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Modeling of Flow Behavior Through Porous Media by Fractal Geometry and Fractional Calculus

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Abstract
Introduction:The use of models in the equations of transport phenomena in the flow of fluids in porous beds, is made difficult due to the highly complex nature of the geometry of the systems of the links, in which the liquid moves through the irregular channels formed between the solid particles that form the bed. In this context, the deterministic models applied to these systems are based on the determination of the empirical constants, which considers these factors that are not considered explicitly.
Method: In this work, it is presented a model that describes the flow behavior as a function of the bed dimensions and the properties of the fluid as laminar regime, whose obtention is based on a formalism in which they are used together the equations of conservancy of motion quantity, fractal geometry, and differential fractional calculus.
Results: The model obtained is equal to that proposed by Blake and Kozeny, differentiated by the fact that all the parameters are theoretically determined from the images that show the real morphology of the porous bed.
Discussion orConclusion: Results show that the model proposed predicts, for the same pressure gradient, an increase in velocity profile as the diameter particles and porosity increase, and a decrease of the velocity as the fractal dimension increase. These predictions correspond each other in magnitude order and tendency with those estimated by the Blake and Kozeny model.

***Keywords* - Flow In Porous Beds; Flow Behavior; Pressure Drop; Fractal Dimension; Porous Morphology.**

I. INTRODUCTION

The flow of fluids through porous media is a process that involves several unit operations related to some industrial systems such as continuous adsorption, heterogeneous catalysis, filtration, and ionic exchange [1]. This process is also observed in the petroleum industry, mainly in the extraction of crude from porous deposits or even in the transportation of underground water [2].

When particles are present in the bed, the shear strength on their walls produces a significant pressure drop. This is the reason why the flow behavior in the system for a focused force is much lower than that reached in a pipeline. Modeling of these kinds of systems is based on two fundamental theories [3]: the first one considers that the fluid pass through immersed particles and pressure drop due to friction is determined by summing the resistance of every single particle. The second one visualizes the system as a conjunction of random-shaped channels and uses the transport phenomena basic equations to describe the flow through these channels. This last one is relatively simpler and more used, being the Blake-Kozeny model the most applied to the laminar regime [4]:

$$V_0 = K\frac{\Phi}{\mu}\varepsilon^3\frac{d^2}{(\varepsilon-1)^2} \qquad (1)$$

Where $V_0$is the flow velocity (ms^{-1}) determinedas the flow per surface unit of the transversal section of the bed, d (m) is the average diameter of the particles, εis the bed porosity,µis the fluid viscosity (Pas), Φis the pressure gradient (Pam^{-1}), and$K$is a non-dimensional constant that involves the irregular geometry of the system and which value is experimentally determined. Because of the lack of experimental data, Blake-Kozeny model propose:

$$K = 1.3333\times10^{-2}$$

That is one the limitations of the model. To overcome this issue, in the present work it is proposed a model based on the movement of the flow through channels with a morphology studied under fractal geometry, applying the transport phenomena equations and fractional calculus to predict the velocity behavior with respect to pressure gradient for laminar regime.

II. Method

1.1. Considerations set up to obtain the model.

The model proposed was obtained accordingly to the next conditions: 1) the area of flow is formed by a group of circular pores with irregular perimeter, which is characterized for an average radius value R, and a fractal dimension f; 2) the regime flow is laminar; 3) the system is in steady state, and 4) the fluid is Newtonian with constant density. The fractal dimension of such channels is obtained by the direct observation of the bed morphology in an equivalent size to the radius of the particles that constitute the bed, for which, a picture of the bed is analyzed by image treatment software. In this sense, even when all the channels have different shapes and sizes, it is considered an average value for both R and f, that are parameters depending on a plenty of factors (particles shape and size, particle bulk, bed porosity).

1.2. Bedmorphologyanalysis.

Figure 1 shows the pictures of two different porous beds, one constituted by naturally rounded stones (A) and the other by mechanically triturated stones (B). For each image, the red square indicates the work area for the pore morphology characterization[5]. Figure 2 shows the corresponding images to the pore contours from where porosity and fractal dimension values were determined by ImageJ software[6] [7].

