.

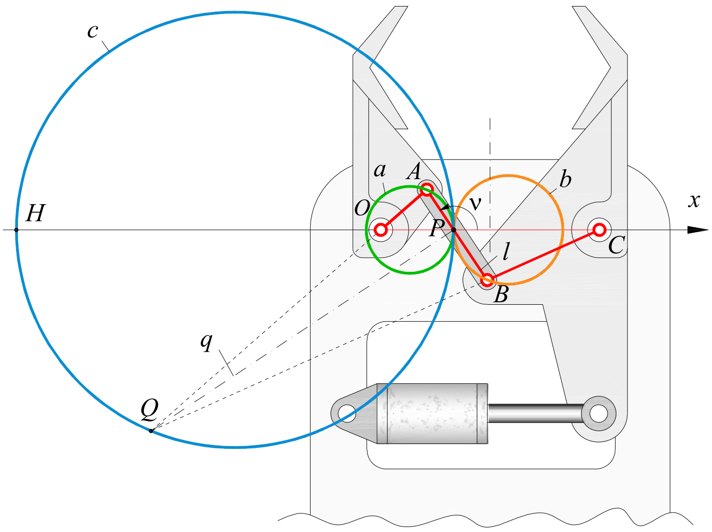

Fig. 2. Four-bar linkage *OABC* for asymmetric opening of the gripper jaws

3.2. Gripper for transferring plates from one position to another with different technologies in microelectronics

One of the main requirements for these grippers is to provide and perform supple, shock-free and smooth stopping movement of the jaws in their working position (Fig. 3) when gripping the plates. The gripper consists of a gripping mechanism constructed by levers *2* and *3* movably connected to the connecting rod *1*. Separator-shaped blocks (jaws) for gripping the plates and a transmission mechanism with a higher kinematic pair (inverse cam mechanism) are attached to them. The cam mechanism is reduced to a conditionally kinematically equivalent four-bar linkage *OABC*. The levers *2* and *3* are opened symmetrically thanks to the four-bar linkage *CDEF* formed by the frame *0* and the links *2* and *3* linked to the connecting rod *1* (Figure 3b).

The requirement for smooth and shock-free access of the jaws to the plates is reduced to three main conditions:

(a) $\psi' = 0$ - takes value of 0 of the velocity $\dot{\varphi}$ of the levers;

(b) $M_\psi = 0$ - takes value of 0 of the reduced moment M_ψ of inertial forces to the axis of link *3*;

(c) $M_\psi' = 0$ - takes value of 0 of the derivative M_ψ' of the reduced moment M_ψ.

Condition (a) is fulfilled when the input angular velocity $\dot{\varphi}$ and the first transfer function $\psi' = d\psi / d\varphi = \dot{\psi} / \dot{\varphi}$ are reset individually or together because $\dot{\psi} = \dot{\varphi}\psi'$. If only $\psi' = 0$, the functional generator mechanism is dead-center position, but the reduced mass moment of inertia $I_\psi = I_c\psi'^{-2} + m_c l_{OS_c}^2 + I_2 + I_3 + m_2 l_{OS_2}^2 + m_3 l_{OS_3}^2$ relative to the C axis theoretically increases infinitely. This leads to a corresponding infinite increase in the moment of inertial forces M_ψ = 0 even at small angular acceleration values $\ddot{\psi}$. This provokes (generates, gives rise to,) a correspondingly infinite increase in the moment of the inertial forces M_ψ = 0 even at small angular acceleration values $\ddot{\psi}$. Infinitely close to the operating position of the mechanism, a large inertial load and corresponding vibrations of the gripper links will occur. In the infinitesimal neighborhood of the working position of the mechanism will be a large inertial load and corresponding vibrations of the gripper links. Therefore, condition (a) is best met if $\dot{\varphi} = 0$ at $\psi' \neq 0$.

