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N-Soft Set Approach Methods For More General Cases

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Abstract – The concept of N-soft set has been introduced in previous studies where the concept is a development of fuzzy sets and soft sets. In its application, the concept of the N-soft set can only be done if the data from the case to be solved are in the form of non-negative integers only. However, in everyday life, a case is not always obtained in the form of integers. This article provides an approach where data from non-integer cases can be solved. This matter opens the opportunity for studying a more general concept than N-soft set by theoretical.

Keywords – Fuzzy Set, Soft Set; N-Soft Set

I. INTRODUCTION

Everyday issues such as social, economic, and other areas affect us in various ways of life. One type of issue that frequently arises is when someone makes a decision that will made with various considerations. We occasionally struggle with uncertainty while making decisions and assigning value to objects. Fuzzy Set (*FS*) is a notion that Prof. LA Zadeh [5] proposed in 1965. This idea helps in decision-making when the data being used has some element of uncertainty about an object related to a parameter. *FS* theory studies about the membership values of an object to the parameters, where the membership values are in the interval [0,1]. However, in 1999 Molodtsov [1] introduced a theory called the *Soft Set (SS)* which was able to overcome the shortcomings of the *FS theory* as the *FS theory* could only solve decision-making problems with only one parameter. In *SS* theory, decision-making can involve more than one parameter where objects are grouped based on the parameters related to these objects. If an object is associated with a certain parameter, it will be given a value of 1, while for objects that are not associated with a certain parameter, it will be given a value of 0.

Later in 2018, Fatimah et al [2] introduced a theory known as the N-Soft Set (NSS) which is a generalization of the SS theory . Because the numbers 0 and 1 are not always able to assess current issues in daily life, the NSS hypothesis was developed. Therefore, the term ranking is introduced in the form of a whole number given to an object with respect to certain parameters. However, in real-world situations, it frequently happens that the data are represented by non-negative real numbers. Thus, it would be interesting to carry out a study where any more general case may be resolved by NSS.

II. FUZZY SET AND SOFT SET

In this section, we will review some of the definitions from the previous article related to the discussion in this article.

Definitions 2.1. [5] *Let X be a set of objects. A Fuzzy Set (FS) over X can be defined as follows :*

$$F = \{(x, \mu(x)) | \, x \in X \},$$

where $\mu : X \to [0,1]$ is a membership function , and $\mu(x)$ is the degree of membership of $x \in X$ on Fuzzy Set F.

Definitions 2.2. [1] *Suppose X is a set of objects, P(X) is a power set over X and E is a set of attributes. Couple (F, E) is called a soft set (SS) over X if and only if F is a mapping that maps E to P(X) or F : E→P(X), which can be expressed as the set of ordered pairs.*

$$(F,E)=\{(e_i,F(e_i))|e_i\in E,F(e_i)\in P(X)\}.$$

A soft set can also be represented in the form of a table as in Table 1, where $\mu_{e_j}(x_i)=1$ if $x_i\in F(e_j)$ and $\mu_{e_j}(x_i)=0$ if $x_i\notin F(e_j)$. SS (F,E) can also be expressed in the form $(F,E)=\{(e_j,\{(x_i,\mu_{e_j}(x_i))|x_i\in X\})|e_j\in E\}$.

Table 1 : Representation of a soft set

(F,E)	e_1	e_2	□	e_i	□	e_n
x_1	$\mu_{e_1}(x_1)$	$\mu_{e_2}(x_1)$	□	$\mu_{e_i}(x_1)$	□	$\mu_{e_n}(x_1)$
x_2	$\mu_{e_1}(x_2)$	$\mu_{e_2}(x_2)$	□	$\mu_{e_i}(x_2)$	□	$\mu_{e_n}(x_2)$
□						
x_j	$\mu_{e_1}(x_j)$	$\mu_{e_2}(x_j)$	□	$\mu_{e_i}(x_j)$	□	$\mu_{e_n}(x_j)$
□						
x_m	$\mu_{e_1}(x_m)$	$\mu_{e_2}(x_m)$	□	$\mu_{e_i}(x_m)$	□	$\mu_{e_n}(x_m$

Definition 2.3. [4] *Suppose (F,A) and (G,B) are the two soft sets X. The (F,A) is said to be the soft subset of (G,B) which is denoted by $(F,A)\subseteq(G,B)$ if*

1. *$A\subset B$,*
2. *$\forall e\in E, F(e)\subseteq G(e)$.*

Definition 2.4. [3] *A Soft Set (F,A) above X is said to be Null Soft Sets which is denoted by $\emptyset_A$ if for each $e\in A$, $S(e)=\emptyset$.*