Due to the pore perimeter has an irregular morphology, the pore surface value is determined through the fractional integral of order equal to the fractal dimension:

$$a = k^{1-f}\int_0^{2\pi}\mathbf{D}_r^{-f}[r]\,d\theta \qquad (2)$$

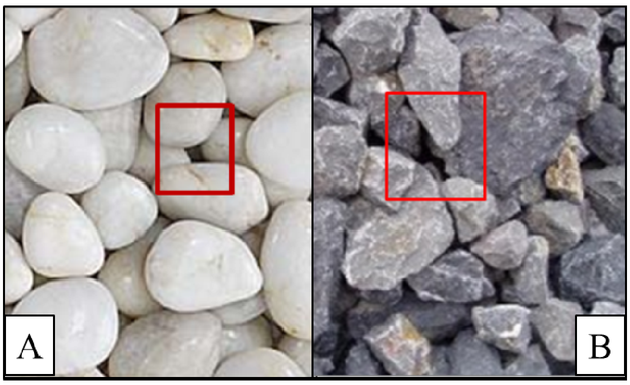


Figure 1. Images of the particles beds: A)rounded particles, and B)triturated particles.

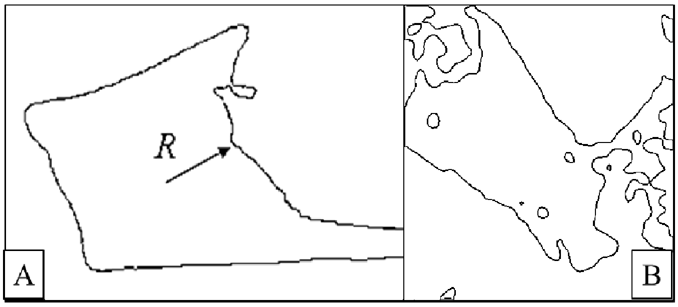


Figure 2. Pore morphology of a polished particles bed (A, f = 1.0424), and triturated particles bed (B, f = 1.1256)

Where k (m) is a parameter related to the measure precision of the pore size, $\mathbf{D}_r^{-f}$ is the factor representing the fractional integral which is determined by:

$$D_r^{-f}\left[f(r)\right]=\frac{1}{\Gamma(f)}\int_0^x (r-t)^{f-1} f(t)dt \tag{3}$$

and $\Gamma(f)$ is the Gamma function:

$$\Gamma(f)=\int_0^\infty e^{-x}x^{f-1}dx \tag{4}$$

The solution of the fractional integral (2) is [5] (Suárez-Domínguez et al. 2018, 79):

$$a=k^{1-f}\frac{2\pi}{\Gamma(f+2)}R^{f+1} \tag{5}$$

Where Rrepresents the pore radius (m) identified as the minimum distance from the center to the perimeter. In this case, $R$is considered a deterministic variable, while the flow channel perimeter is a random variable that is analyzed and characterized through fractal dimension.

1.3. Velocity profile and total flow determination through the bed.

Velocity profile was obtained from the quantity of movement equation for the cylindrical coordinates system under the considerations established getting the next partial differential equation:

$$0=\Phi+\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) \tag{6}$$

for the limit conditions:

$$\left(\frac{\partial v}{\partial r}\right)_{r=0}=0 \qquad v(R)=0 \tag{7}$$

The exact analytical condition is:

$$v(r)=\frac{1}{4}\frac{\Phi}{\mu}\left(R^2-r^2\right) \tag{8}$$

To obtain the flow$q$ ($m^{-3}s$) going through a channel, Eq. (2) is integrated with respect to the flow area considering the irregular shape of the area by applying a fractional integral of order f:

$$q=\int_0^{2\pi}\mathbf{D}_r^{-f}\left[v(r)r\right]d\theta \tag{9}$$

$$q=\frac{\Phi\pi}{2\mu}\frac{f(f+5)}{\Gamma(f+4)}k^{1-f}R^{f+3} \tag{10}$$

The total flow Q ($m^{-3}s$) is estimated as the final product of q and the n flow channels, which is estimated as the available area for flow divided by a single channel area:

$$n=\frac{\varepsilon A_t}{a}=\frac{\varepsilon A_t}{k^{1-f}\frac{2\pi}{\Gamma(f+2)}R^{f+1}} \tag{11}$$

Such that it is obtained:

$$Q=\frac{\Phi}{\mu}\varepsilon\frac{f(f+5)\Gamma(f+2)}{4\Gamma(f+4)}A_tR^2 \tag{12}$$