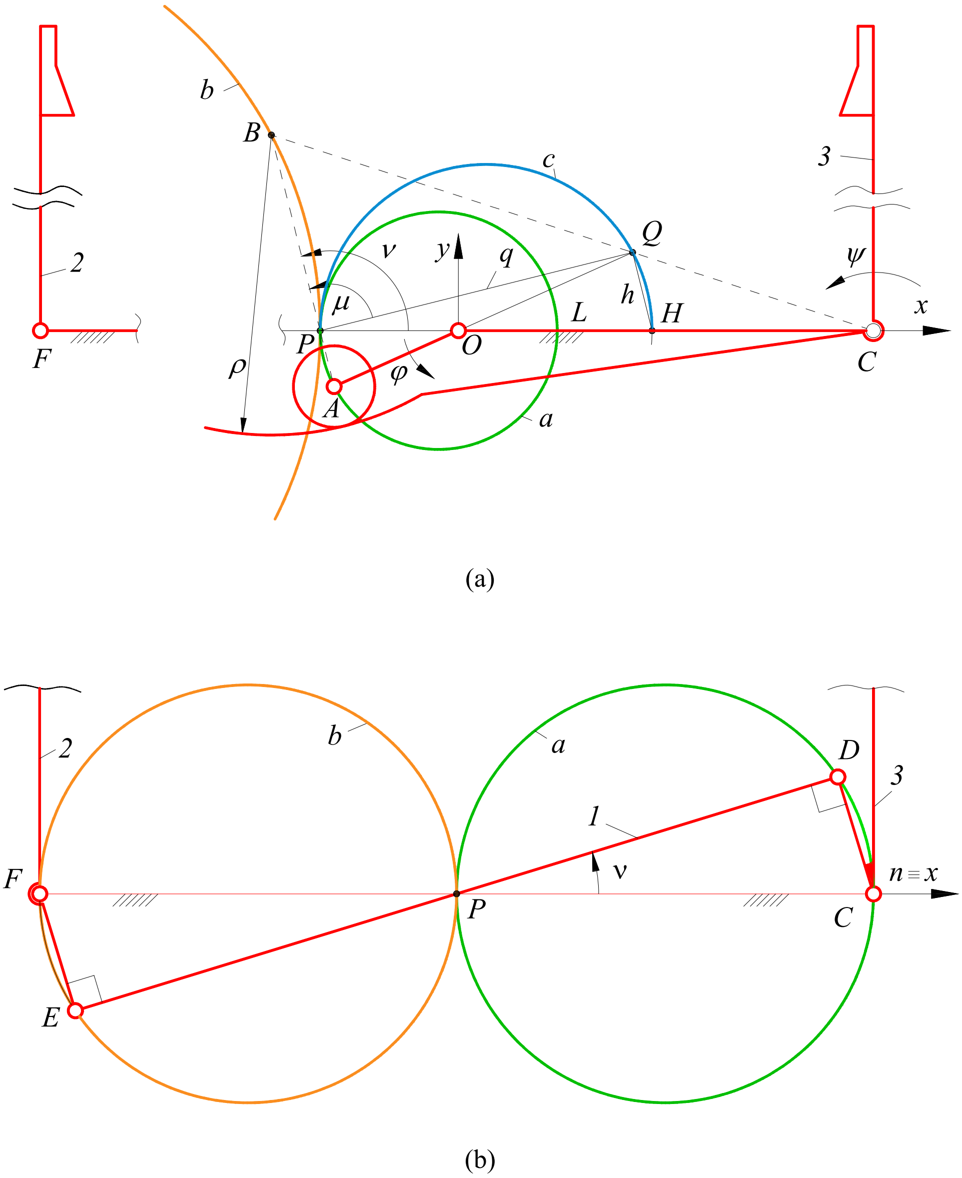

Fig. 3. Plate gripper for microelectronics and graphic constructions related to the synthesis of:

(a) gripping mechanism; (b) a functional generator mechanism connecting the levers 2 and 3

The entered notations are: I_c and m_c - mass moment of inertia and mass of the crank *OA* together with the roller; I_2, I_3 and m_2, m_3 - mass moments of inertia and masses of links *2* and *3* to the axes of which the mass of link *1* is concentrated.

Condition (b) $M_\psi = 0$ will be fulfilled if $\ddot{\psi} = \dot{\varphi}^2\psi'' + \ddot{\varphi}\psi' = 0$. Then the input angular velocity $\dot{\varphi} = 0$ and the input angular acceleration $\ddot{\varphi} = d^2\varphi / dt^2 = 0$. Smoother stopping of the jaws will be achieved if the condition is additionally met $\psi'' = 0$.

