International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

Vol. 31 No. 2 March 2022, pp. 238-249

Illumination Waveform Design For Non-Gaussian Multi-Hypothesis Target Classification In Cognitive Radar

RANDRIANANDRASANA Marie Emile[1], RANDRIAMITANTSOA Paul Auguste[2], RANDRIAMITANTSOA Andry Auguste[3]

[1]Dept. of Telecommunication, Antsirabe Vankinankaratra High Education Institute,
University of Antananarivo, Madagascar,

[2]Dept. of Telecommunication, High School Polytechnic of Antananarivo,
University of Antananarivo, Madagascar,

[3]Dept. of Telecommunication, High School Polytechnic of Antananarivo,
University of Antananarivo, Madagascar,

Abstract – The cognitive radar system is generalized to deal effectively with arbitrary non-Gaussian distributed target responses via two key contributions: (1) an important statistical expected value operation that is usually evaluated in closed form is evaluated numerically using an ensemble averaging operation, and (2) a powerful new statistical sampling algorithm and a kernel density estimator are applied to draw complex target samples from target distributions specified by both a desired power spectral density and an arbitrary desired probability density function. Simulations using non-Gaussian targets demonstrate very effective algorithm performance. As expected, this performance gain is realized at the expense of increased computational complexity

Keywords – Cognitive Radar, Non-Gaussian, target classification.

I. INTRODUCTION

Cognitive radar (CR) is a concept proposed in 2006 by S. Haykin to improve the performance of radar systems in resource-constrained and interference-limited environments. A CR system is one that has the ability to observe and learn from the environment, then use a dynamic closed-loop feedback mechanism to adapt the illumination waveform so as to provide system performance improvements over traditional radar systems.

This article is focused on finding a general classification algorithm that is suitable for targets with any statistical distributions. This research requires non-Gaussian target simulation, probability density function (PDF) estimation, and target classification.

II. MATCHED WAVEFORM DESIGN

A. Signal and system model

The complex stochastic radar target impulse response $g(t)$ is, at times in practice, a finite-length signal. This $g(t)$ can be modeled as a stochastic process multiplied by a rectangular window of duration T_g [3]. It is called "extended," because it has finite temporal extent.

$$y(t) = x(t) * g(t) + n(t) \tag{1}$$

The theoretical radar target and measurement model is illustrated in Figure 1.

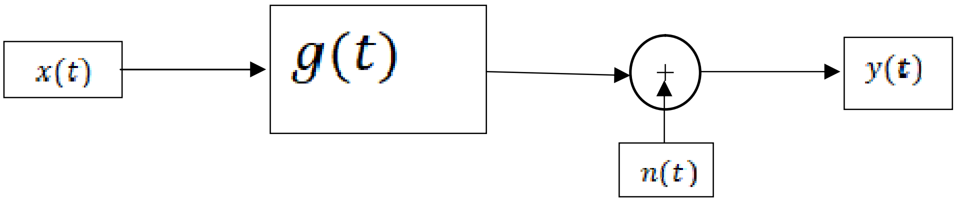


Fig1: Complex-valued baseband signal and system model with a stochastic target.

B. Optimal Snr Waveform Design For A Single Target

The optimal SNR waveform $\hat{x}(t)$ is given as the eigenvector corresponding to the maximum eigenvalue of the eigenfunction.

$$\lambda_{max}\hat{x}(t) = \int_{-\frac{T}{2}}^{\frac{T}{2}} \hat{x}(\tau) R_g(t-\tau) d\tau \tag{2}$$

where the kernel $R_g(t)$ is

$$R_g(t) = \frac{1}{N_0}\int_{-\infty}^{\infty} \sigma_G^2(f) e^{j2\pi f}\, df \tag{3}$$

III. Multiple Hypothsis Target Classification

We assume that we have available a set of M known representative target responses responses $\{g_i(t)\}_{i=1}^{M}$ that have been measured or simulated in advance. Also, assume that we calculate in advance the M optimal illumination waveforms $\{x_i^{opt}(t)\}_{i=1}^{M}$associated with the M target classes. We then use radar measurement signal y(t) in an iterative scheme to produce an illumination waveform that provides high classification performance measured by probability of correct classification.