Defining the velocity V_0as the ratio between total flow and bed total area, it is obtained:

$$V_0=\frac{\Phi}{\mu}\times\frac{f(f+5)\Gamma(f+2)}{4\Gamma(f+4)}\times\varepsilon\times R^2 \tag{13}$$

The value of$R$relays on the bed porosity and in the particles size. Toestimate the Rvalue, Eq. (5) and (13) are expressed in function of non-dimensional variables:

$$\phi = \frac{R}{R_p} \qquad \kappa = \frac{k}{R_p} \tag{14}$$

where R_p (m) is the average particles radius, so that:

$$a = \frac{2}{\Gamma(f+2)} \times (\kappa)^{1-f} \times (\phi)^{f+1} \times \pi R_p^2 \tag{15}$$

$$V_0 = \frac{\Phi}{\mu} \frac{f(f+5)\Gamma(f+2)}{4\Gamma(f+4)} \varepsilon \phi^2 R_p^2 \tag{16}$$

Considering that there is an equivalence between the pores area and the area occupied by the particles (at the macro and microscopic levels), it will be that:

$$\frac{\varepsilon}{(1-\varepsilon)} = \frac{a}{\pi R_p^2} \tag{17}$$

and from equations (15) and (17) it will be obtained:

$$\phi^2 = \frac{\varepsilon^{\frac{2}{f+1}}}{(1-\varepsilon)^{\frac{2}{f+1}}} (\kappa)^{2\frac{f-1}{f+1}} \left(\frac{\Gamma(f+2)}{2} \right)^{\frac{2}{f+1}} \tag{18}$$

Substitutingequation (18) onequation (16):

$$V_0 = \frac{\Phi}{\mu} K_0 \frac{\varepsilon^{\frac{2}{f+1}+1}}{(1-\varepsilon)^{\frac{2}{f+1}}} d_p^2 \tag{19}$$

$$K_0 = \frac{f(f+5)\Gamma(f+2)}{16\Gamma(f+4)} \left(\frac{\Gamma(f+2)}{2} \right)^{\frac{2}{f+1}} (\kappa)^{2\frac{f-1}{f+1}} \tag{20}$$

where d_p (m) represents the average particle diameter and the precision κis recommended to have a value of:

$$\kappa \leq 0.1 d_p \tag{21}$$

1.4. Pressure behavior estimation with respect to the bed length.

Flow velocity profile stems from equation (19):

$$\Phi = \frac{V}{d_p^2} \frac{\mu}{K_0} \frac{(1-\varepsilon)^{\frac{2}{f+1}}}{\varepsilon^{\frac{2}{f+1}+1}} \tag{22}$$

Due to the irregular morphology of flow channels, the longitudinal distance traveled for the flow is considered to be longer than the lineal distance between the entrance and the exit of the system in such a way that the pressure gradient is represented by the fractional derivative of the length bed:

$$ {}_0\mathbf{D}_z^f (P) = -\frac{V}{d^2} \frac{\mu}{K_0} \frac{(1-\varepsilon)^{\frac{2}{f+1}}}{\varepsilon^{\frac{2}{f+1}+1}} \qquad P(0) = P_0 \tag{23}$$

whichsolutionis:

$$P(z) = P_0 - \frac{V_0}{d_p^2} \frac{\mu}{K_0} \frac{(1-\varepsilon)^{\frac{2}{f+1}}}{\varepsilon^{\frac{2}{f+1}+1}} \frac{z^f}{\Gamma(f+1)} z_0^{1-f} \tag{24}$$

Where z_0 (m) is the measure precision of the length bed. From the above relation, the pressure drop is obtained for a porous bed with length$L$ (m) by:

$$\Delta P = \frac{V_0}{d_p^2} \frac{\mu}{K_0} \frac{(1-\varepsilon)^{\frac{2}{f+1}}}{\varepsilon^{\frac{2}{f+1}+1}} \frac{z_0^{1-f}}{\Gamma(f+1)} L^f \tag{25}$$

where it must be accomplished that:

$$z_0 \leq 0.1L \tag{26}$$

III. Results and Discusion

Figure 3 shows the model predictions for the flow velocityV_0changes with respect to porosity considering as parameters the channel fractal dimension and the average diameter particles for a fluid with viscosity of 2 Pa s and a pressure gradient of 1000 Pam^{-1}, where slashed represents the behavior of the predictions made for the Blake-Kozeny model with the parameter K was experimentally determined (BK). It can be observed that predicted velocity values for the model on this work are equal in magnitude order to those obtained by the Blake-Kozeny model.As was expected, for a constant value of pressure drop and fluid viscosity, velocity is increased with higher values of porosity and average particle size, which corresponds to experimental observations. This behavior is shown in Figure 4, where it can be observed the velocity profile with respect to particle size, considering as parameters the fractal dimension and bed porosity.