Definition 2.5. [3] *A Soft Sets (F,A) above X is said to be Absolute Soft Sets which is denoted by X_A if for each $e\in A$, $S(e)=X$.*

Definition 2.6. [3] *A Soft Sets (F,A) over X is said to be Deterministic Soft Sets if for $e_1,e_2\in A$ and $e_1\neq e_2$, $\cup_{e\in A}S(e)=X$ and $S(e_1)\cap S(e_2)=\emptyset$.*

Definition 2.7. [4] *Suppose X is a set of objects, E is a set of attributes, $A,B\subseteq E$ and $A\cap B\neq\emptyset$. A restricted intersection of (F,A) and (G,B) denoted by $\cap_R$ is expressed as :*

$$(F,A)\cap_{\boldsymbol{R}}(G,B)=(H,A\cap B)$$

where for each $e\in A\cap B$ and $x\in X$, $H(e)=F(e)\cap G(e)$.

Definition 2.8. [4] *Suppose X is a set of objects, E is a set of attributes, $A,B\subseteq E$ and $A\cap B\neq\emptyset$. An extended intersection of (F,A) and (G,B) denoted by $\cap_E$ is expressed as :*

$$(F,A)\cap_{\boldsymbol{E}}(G,B)=(H,A\cup B)$$

where for each $e\in A\cup B$ and $x\in X$, $H(e)$ is given by

$$H(e)=\begin{cases}F(e) & ,if\ e\in A-B\\ G(e) & ,if\ e\in B-A\\ F(e)\cap G(e) & ,if\ e\in A\cap B\,.\end{cases}$$

Definition 2.9. [4] *Suppose X is a set of objects, E is a set of attributes, $A,B\subseteq E$ and $A\cap B\neq\emptyset$. A restricted union of (F,A) and (G,B) denoted by $\cup_R$ is expressed as:*

$$(F,A)\cup_{\boldsymbol{R}}(G,B)=(J,A\cap B)$$

where for each $e\in A\cap B$ and $x\in X$, $J(e)=F(e)\cap G(e)$.

Definition 2.10. [4] *Suppose X is a set of objects, E is a set of attributes, $A, B \subseteq E$ and $A \cap B \neq \emptyset$. An extended union of (F, A) and (G, B) denoted by $\cup_E$ is expressed:*

$$(F, A) \cup_{\boldsymbol{E}} (G, B) = (J, A \cup B)$$

where for each $e \in A \cup B$ and $x \in X$, $J(e)$is given by:

$$J(e) = \begin{cases} F(e) & , if\ e \in A - B \\ G(e) & , if\ e \in B - A \\ F(e) \cup G(e) & , if\ e \in A \cap B\,. \end{cases}$$

III. Results and Discussion

In this section, the definition of NSS and their algorithms in decision-making will be reviewed. Then a method that is able to make the more general cases in decision- making into simpler cases will be constructed so that these cases can be solved using the algorithms in the *NSS*.

Definitions 3.1. [2] *Suppose X is a set of objects, E is a set of attributes, $A \subseteq E$ and $S = \{0, 1, 2, \dots, N-1\}$ is the set of classes (grades) where $N \in \{2, 3, \dots\}$. An N-Soft Set (NSS) over X is defined as:*

$$(F, A, N) = \{(e_i, F(e_i))|e_i \in A\}.$$

where $F : A \to 2^{X \times S}$and $2^{X \times S}$is the collection of all subsets of inner products of X and S such that for each $e_i \in A$ has exactly one $F(e_i) = \{(x_j, s_{e_i x_j})|x_j \in X, s_{e_i x_j} \in S\}$ or $F(e_i)(x_j) = s_{e_i x_j} = s_{e_i}(x_j)$.

An *NSS* over X can also be represented in the form of a table as in Table 2 as follows:

Table 2 : Representation of a N- soft set

(F, E)	e_1	e_2	□	e_i	□	e_n
x_1	$s_{e_1}(x_1)$	$s_{e_2}(x_1)$	□	$s_{e_i}(x_1)$	□	$s_{e_n}(x_1)$
x_2	$s_{e_1}(x_2)$	$s_{e_2}(x_2)$	□	$s_{e_i}(x_2)$	□	$s_{e_n}(x_2)$
□						
x_j	$s_{e_1}(x_j)$	$s_{e_2}(x_j)$	□	$s_{e_i}(x_j)$	□	$s_{e_n}(x_j)$
□						
x_m	$s_1(x_m)$	$s_{e_2}(x_m)$	□	$s_{e_i}(x_m)$	□	$s_{e_n}(x_m)$