$$y(t) = x(t) * g(t) + n(t) \tag{4}$$

A. Discrete-Time Measurement Model

we need a discrete time vector formulation which we describe next. For a given target hypothesis H_i we can write the measurement equation as:

$$\underline{y} = x\underline{g}_i + \underline{n} \tag{5}$$

where $\underline{g}_i$ is $L \times 1$, $\underline{y}$ is $L_y \times 1$ and $\underline{n}$ is $L_y \times 1$. The input illumination signal $x(t)$ can be written as an $L \times 1$ input vector$\underline{x}$. This input vector can then be used to construct the circulant $L_y \times L$ circulant convolution matrix X_k:

$$x_k = \begin{bmatrix} x_k(1) & 0 & \cdots & \cdots & 0 \\ x_k(2) & x_k(1) & \ddots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x_k(L_x) & x_k(L_x-1) & \cdots & x_k(1) & 0 \\ 0 & x_k(L_k) & x_x(L_x-1) & \cdots & x_k(1) \\ \vdots & 0 & x_k(L_x) & \cdots & x_k(2) \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & x_k(L_x) \end{bmatrix} \tag{6}$$

For the classification problem, we use multiple illuminations indexed by *k*. For this case, we write the measurement for the k^{th} illumination as

$$\underline{y}_k = X_k \underline{g}_i + n_k \tag{7}$$

B. PWE transmission technique

In a multiple target hypothesis classification setting, the individually optimal illumination waveforms $\{g_i(t)\}_{i=1}^{M}$ are linearly combined in a weighted sum to form a single designed illumination waveform called PWE, where the weights are the prior probabilities (or priors) p_i of the *M* corresponding hypotheses p_i=p(H_i). For each iteration, a new illumination waveform is transmitted to illuminate the targets. The desired number of iterations is chosen in advance by the user, so no additional stopping criterion is used. In this scheme, the transmitter tends to place the illumination waveform energy in the spectral bands where the true target most likely resides. This multiple transmission technique is termed the PWE technique [4] and is illustrated in Figure 3.

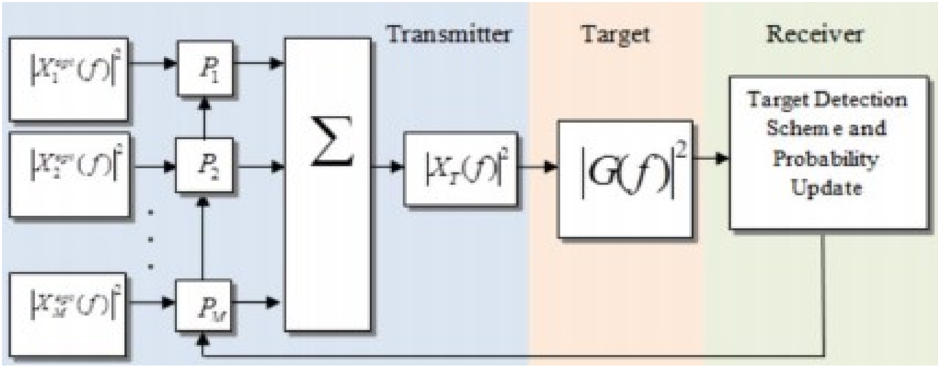


Fig2: PWE multiple technique for one iteration

C. Iterative Algorithmes For Updating The Prior Probabilites

In the closed-loop radar multiple hypothesis classification scheme, for the i^{th} target for $(K+1)^{th}$ transmission, the normalized prior probability

$$P_i^{k+1} = \frac{P_i^k(y_1 y_2 \dots y_k) P_0}{\sum_{i=1}^{M} P_i^k(y_1 y_2 \dots y_k) P_0} \qquad (8)$$

1. The MAP Scheme for Updating the Priors in the GCCR System

The Bayesian MAP approach requires prior knowledge of the prior PDFs $\{p_i(g_i)\}_{i=1}^{M}$, where *M* is the number or target classes. These priors are available. We also assume that the *M* target responses are known in advance because we have measured them or modeled them. For the GCCR system, the PDFs have been modeled as zero mean Gaussian with target covariance matrix K_{g_i}.