Condition (c) $M_\psi' = 0$ will be executed successfully if both addends to $M'_\psi = I'_\psi\psi'' + I_\psi\dddot{\psi} / \dot{\varphi}$ are with value of 0. Obviously, this is possible when $\psi'' = 0$ and $\dddot{\psi} = 0$. The condition $\dddot{\psi} = 0$ is fulfilled if the three addends of the derivative of the output angular acceleration $\dddot{\psi} = \dot{\varphi}^3\psi''' + 3\dot{\varphi}\ddot{\varphi}\psi'' + \dddot{\varphi}\psi'$ are with value of 0. This is possible if $\dot{\varphi} = \ddot{\varphi} = \dddot{\varphi} = 0$. The change of M_ψ' infinitely close to the working position of the gripper will be smaller if the derivative $I'_\psi = -2I_1\psi''\psi'^{-3}$ is reset, which is possible if again $\psi'' = 0$ and $\psi' \neq 0$. The deviation of M_ψ' from zero in the immediate vicinity the working position of the mechanism will be less if additional $\psi''' = 0$.

From the general conditions (a), (b) and (c) we come to the specific conditions:

(d) $\dot{\varphi} = \ddot{\varphi} = \dddot{\varphi} = 0$

(e) $\psi' \neq 0,\ \psi'' = 0,\ \psi''' = 0$.

The ***conditions (d)*** require a law of motion of the input unit where, at the moment of stopping, the first three derivatives of φ with respect to time t are with value 0 simultaneously. This means that the function $\varphi(t)$ needs to be a polynomial of exponent four or higher, or another trigonometric or power-trigonometric polynomial. This polynomial must satisfy conditions (d) [19] at the moment of reaching the working position of the gripper, in which the jaws grip the plates.

Conditions (e) can be fulfilled by an oscillating roller follower cam mechanism, which corresponds to a conditionally kinematically equivalent four-bar linkage *OABC* (Fig. 3b).

Initial data for the synthesis of the kinematic diagram of the functional generator mechanism of the gripper: $L = \overline{OC} = 51$, $\psi' = 0.25$, $\psi'' = \psi''' = 0$.

From (10) is obtained $x_P = -17$, $\mu = 90°$, $x_H = 23.8$ ($d_c = 40.8$). The Burmester curves disintegrated to polar normal $n \equiv Ox$ and two circles ***a*** and ***b***, symmetrically located with respect to the *Ox* axis. The circles have diameters, respectively $d_a = 29.143mm$, $d_b = 102mm$.

The circles are tangent to the center *P* and their centers are on the axis *Ox* at points with ordinates, respectively $x_a = 12.143$ and $x_b = -119$.

At $k_v = -4$ (value calculated at $|\theta| \approx 10°$) the coordinates $x_A = -15.286$ and $y_A = -6.857$ are obtained from the intersection of lines through the center *P* respectively with the circles ***a*** and ***b***. The use of the equations of the circles ***a*** and ***b*** can be avoided if (4) to (9) of the linear mathematical model for synthesis are applied.

These results make it possible to synthesize two kinematically equivalent mechanisms:

- four-bar linkage in the design position of the crank *OA*, determined by the angle $\varphi = arctg(k_{OQ}) = 24.161°$, with parameters: $l_{OA} = 16.754$, $l_{AB} = 31.807$, $l_{BC} = 77.784$;

- inverse cam mechanism with parameters $l_{OA} = 16.754$ and $\rho = 36.807$ also for roller radius $r = 5$. This mechanism was preferred due to smaller dimensions and gaps.

The four-bar mechanism *CDEF* (Fig. 3b) was synthesized under conditions $L = \overline{CF} = 120$, $\psi' = -1, \psi'' = \psi''' = 0$, which provides symmetrical opening of the jaws of the gripper in a limited interval. The Burmester curves disintegrate into the polar normal and into two circles ***a*** and ***b***, mirror situated with respect to the polar tangent *t*.

The diameters of the circles are $d_a = d_b = L / 2$. At a definite value of the angle $\nu = 20°$, the dimensions are obtained: $l_{CD} = l_{EF} = 0.5L\sin\nu = 20.521$ and $l_{DE} = L\cos\nu = 112.763$.

IV. Conclusion

The derived new computer-applied linear mathematical model would facilitate the engineers in the synthesis of four-bar linkages for grippers and other devices in the proximity of a given project position of the gripping levers. The model is based on the *theory of stationary curvature* (cubic of stationary curvature) and in particular on the *Carter-Hall circle* for generating *Burmester curves*. The model also includes a condition for achieving a certain value of the pressure angle, whereupon is uniquely determined the kinematic diagram of the mechanism.

