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Social-Economic Advantages Of Weighted C-Centers And M-Centers In North Macedonia

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Abstract – Graph theory has many applications, because with the help of graphs we can model various complex problems, such as the placement of roads and intersections, the placement of electricity networks, computer networks, the placement of facilities of public importance, etc. Starting from the chaos that appears in the Capital, especially in front of the University Clinic or facilities of public importance, we decided to give a mathematical solution to these problems. In the paper, in addition to the theoretical elaboration on the concepts of graph theory, Floyd's Algorithm for minimum distances, concepts for weighted c-centers and m-centers, we will build the graph of the most important roads in North Macedonia, where as vertices we will use cities and crossroads in North Macedonia, then we will find c-centers and m-centers without and with weight in North Macedonia, and we will give some reasoning on the socio-economic advantages of these centers.

Keywords – Graph, algorithm, c-center, m-center, weight, North Macedonia, socio-economic favors

I. Introduction

The object location problem is a classic problem that has been studied since the Weber Location Problem in 1909. Such problems require special attention. Thus, the best locations for a group of objects should be positioned in those places that should meet the most of the service requirements for a group of customers. In a broader sense we will use "object". This means that it is intended to include entities such as factories, hospitals, electronic communication centers and emergency warning sirens, customs, etc. The inclusiveness of location decision-making has led in a particular interest to analyze models, different algorithms for finding optimal solutions, to decide where to place the objects, along with determining how to assign the demand to the placed objects in order to use the resources in more effective way.

II. The main concepts of Graph Theory

2.1. Definition of graph. Examples

Let V be a nonempty finite set: $V=\{v_1, v_2, \ldots, v_n\}$. Such a set can be represented by a diagram, where its elements are marked with dots. Each such point is called a *vertex*. If two of its elements v_i, v_j, not necessarily different, are examined together, then those points are connected in the diagram with rectilinear or arcuate segments, where one end is point v_i and the other is point v_j. Such a segment is called a *edge* and is denoted v_iv_j, where the order does not matter. Note that two such points can be connected by more than one edge, when the vertices are the same point, the edge is represented by a loop.

Let E be a set of edges with endpoints at the vertices of the set V.

Definition 2.1.1. . In the above conditions, the pair G=(V, E), where V is the finite non-empty set of vertices, while E is the finite set of edges, is called a *graph*.

Example 2.1.2. The graph G=(V, E) is given in the diagram below (Fig. 2.1.1). Find V and E.

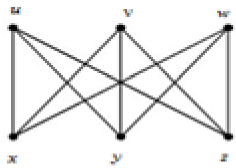


Fig. 2.1.1.

Solution. V = {u, v, w, x, y, z}; E= {$ux, uy, uz, vx, vy, vz, wx, wy, wz$}.

Definition 2.1.3.

- The vertices of an edge are called *ends*;
- Two vertices are called neighbors, if there is an edge that connects them;
- Two edges are called *incidents*, if they have a common vertex;
- The graph is called *simple*, if there are no loops;
- A graph that has vertices but no edges is called an *empty* graph.

Definition 2.1.4. Graph H=(V_H, E_H) is called *a subgraph* of a graph G=(V_G, E_G), if $V_H \subseteq V_G$ and $E_H \subseteq E_G$. Marked $H \leq G$.

Definition 2.1.5. ([4], page 225) Oriented graph is called a ordered pair G=(V, E), where V is the finite nonempty set, while E is the subset of the Cartesian product $V \times V$.

If the element (x_i,x_j) took part p- times in E, then the graph G is called a p-graph. Consequently, if every pair (x_i,x_j) is part of E only once, then the graph G is called a 1-graph.

Definition 2.1.5. An ordered system of vertices (x_0, x_1, x_2, ..., x_r), $r \geq 1$ of graph G=(V, E) such that, each of the edges $x_i x_{i+1}$, for i = 0, 1, ..., r-1, belongs to graph G, it is called a *path* in G. The vertex x_0 is called the *beginning* of the path, while x_r it is called the *end* of the path.