We also require the joint conditional PDF $P_i^k(y_1, y_2, \dots, y_k | g_i)$ for the k^{iem} iteration. The *k* measurements conditioned on the target realization g_i are independent, so their joint conditional density can be written as the product of the individual conditional densities $P_i^k(\underline{y_i} | \underline{g_i})$as follows:

$$P_i^k\left(\underline{y_1}, \underline{y_2}, \dots, \underline{y_k} \middle| \underline{g_i}\right) = \prod_{k=1}^{K} (\underline{y_i} | \underline{g_i}) \qquad (9)$$

The key to the GCCR system is the formulation of the individual posterior conditional densities $\{P_i^k(\underline{y_i} | \underline{g_i})\}_{k=1}^{M}$ as functions of the true measurements $\{\underline{y_k}\}_{i=1}^{M}$ and the estimated measurements $\{\underline{\hat{y}_{i,k}} = X_k \underline{\hat{g}_i}\}_{i=1}^{M}$. consider the posterior conditional densities as Gaussian PDFs and mode them as such:

$$P_i^k(\underline{y_k} | \underline{g_i}) = \frac{1}{\pi^L |K_N|} \exp[-(\underline{y_k} - X_k \underline{\hat{g}_i})^H K_N^{-1} (\underline{y_k} - X_k \underline{\hat{g}_i})]$$

In this expression, the estimated measurement $\{\underline{\hat{y}_{i,k}} = X_k \underline{\hat{g}_i}\}_{i=1}^{M}$ takes on the role of the mean of the Gaussian random variable $\underline{y_k}$, and the covariance matrix K_N takes on the role of the covariance K_y .

Continuing with the GCCR system development, we can write the posterior conditional PDF for the set of k independent measurements as:

$$P_i^k\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k \,\middle|\, \underline{g}_i\right)=\prod_{k=1}^{K}\left(\underline{y}_k \,\middle|\, \underline{g}_i\right)=\frac{1}{\pi^L|K_N|^k}\exp\left[-\sum_{k=1}^{K}(\underline{y}_k-X_k\underline{\hat{g}}_i)^H K_N^{-1}(\underline{y}_k-X_k\underline{\hat{g}}_i)\right]$$

Given the quantities described above, the joint density of the multiple measurements can be written as the expected value of the joint density of the multiple measurements conditioned on the target:

$$P_i^k(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k)=E_g\{P_i^k(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k|\underline{g}_i)\}$$

Since the target signal is based on a Gaussian assumption, the prior density of the target $p(g_i)$ is assumed to be a zero-mean complex Gaussian signal, and its PDF is given by

$$P_i^k\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k\right)=\int_{\underline{g}_i}P_i^k\left(\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k\right)\middle|\underline{g}_i\right)p(\underline{g}_i)\,d\underline{g}_i$$

Since the target signal is based on a Gaussian assumption, the prior density of the target $p(\underline{g}_i)$ is assumed to be a zero-mean complex Gaussian signal, and its PDF is given by:

$$p\left(\underline{g}_i\right)=\frac{1}{\pi^L|K_{g_i}|}\exp(\underline{g}_i{}^H K_g^{-1}\underline{g}_i)$$

We can simplifie l'eqaution 12 :

$$P_i^k\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k\right)=\frac{|Q^{-1}|}{(|K_{g_i}|\pi^{LK}|K_N|^K)}\times\exp\left[-\sum_{k=1}^{K}\underline{y}_k^H K_N^{-1}\underline{y}_k\right]\exp\left[\left(\sum_{k=1}^{K}\underline{X}_k^H K_N^{-1}\underline{y}_k\right)^H Q^{-1}\sum_{k=1}^{K}X_k^H K_N^{-1}\underline{y}_k\right]$$

2. Classification algorithm for the NGCCR system

The fundamental Bayesian MAP approach for the NGCCR system is the same as that for the GCCR system. We desire to evaluate the joint density of the multiple measurements, which can be written as the expected value of the joint density of the multiple measurements conditioned on the target:

$$P_i^k\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k\right)=E_g\{P_i^k\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k|\underline{g}_i\right)\}$$

Recall that for the GCCR system, this can be written in closed form using the expected value operation in integral form the eq 13

However, for the NGCCR system, we assume that the target responses have arbitrary distributions, so the desired PDFs cannot be written in closed form. As a result, we cannot use the integral form of the expected value $E_{g_i}\{\bullet\}$. Nonetheless, the posterior joint conditional density of the multiple measurements given a realization of a target is similarly given by:

$$P_i^k\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k\right)\approx\frac{1}{N_g}\sum_{j=1}^{N_g}P_{i,j}^k\left(\underline{y}_1,\underline{y}_2,\dots,\underline{y}_k|\underline{g}_i\right)$$

Once we have these joint densities, we use them to update the prior probabilities. We use the updated priors to update the PWE illumination signal, which is then transmitted by the radar to illuminate the targets. The user defines in advance the desired number of iterations to use. When that number of iterations is complete, the final target classification decision is made.