The use of the mathematical model is illustrated by two examples for the synthesis of the mechanisms of two grippers with practical application. The first gripper refers to the synthesis of a mechanism with asymmetric opening of the jaws, and the second - to the gripper for transferring plates from one position to another in various technologies in microelectronics.

References

[1] Galabov V., Mimmi G., Rovetta A. Sulle caratteristiche funzionali di sistemi meccanici dipresa in robots industriali. III Congr. nat. dell' Assoc. Italiana di mecc. teor. e appl., Cagliari, Italia, 1976, Sez. III 18.1 - 18.12.

[2] Konstantinov M., Galabov W., Kriterien zum Entwurf von Greifermechanismen fuer Manipulatoren und Industrieroboter, Maschinenbautechnik, vol.27(12), 1978, pp. 532-536.

[3] Chen F. Y., Force Analysis and Design Considerations of Grippers, The Industrial Robots, Dec., 1982, pp. 243-249.

[4] Kato I., Sadamto K., Mechanical Hands Illustrated (Revised Edition). Hemisphere Publishing Corporation, W, New York, L., 216 p., 1985.

[5] Slavov G. Stoyanova Ya. Galabov V. Synthesis of a gripper with parabolic change of the gripping force. Mechanics of Machines, No. 82, 2009, pp. 36-40.

[6] Stoyanova J. Approximate synthesis of a gripper with parabolic change of the gripping force by means of method of least squares and minimax method. Mechanics of Machines, No. 82, 2009, pp. 18-22.

[7] Stoyanova J. Approximate synthesis of a gripper with parabolic change of the gripping force by least squares method. Mechanics of Machines, No. 82, 2009, pp. 23-26.

[8] Stoyanova J. Approximate synthesis of a gripper with parabolic change of the gripping force by the method of maximally contracted evolute. Mechanics of Machines, No. 82, 2009, pp. 27-31.

[9] Roth B., Yang A. T., The Application of the Instantaneous Invariants to the Analysis and Synthesis of Mechanisms, Journal of Engineering for Industry, vol. 99 B , No. 1, 1977, pp. 97-103.

[10] Dijksman E.A., & Herve J. M. Instantaneous four-bar kinematics based on the Carter-Hall circle. Proceedings of the first international applied mechanical systems design conference: with tutorial workshops; June 11-14, 1989, Nashville, Tennesee, Technological University, vol.1, pp.P1.1-P1.12.

[11] Uicker J.J., Pennock G.R., Shigley J.E., "Theory of Machines and Mechanisms", Fifth Edition, Oxford University Press, New York, 950 p., 2017.

[12] Carter W. J., Kinematic Analysis and Synthesis Using Colleneation-Axis Equation, Transactions of the American Society of Mechanical Engineers (Trans. ASME), vol. 79, No. 6, pp.1305-1312, 1957, with discussion by A. S. Hall, Jr.

[13] Lakshminarayana K., On the Carter-Hall Circle and its Application, Journal Mechanisms, vol. 6, 1971, pp. 517-532.

[14] Hirschorn J., Kinematics and Dinamics of Plane Mecahnisms, McGraw - Hill, New York, 365 p., 1972,.

[15] Galabov V., Synthesis of mechanisms in robotics. TU - Sofia, 264 p., 1992.

[16] Galabov V., Structural-metric synthesis of mechanisms, Dissertation for DSc, Technical University - Sofia, 491 p., 1998.

[17] Galabov V., Andonov I., Determining the area of the initial position of the output link in the synthesis of cam, belt and lever mechanisms. Part I - Mechanisms with rotation of the input and oscillating roller follower at the output. Mechanics of Machines, No. 105, 2014, pp. 69-75.

[18] Freudenstein F., On the Maximum and Minimum Velocities and Accelerations in Four-Link Mechanisms, Transactions of the American Society of Mechanical Engineers (Trans. ASME), 78, No. 4, 1956, pp. 779-787.

[19] Galabov V., Roussev R., Paleva-Kadiyska Bl., Synthesis of Cam Mechanisms - I, Monograph, New Man Publishing House, ISBN 978-954-407-506-4, 2020.