Definitions 3.2. [2] *Suppose X is a set of objects, E is a set of attributes, $A, B \subseteq E$ and $A \cap B \neq \emptyset$. $S_1 = \{0, 1, 2, \dots, N_1 - 1\}$ and $S_2 = \{0, 1, 2, \dots, N_2 - 1\}$ is the set of classes (grades) where $N_1, N_2 \in \{2, 3, \dots\}$. A restricted intersection of (F, A, N_1) and (G, B, N_2) denoted by $\cap_R$ is expressed as:*

$$(F, A, N_1) \cap_{\boldsymbol{R}} (G, B, N_2) = (K, A \cap B, \min\{N_1, N_2\})$$

where for each $e \in A \cap B$ and $x \in X$, $(x, s_{ex}) \in K(e)$ with $s_{ex} = \min\{s_{ex}^F, s_{ex}^G\}, (x, s_{ex}^F) \in F(e)$and $(x, s_{ex}^G) \in G(e)$.

Definitions 3.3. [2] *Suppose X is a set of objects, E is a set of attributes, $A, B \subseteq E$ and $A \cap B \neq \emptyset$. $S_1 = \{0, 1, 2, \dots, N_1 - 1\}$ and $S_2 = \{0, 1, 2, \dots, N_2 - 1\}$ is the set of classes (grade) where $N_1, N_2 \in \{2, 3, \dots\}$. An extended intersection of (F, A, N_1) and (G, B, N_2) denoted by $\cap_E$ is expressed as:*

$$(F, A, N_1) \cap_{\boldsymbol{E}} (G, B, N_2) = (K, A \cup B, \max\{N_1, N_2\})$$

where for each $e \in A \cup B$ and $x \in X$, $K(e)$ is given by:

$$K(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \{(x, s_{ex}) | x \in X\}, & if\ e \in A \cap B, with\ s_{ex} = \min\{s_{ex}^F, s_{ex}^G\}, \\ & where\ (x, s_{ex}^F) \in F(e)\ and\ (x, s_{ex}^G) \in G(e). \end{cases}$$

Definitions 3.4. [2] *Suppose X is a set of objects, E is a set of attributes, $A, B \subseteq E$ and $A \cap B \neq \emptyset$. $S_1 = \{0, 1, 2, \dots, N_1 - 1\}$ and $S_2 = \{0, 1, 2, \dots, N_2 - 1\}$ is the set of classes (grades) where $N_1, N_2 \in \{2, 3, \dots\}$. A restricted union of (F, A, N_1) and (G, B, N_2) denoted by $\cup_R$ is expressed as:*

$$(F, A, N_1) \cup_{\boldsymbol{R}} (G, B, N_2) = (L, A \cap B, \max\{N_1, N_2\})$$

where for each $e \in A \cap B$ and $x \in X$, $(x, s_{ex}) \in L(e)$ with $s_{ex} = \max\{s_{ex}^F, s_{ex}^G\}$, $(x, s_{ex}^F) \in F(e)$ and $(x, s_{ex}^G) \in G(e)$.

Definitions 3.5. [2] *Suppose X is a set of objects, E is a set of attributes, $A, B \subseteq E$ and $A \cap B \neq \emptyset$. $S_1 = \{0, 1, 2, \dots, N_1 - 1\}$ and $S_2 = \{0, 1, 2, \dots, N_2 - 1\}$ is the set of classes (grades) where $N_1, N_2 \in \{2, 3, \dots\}$. An extended union of (F, A, N_1) and (G, B, N_2) denoted by $\cup_E$ is expressed as :*

$$(F, A, N_1) \cup_{\boldsymbol{E}} (G, B, N_2) = (L, A \cup B, \max\{N_1, N_2\})$$

where for each $e \in A \cup B$ and $x \in X$, $L(e)$ is given by:

$$L(e) = \begin{cases} F(e), & if\ e \in A - B \\ G(e), & if\ e \in B - A \\ \{(x, s_{ex}) | x \in X\}, & if\ e \in A \cap B, with\ s_{ex} = \max\{s_{ex}^F, s_{ex}^G\}, \\ & where\ (x, s_{ex}^F) \in F(e)\ and\ (x, s_{ex}^G) \in G(e). \end{cases}$$

Furthermore, the algorithm used in the NSS will be defined in decision-making as follows:

Algorithm. [2] *Algorithm of extended choice values (ECVs)*

1. *Inputs $X = \{x_1, \dots, x_m\}$ and $A = \{e_1, \dots, e_n\}$.*
2. *Input the N-Soft Set (F, A, N), with $S = \{0, 1, \dots, N - 1\}$, $N \in \{2, 3, \dots\}$ for each $x_i \in X$, $e_j \in A$, $\exists!\, s_{e_j}(x_i) \in S$.*
3. *For a x_i, will be calculated ECVs with the formula $\sigma_i = \sum_{j=1}^{n} s_{e_j}(x_i)$.*
4. *Search from every σ_i and choose the one σ_i with the largest value.*
5. *Choose x_i the one associated with the σ_i largest in the Step. 4.*