IV. Generation Of Stochastic Complex Signal

A. Motivation for using a complex data model

In radar, sonar, and many communication systems, complex signals are used extensively to represent the in-phase and quadrature components of the demodulated data. The complex envelope represents the concatenation of the in-phase and quadrature components into the form of real and imaginary quantities. Using the complex data model simplifies analysis. This method is similar to the simplification of using a Fourier series of complex exponentials to represent sines and cosines.

B. Conception of complex density

The goal for the complex density is to express the PDF, CDF, and all related statistical measures as a function of the complex variable. Let the variable $\hat{x}$ be a $2N \times 1$ vector of real and imaginary components as two individual random variables defined as:

$$\hat{x} = [x_R, x_i]^T$$

and let the variable $\tilde{x}$ be a single complex random variable defined as:

$$\tilde{x} = x_R + jx_i$$

If the covariance matrix $K_{\hat{x}}$ of the original real $2N \times 1$ vector is constrained such that:

$$k_R = k_i = k_R/2$$

And

$$k_{iR} = -k_{Ri}^T = k_i/2$$

then the duality of the density functions for the real random variables and the complex random variables can be written as:

$$f_{\hat{x}}(u, v) = f_{\tilde{x}}(u + jv)$$

V. Generation Of Complex Signals With Specified Spectra And PDF

A. Generation of a Complex Gaussian Signal with a Specified PSD

In previous research, the target classification scheme for the cognitive radar system was focused on detecting complex Gaussian distributed signals with certain power spectral densities. A traditional method used to generate this type of signal was developed by Steven Kay. This algorithm is based on the concept that for a Gaussian random process, the PSD is specified by the covariance matrix **R.**

Let $S_{xx}(f)$ be the PSD of x[n] ; then as $N \to \infty$, the eigenvalues λ_i and the eigenvectors v_i are:

$$\begin{cases} \lambda_i = S_{xx}(f_i) \\ v_i = \frac{1}{N}[1 \exp(j2\pi f_i) \exp(j4\pi f_i) \dots . \exp(j2\pi(N-1)f_i]^T \end{cases}$$

for $i = 0,1,\ldots,N-1$ and $f_i = \frac{1}{N}$. Then covariance matrix **R** is defined as:

$$R = \sum_{i=0}^{N-1} \lambda_i v_i v_i^H$$

If the data set {x[0],x[1],…,x[N-1]} is randomly generated based on a Gaussian distribution where x[n] : N(0,σ^2), then the zero-mean complex Gaussian discrete target signal g[n] with a specified covariance matrix **R** can be created by

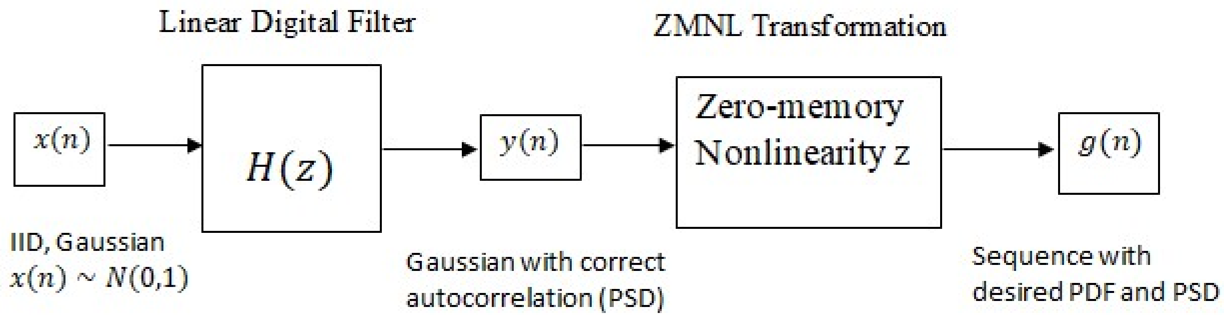
$$g[n] = \sqrt{R}\, x[n]$$

for $n = 0,1,\ldots,N-1$.