Definition 2.1.6. Let be given the graphs G_1 = (V_1, E_1) and G_2 = (V_2, E_2), where $V1 \cap V2 = \varnothing$ (are disjoint). Graph $G = (V, E) = G_1 \cup G_2$, where $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$ is called graph *union* of G_1 and G_2.

Definition 2.1.7. A graph is called *connected*, if it is not a union of two graphs. In the opposite case, we say that the graph is *disconnected*.

2.2. Weighted Graph. The minimum distance-shortest path problem

Let G=(V, E) be a directed graph, about which a function is given W:$E \to R_0^+$, that each of his edge $e \in E$ is accompanied by only one non-negative real number W (e), which is called weight of edge e or *weighted length* (short, length) of edge e or *weighted distance* (short, distance) between the ends of edge e. If the edge is e=(vi, vj), then we note its weighted length with

$W_{i,j}$: $W(v_i, v_j) = W_{i,j}$.

Definition 2.2.1. A graph G equipped with a function W of the weights of its edges is called a *weighted graph* and denoted by G^W.

For the subgraph H of graph G^W, the number $W(H) = \sum_{e \in E(H)} w(e)$ is called the *overall weight* of the subgraph H.

In particular, if H is a path $P = (v_{i_1}, v_{i_2}, ..., v_{i_p})$ from G^W, then the sum $W(P) = \sum_{k=1}^{p-1} W_{i_k, i_{k+1}}$ is called *weighted length* (short, *length*) of the path P. Each individual graph vertex G^W we consider it as a path of length 0.

In a directed graph there may be several paths starting at *vi* and ending at *vj*. Of interest is knowing those paths that have the smallest length, otherwise we say the search for the shortest path starting at a vertex *vi* and ending at a vertex *vj*. The length of the shortest path starting at *vi* and ending at *vj* is called the *minimum distance* between vertices *vi* and *vj* and is denoted by *d*(*vi, vj*), ie.

$$d(v_i, v_j) = min\{W(P)|P \text{ it is path with a beggining } v_i \text{ and ending } v_j\} .$$

It is clear that shortest paths are required between elementary paths. Since the number of graph vertices is finite, the number of different elementary paths between two vertices is also finite. This makes it possible to find the shortest paths, but aim for the best way, which is given by certain algorithms.

Note. When the graph is undirected, as for example in the graph of the mathematical model of building a highway at minimum cost between two cities (see [4], example on page 270), taking the cost as a weight, and considering that the highway works in both directions, such a graph is considered as a directed graph with two-way edges [8].

2.3. Floyd's algorithm

This algorithm solves the problem of minimum distances and shortest paths between any two vertices of a 1-graph with a discrete distance matrix A=$(l_{ij})_{nxm.}$

The initial step: Let's take the matrices $A_0=(l_{ij}^{(0)})_{nxm}$ and $S_0=(s_{ij}^{(0)})_{nxm}$ such that $l_{ij}^{(0)}= l_{ij}$ and $s_{ij}^{(0)}$=j for each i,j=1,2,…,n. We take *k*=l and we go to the general step.

General step (*k*): We find the matrices $A_k=(l_{ij}^{(k)})_{nxm}$ and $\mathbf{S_k}=(s_{ij}^{(k)})_{nxm}$ where:

$$l_{ij}^{(k)} = \begin{cases} \min(l_{ij}^{(k-1)}, l_{ik}^{(k-1)} + l_{kj}^{(k-1)}) & for\ i \neq k, j \neq k \\ l_{ij}^{(k-1)} & for\ i = k\ and|or\ j = k \end{cases}$$

$$S_{ij}^{(k)} = \begin{cases} k\ for\ i,j \neq k\ such\ that\ l_{ik}^{(k-1)} + l_{kj}^{(k-1)} < l_{ij}^{(k-1)} \\ \quad S_{ij}^{(k)} \qquad otherwise \end{cases}$$

We make *k* equal to (*k*+1) and repeat the general step until *k* takes the value *n*. The matrix *An* found at the end of the *n*-steps is the matrix of minimum distances between graph vertices, so $An=A^*$.[2],[3]