The proposed model predicts that fractal dimension affects significantly on the velocity and that (as well for pressure gradient) velocity is decreased while fractal dimension is increased. Pore morphology is closely related to the particles morphology and is expected that this is increased when particles are more irregular (deduced from results in figures 1 and 2). On the other hand, pore contour represents the solid surface in contact with the fluid. If fractal dimension is increased, then the superficial area (of this surface) will be bigger, effect that entails pressure drops caused by friction. Therefore, for the same pressure gradient, it can be inferred a lower flow.

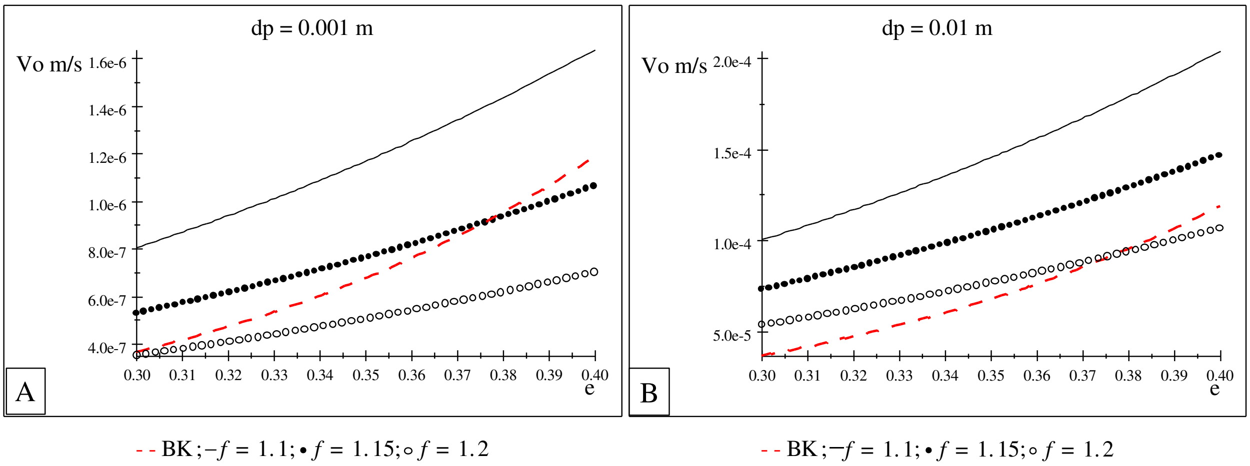


Figure 3. Velocity profile with respect to bed porosity with a particle size of 0.001 m (A) and 0.01 m (B) considering μ=2 Pas, Φ=1000 Pam^{-1},and different values of fractal dimension *f*against Blake-Kozeny model (BK).