One of the problems that often arise in everyday life is how to make a decision where the decision-making depends on certain attributes and involves many objects. One example of the problems in making these decisions can be seen in Figure 1 below:

Figure1 : The results of the assessment of several films on a web

Based on the picture above, it can be seen that the ratings of these films are not only in the form of natural numbers, but there are several films that have ratings in the form of non-negative real numbers. In mathematics, we know that there is a way to convert real numbers into natural numbers, namely by estimating or rounding numbers. Next, a method will be introduced using the concept of estimation or rounding the number.

Number Estimation Method or Number Rounding Suppose X is a set of objects, E is a set of attributes, A ⊆ E and $S = \{0, 1, 2, \dots, N-1\}$ is a set of classes (grades) where $N \in \{2, 3, \dots\}$ and p ∈ ℕ. If a non-negative real number $r_{e_j}(x_i)$ is a grade of x_i related to e_j, then the new value $s_{e_j}(x_i)$ obtained from the estimation or rounding of $r_{e_j}(x_i)$ can be expressed as

$$s_{e_j}(x_i) = \begin{cases} p & , if\ p \leq r_{e_j}(x_i) < p + 0.5 \\ p+1 & , if\ r_{e_j}(x_i) \geq p + 0.5. \end{cases}$$

The above method aims to change the rating or class (grade) from non-negative real numbers into natural numbers to be defined in the NSS. Next is an example of the application of the NSS through the algorithm that has been given along with the use of the above method with the data used in the form of illustrative data.

Case A car company will open a new branch in a city and will recruit new employees who will be placed in the new branch. Before becoming an employee, an internship process must be done for several days. Suppose $X = \{x_1, x_2, x_3, x_4, x_5\}$ is a set of interns who follow the apprenticeship process. In the internship process, an assessment will be carried out based on several attributes. For example, $A = \{e_1, e_2, e_3\}$, the set of attributes that are used as a reference in assessing interns, where "e_1" states how polite the interns are in serving buyers, then " e_2" states how neat the clothes of the interns are in serving buyers and " e_3" states how disciplined the interns are during the internship process. Then given the provisions of the grades that will be used in the assessment as follows:

1. “5” for the very good predicate.
2. “4” for the good predicate.
3. "3" for the standard predicate.
4. "2" for the predicate passable.
5. “1” for the bad predicate.
6. “0” for the very bad predicate.

After observation based on the stipulated provisions, for example, the data obtained are as follows:

Table 3: observational data from interns

$(F, A, 6)$	e_1	e_2	e_3
x_1	4.6	2.4	3
x_2	1.2	3.3	5
x_3	2.9	3.5	4.6

x_4	2.7	1.1	3.5
x_5	3.4	2.5	1.1

It can be seen from the table above that the NSS algorithm cannot directly resolve the data in this case. Therefore, the above data will be changed first by using the estimation method or rounding numbers as previously defined so that the data in Table 3 will change to the following.

Table 4: $(F, A, 6)$

$(F, A, 6)$	e_1	e_2	e_3
x_1	5	2	3
x_2	1	3	5
x_3	3	4	5
x_4	3	1	4
x_5	3	3	1

Furthermore, after obtaining data such as the table above, the above data can be processed with the NSS algorithm for solving this case. Based on the NSS algorithm, the next step is to find the value of *ECVs* with the formula $\sigma_i = \sum_{j=1}^{n} s_{e_j}(x_i)$ which can be presented in the following table.

Table 5: data value of *ECVs*

$(F, A, 6)$	e_1	e_2	e_3	σ_i
x_1	5	2	3	10
x_2	1	3	5	9
x_3	3	4	5	12
x_4	3	1	4	8
x_5	3	3	1	7

Based on the NSS algorithm, the last step is to determine the best object or intern based on Table 5 by sorting the values from σ_i. It can be seen from Table 5 above that $\sigma_3 > \sigma_1 > \sigma_2 > \sigma_4 > \sigma_5$ so that in decision-making in this case, it was chosen x_3 as the main choice and for the next choice to follow from $\sigma_3 > \sigma_1 > \sigma_2 > \sigma_4 > \sigma_5$.

IV. Conclusion

As demonstrated in this paper, the method of estimating numbers or rounding numbers can assist in simplifying more common scenarios where the rating or grade is not always in the form of natural numbers but may also be in the form of non-negative real numbers, allowing *NSS* to be used in more general cases.

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