2.4. Concepts of centers in Graphs

In practice, it is often required to find the "most suitable" place to build a facility that serves several peripheral points which may be residential centers or different points that require service. Example, in a given region where it is better to place a health, social-cultural facility, or a shopping center, serving the residential centers, for a telegraphic network where it is better to place the information processing center (central). Depending on the criterion of optimality for the position of this object, we consider two types of such problems:

- The first includes those problems where the most suitable place for the object is considered the one from which the minimum distance from the object to the peripheral points is the smallest possible. This type includes the problem of placing the social-cultural or health facility.
- In problems of the second type, the most suitable place for the object is the one from which the sum of the lengths of all the shortest paths connecting the object with peripheral points is the smallest possible. It is more economical that the total length of the conductors connecting the points of the telegraph network to the switchboard to be minimal, therefore this problem belongs to the problems of the second type. [2]

2.4.1. Centers in Graphs

If we take a graph *G*=(*V*,*E*) which is connected with *n* vertices, we put each edge *u*∈*E* in correspondence with a non-negative number *l*(*u*). We denote by $l^*(x_i,x_j)$ the minimum distance from x_i to x_j.

- The vertex $x_c \in V$ such that for each i=1,2,..,n satisfies the inequality:

$max_{1\le j\le n} l^*(x_c, x_j) \le max_{1\le j\le n} l^*(x_i, x_j)$ it is called the *c-center of graph*. In other words c-center is the vertex where it is reached

$$min_{1\le i\le n}\{max_{1\le j\le n} l^*(x_i, x_j)\}$$

- The vertex $x_m \in V$ such that for every i=1,2,..,n satisfies the inequality:

$\sum_{j=1}^{n} l^*(x_m, x_j) \le \sum_{j=1}^{n} l^*(x_i, x_j)$, it is called the m-center of graph.

In other words m-center is the vertex where it is reached

$$min_{1\le i\le n}\{\sum_{j=1}^{n} l^*(x_i, x_j)\}$$

We consider that for each vertex$x_j \in V$ we have given a positive number w_j which we call its weight. In different problems w_j has different meanings, for example, in placing a service center in an area,, w_j can also indicate the number of inhabitants of the i-th settlement.

- weighted c-center of graph *G* is called every vertex$x_c \in V$ such that:

$max_{1\le j\le n}\{w_j\ l^*(x_c, x_j)\} \le max_{1\le j\le n}\{w_j\ l^*(x_i, x_j)\}$

for each i=1,2,..,n

- weighted m-center of graph *G* is called every vertex$x_m \in V$ such that:

$\sum_{j=1}^{n} w_j\ l^*(x_m, x_j) \le \sum_{j=1}^{n} w_j\ l^*(x_i, x_j)$ for each i=1,2,..,n

In the above definitions, the center exists only when the left sides of the inequalities are finite numbers. On the contrary, it is said that the graph has no center, namely c-center or m-center. It is clear that *G* will have a center, if and only if there is at least one vertex from which every other vertex can be traversed by paths in *G*. In the above definitions, the vertices are considered with the same "rights". In practice this consideration is not always appropriate. For example, in the establishment of a health or social-cultural facility that serves several residential centers, the number of residents of each residential center should also be taken into in consideration, which means; that the service facility should be as close as possible to centers with more residents. [2],[3]

2.4.2 A way to find centers

All four types of centers defined above are very easily found through the matrix *A** which we find with the help of Floyd's Algorithm.

i) For simple centers, we add to the matrix *A** a column where we place the largest element of each row and a column where we place the sum of each row. The vertex belonging to the smallest element in the first added column represents the c-center, while the vertex belonging to the smallest element in the second added column represents the graph m-center.

ii) To get the weighted c-center and the weighted m-center, first the columns of the matrix *A** are multiplied by the weights *wj* of the vertices respectively and then act in the same way as in point i).