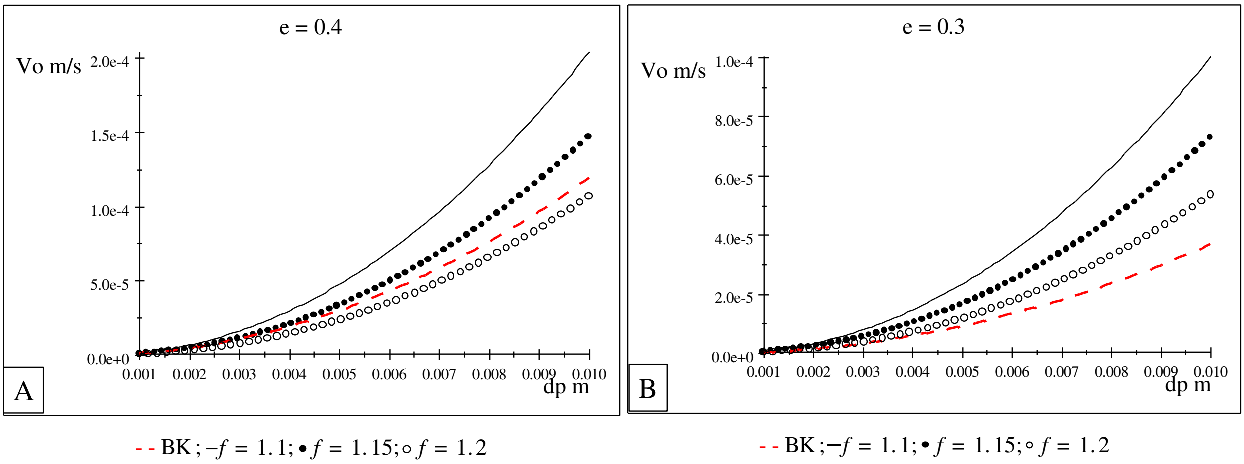
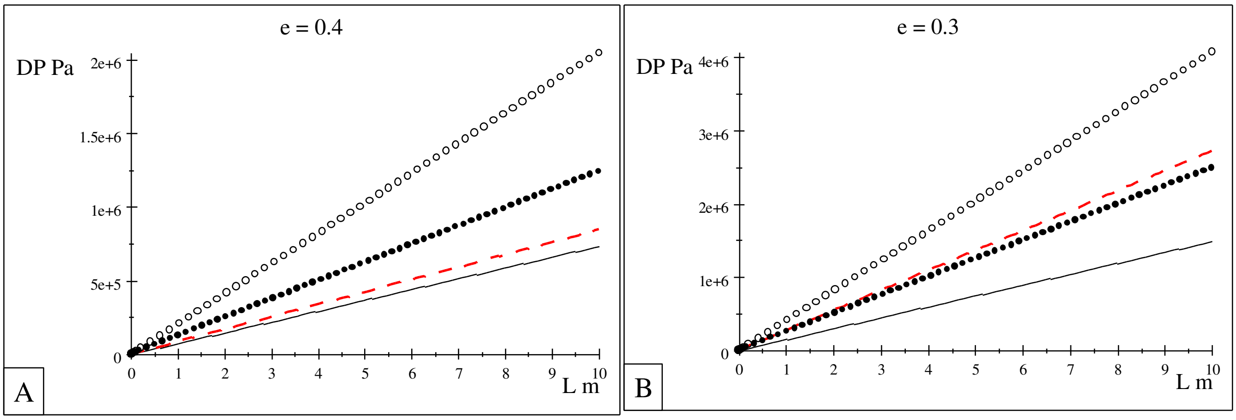


Figure 4. Velocity profile with respect to the particle diameter with a porosity of 0.4 (A) and 0.3 (B) consideringμ= 2 Pas,Φ=1000 Pam^{-1}, and different values of fractal dimension *f* against Blake-Kozeny model (BK).

Figure 5 shows pressure losses with respect to the bed lineal length with a flow of 0.0001 ms^{-1}, and viscosity equal to 2 Pas for different values of porosity and fractal dimension. In this case, it is observed that the values predicted for the model are of the same magnitude order of those estimated by Blake-Kozeny model, where the pressure losses due to friction are increased when viscosity is increased.



- - BK; $-f$=1.1;• f=1.15;∘ f =1.2   - - BK ;$-f$ = 1.1; • f = 1.15; ∘ f = 1.2

Figure 5. Pressure losses by friction with respect to bed lineal length with porosity of 0.4 (A) and 0.3 (B) consideringμ=2 Pas, V_0=0.0001m s^{-1}, and different values of fractal dimension$f$ against Blake-Kozeny model (BK).

Eventhough the obtained model has the advantage to describe the flow behavior and the pressure drop in porous bed without needing an empirical constant (like the BK model), it requires the images of bed pores and the determination of the fractal dimension of the contours. This entails the possibleoccurrence of mistakes as a result of this determination that depends on resolution and image magnification, as well as the correct image processing.

IV. Conclusions

Fromtheapplicationoftheformalismbasedon fractal geometry, fractionaldiferential calculus and transport phenomena equations, it was obtained a model to predict the flow behaviorthroughporous media. The proposed model predicts that for the same pressure gradient, velocity profile increases with porosity and particle size, and decreases when fractal dimension is high.Behaviors predicted for corresponds to tendency and magnitude order to those estimated by BK model, being interesting that, (different from the last one) is not necessary to adjust an empirical parameter because of all the parameters in the proposed model can be determined theoretically from the real morphology of the bed.

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