III. C-CENTER AND M-CENTER IN NORTH MACEDONIA (RESULTS)

3.1. Simple c-center and m-center

At the beginning, we give the graph of the most important roads of North Macedonia (fig. 3.1.1)

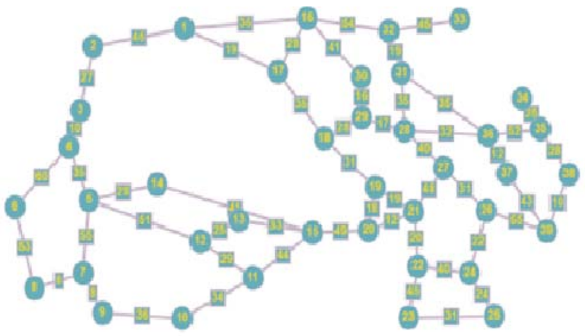


Fig.3.1.1

Where the circles with numbers represent the cities or crossroads, while the rectangles represent the distances or weights of the edges.

In table 3.1.1 we present the data on the ordinal numbers of the vertices and the population of each settlement.

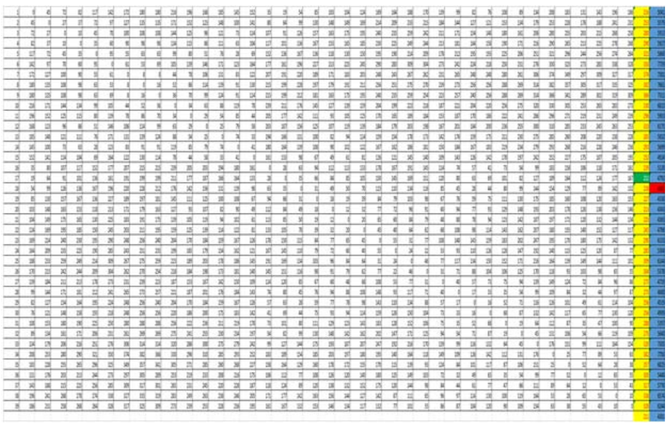
Tab.3.1.1.

Nr	Settlement	Population
1.	Skopje	607000
2.	Tetovo	185000
3.	Gostivar	106000
4.	Mavrovo	8000
5.	Kicevo	56000
6.	Debar	26000
7.	Podmolje	330
8.	Struga	65000
9.	Ohrid	61000
10.	Resen	16500
11.	Bitola	105000
12.	Demir Hisar	9400
13.	Krushevo	9600
14.	Makedonski Brod	11600
15.	Prilep	96000
16.	Kumanovo	127000
17.	Petrovec	8200
18.	Veles	64000
19.	Gradsko	2200

20.	Kavadarci	42000
21.	Negotino	20000
22.	Demir Kapija	3000
23.	Gevgelija	23000
24.	Valandovo	11500
25.	Dojran	3400
26	Strumica	66000
27.	Radovish	31000
28.	Shtip	51000
29.	Kadrifakovo	160
30.	Sveti Nikole	21000
31.	Probishtip	16000
32.	Kratovo	10000
33.	Kriva Palanka	24000
34.	Makedonska Kam.	5000
35.	Delcevo	20000
36.	Kocani	48000
37.	Vinica	19000
38.	Pehcevo	3000
39.	Berovo	16000

We use the Floyd-id Algorithm for finding A^* and act according to the explanations we have in 2.4.1. and 2.4.2.i), we get table 3.1.2.

Tab.3.1.2



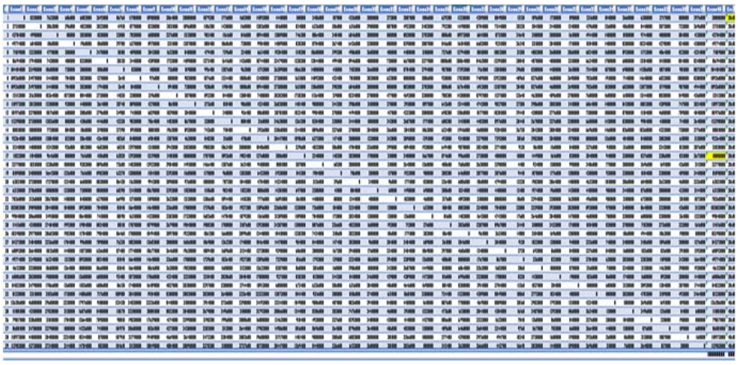
Which is given as appendix 1.

From table 3.1.2 it can be seen that the c-center is the vertex 17 belonging to the settlement Petrovec, while the m-center is vertex 18 belonging to the settlement Veles.

3.2 Weighted c-center and weighted m-center in North Macedonia

After the actions as in explanations 2.4.2.ii) we take table 3.1.3.

Tab.3.1.3



Which is given as appendix 2.

From table 3.1.3 it is observed that:

- ***weighted c-center is the vertex 17 which belongs to the settlement Petrovec, while***
- ***weighted m-center is the vertex 1 belonging to the capital Skopje.***

If we analyze appendix 2 further, we notice that:

- the second weighted c-center is the vertex 1 - the city of Skopje
- the third weighted c-center is the vertex 16 – the city of Kumanovo, while
- the second weighted m-center is the vertex 17- Petrovec settlement
- the third weighted m-center is the vertex 2- the city of Tetovo

Remark. In the graph above, we have not considered the permitted speed of vehicle movements on the road. If speed is also considered, then the c,m-centers may change.

IV. THE SOCIAL-ECONOMIC ADVANTAGES OF WEIGHTED C-CENTERS AND WEIGHTED M-CENTERS

Looking at the results we obtained for weighted c-centers and weighted m-centers and considering the theoretical treatment for c-centers and m-centers, then we are listing some economic and social advantages for weighted c-centers and for weighted m-centers:

4.1 Socio-economic advantages of weighted c-centers

The facilities of public importance in North Macedonia of the first type (hospital, university, training facility, etc.) should be placed in vertex 17 (or as an alternative 1 and 16), i.e. in Petrovec (as an alternative Skopje, Kumanovo), for the reason that:

The smallest average distance for each endpoint in North Macedonia turns out to be Petrovec (the second weighted c-center is Skopje and the third weighted c-center is Kumanovo)

- Not far from Petrovec are the big cities of North Macedonia.
- On average, the cost of transportation of goods and services in North Macedonia falls
- The capital is relieved of heavy traffic

- Petroveci (Skopje, Kumanova), develop more economically, for the reason that small commercial services can be opened around these objects
- The weight of the capital is distributed
- New employment opportunities for weighted c-centers are opened
- The internal displacement of the population to the Capital is stopped.

4.2. Socio-economic advantages of weighted m-centers

The facilities of public importance in North Macedonia of the second type (information processing centers, distribution centers should be located in the vertex 1 (or alternatively 17 and 2), i.e. in the city of Skopje (Petrovec, Tetovo), for the reason that:

• The distribution of networks throughout North Macedonia will have a lower cost, if this distribution has its center in Skopje.

• Not far from Skopje are the big cities of North Macedonia.

• On average, the cost of building such networks decreases

• Skopje (Petrovec, Tetovo) develop more economically, for the reason that small commercial services can be opened around these objects

• New employment opportunities are opened for weighted m-centers

V. Conclusions

After a theoretical support that we gave to our aim, we successfully reached some significant results:

• weighted c-center in North Macedonia is Petroveci

• weighted m-centre in North Macedonia is Skopje

• Some socio-economic arguments were given for these centers

We hope that with this paper we have managed to solve some issues on the placement of public facilities in North Macedonia.

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Appendix 1

1	0	45	72	82	117	142	172	180	180	216	196	168	185	145	152	35	19	54	85	103	104	124	169	164	188	170	139	99	82	76	108	89	134	208	183	131	143	196	186	216	5041
2	45	0	27	37	72	97	127	135	135	171	152	123	148	100	141	80	64	99	130	148	149	169	214	209	233	215	184	144	127	121	153	134	179	253	228	176	188	241	231	253	5579
3	72	27	0	10	45	70	100	108	108	144	125	96	121	73	114	107	91	126	157	163	175	195	240	235	259	242	211	171	154	148	180	161	206	280	255	203	215	268	258	280	5913
4	82	37	10	0	35	60	90	98	98	134	115	86	111	63	104	117	101	136	167	153	165	185	230	225	249	244	213	181	164	158	190	171	216	290	265	213	225	278	268	290	5927
5	117	72	45	35	0	95	55	63	63	99	80	51	76	28	69	152	136	167	136	118	130	150	195	190	214	209	178	212	195	193	225	206	251	321	296	244	256	274	264	321	5860
6	142	97	70	60	95	0	61	53	69	105	139	146	171	123	164	177	161	196	227	213	225	245	290	285	309	304	273	241	224	218	250	231	276	350	325	273	285	338	328	350	7739
7	172	127	100	90	55	61	0	8	8	44	78	106	131	83	122	207	191	220	189	171	183	203	248	243	267	262	231	265	248	248	280	261	306	374	349	297	309	327	317	374	7381
8	180	135	108	98	63	53	8	0	16	52	86	114	139	91	130	215	199	228	197	179	191	211	256	251	275	270	239	273	256	256	288	269	314	382	357	305	317	335	325	382	7661
9	180	135	108	98	63	69	8	16	0	36	70	99	124	91	114	215	199	212	181	163	175	195	240	235	259	254	223	257	240	256	288	269	314	366	341	289	301	319	309	366	7311
10	216	171	144	134	99	105	44	52	36	0	34	63	88	119	78	239	211	176	145	127	139	159	204	199	223	218	187	221	204	220	256	275	320	330	305	253	265	283	273	330	6815
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26	170	215	242	244	209	304	262	270	254	218	184	198	173	181	140	145	151	116	98	91	79	62	77	22	46	0	31	71	88	104	106	125	170	118	93	103	98	65	55	304	5378
27	139	184	211	213	178	273	231	239	223	187	153	167	142	150	109	114	120	85	67	60	48	68	108	53	77	31	0	40	57	73	75	94	139	149	124	72	84	96	86	273	4719
28	99	144	171	181	212	241	265	273	257	221	187	201	176	184	143	74	80	45	76	94	88	108	148	93	117	71	40	0	17	33	35	54	99	109	84	32	44	97	87	273	4680
29	82	127	154	164	195	224	248	256	240	204	170	184	159	167	126	57	63	28	59	77	78	98	143	110	134	88	57	17	0	16	52	71	116	126	101	49	61	114	104	256	4519
30	76	121	148	158	193	218	248	256	256	220	186	200	175	183	142	41	69	44	75	93	94	114	159	126	150	104	73	33	16	0	68	87	132	142	117	65	77	130	120	256	4909
31	108	153	180	190	225	250	280	288	288	256	222	236	211	219	178	73	101	80	111	129	123	143	183	128	152	106	75	35	52	68	0	19	64	112	87	35	47	100	90	288	5397
32	89	134	161	171	206	231	261	269	269	275	241	255	230	234	197	54	82	99	130	148	142	162	202	147	171	125	94	54	71	87	19	0	45	131	106	54	66	119	109	275	5640
33	134	179	206	216	251	276	306	314	314	320	286	300	275	279	242	99	127	144	175	193	187	207	247	192	216	170	139	99	116	132	64	45	0	176	151	99	111	164	154	320	7305
34	208	253	280	290	321	350	374	382	366	330	296	310	285	293	252	183	189	154	185	203	197	180	195	140	164	118	149	109	126	142	112	131	176	0	25	77	89	53	63	382	7750
35	183	228	255	265	296	325	349	357	341	305	271	285	260	268	227	158	164	129	160	178	172	155	170	115	139	93	124	84	101	117	87	106	151	25	0	52	64	28	38	357	6825
36	131	176	203	213	244	273	297	305	289	253	219	233	208	216	175	106	112	77	108	126	120	140	180	125	149	103	72	32	49	65	35	54	99	77	52	0	12	65	55	305	5448
37	143	188	215	225	256	285	309	317	301	265	231	245	220	228	187	118	124	89	120	138	132	152	175	120	144	98	84	44	61	77	47	66	111	89	64	12	0	53	43	317	5776
38	196	241	268	278	274	338	327	335	319	283	249	263	238	246	205	171	177	142	163	156	144	127	142	87	111	65	96	97	114	130	100	119	164	53	28	65	53	0	10	338	6574
39	186	231	258	268	264	328	317	325	309	273	239	253	228	236	195	161	167	132	153	146	134	117	132	77	101	55	86	87	104	120	90	109	154	63	38	55	43	10	0	328	6244
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Appendix 2