Using this method, we can generate a complex Gaussian distributed sequence with a specified PSD to simulate the target signals for the existing target detection scheme.

B. Generate a complex signal with a specified PDF and PSD

A block diagram that generally summarizes the procedure of generating signals with both a desired PDF and a PSD is illustrated in Figure 3. First, a spectrally white, Gaussian distributed sequence is linearly filtered by the specified auto-covariance to obtain a sequence with a Gaussian PDF and the specified PSD. This step makes use of the fact that if a Gaussian sequence is filtered by a linear system, the output is a Gaussian sequence as well. The data are then subject to a zero-memory, nonlinear (ZMNL) transformation in order to produce a signal with the desired nonGaussian PDF.



The goal of the algorithm is to produce a sequence of observations [n] n=0,1 ,…,N with PDF p(x) and a PSD PSD $s_{xx}(f)$. First, a desired PSD function $s_{xx}(f)$ is proposed with N discrete frequencies at the bandwidth

$$f_k = (K-N/2)\Delta_f,$$

$k = 0,..., N-1$. The frequency bin Δf is dictated by the temporal sampling interval Δ_t such that

$$\Delta_f = \frac{1}{N\Delta_t}$$

In general, Δ_t should be chosen in accordance with the Nyquist criterion for the maximum resolvable frequency. For the purpose of this research, the frequency domain is normalized to 1 Hz for the analysis of all signal spectra. The discrete Fourier transform and the inverse Fourier transform are defined as:

$$X(K) = FT(x[n]) \equiv \sum_{n=0}^{N-1} x[n]e^{-2j\pi kn/N}$$

And

$$x[n] = FT^{-1}(X[K]) \equiv \frac{1}{N}\sum_{k=0}^{N-1} X[K]e^{j2\pi kn/N}$$

At this point, the relationship between the signal $x[n]$ and its PSD $S_{xx}(f)$ in terms of mean and variance is defined by:

$$\overline{x} = \frac{\sqrt{s_{xx}(0)}}{N\Delta t}$$

And

$$\sigma_x^2 = \frac{N}{N-1}\sum_{k=0}^{N-1} s_{xx}(f_k)\Delta f$$

A data sequence s[n] is generated from a desired distribution $p(x)$. It is natural to have the proposed PDF $p(x)$ to specify the mean of the signal and to have the proposed PSD $s_{xx}(f)$ to specify the variance of the signal. Then the variance of the signal *s[n]* must be adjusted to conform to the variance of the proposed PSD $s_{xx}(f)$. The variance of the signal isσ_x^2. With the variance of the PSD obtained from Eq. (4), the signal $s[n]$ can be adjusted to a scaled sequence $x0[\,]\,n$ that conforms to the variance of $s_{xx}(f)$ using

$$x_0[n] = \frac{\sqrt{\sigma_{\overline{x}}^2}}{\sqrt{\sigma_x^2}} s[n]$$

A new signal $x[n]$ is created with the phases of $x_0[n]$ and the Fourier amplitudes of the proposed PSD from:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} e^{i\phi(k)} X_t[K] e^{i2\pi kn/N} = FT^{-1}(e^{i\phi(k)} X_t(k))$$

VI. Simulation and Interpretation of the Resultant

All proposed PSVs for the experiment are given by the Mixed-Gaussian probability density function.

$$S_{xx}(f) = \frac{P_{signal}}{2\sqrt{2\pi\sigma}}\left(e^{\frac{(f-\mu)^2}{2\sigma^2}} + e^{\frac{(f+\mu)^2}{2\sigma^2}}\right)$$

The non-Gaussian PDFs are Rayleigh, exponential, Gamma, and log-normal distributions. This simulation is conducted mainly to evaluate the performance of the new algorithm for non-Gaussian target classes with four different distributions. The specific parameters of the four different target hypotheses are listed in Table 1.

Table 1: Parameter setup of the four target hypotheses for the third experiment

Target Hypothèse	Target PDFs	μ	σ
Target# 1	Gaussien complexe $\mu = 2$	0.2	0.03
Target # 2	Gaussien complexe $\mu = 2$	0.3	0.02
Target # 3	Gaussien complexe $\mu = 2$	0.35	0.025
Target # 4	Gaussien complexe $\mu = 2$	0.4	0.015

this figure illustrates the four proposed PSVs.

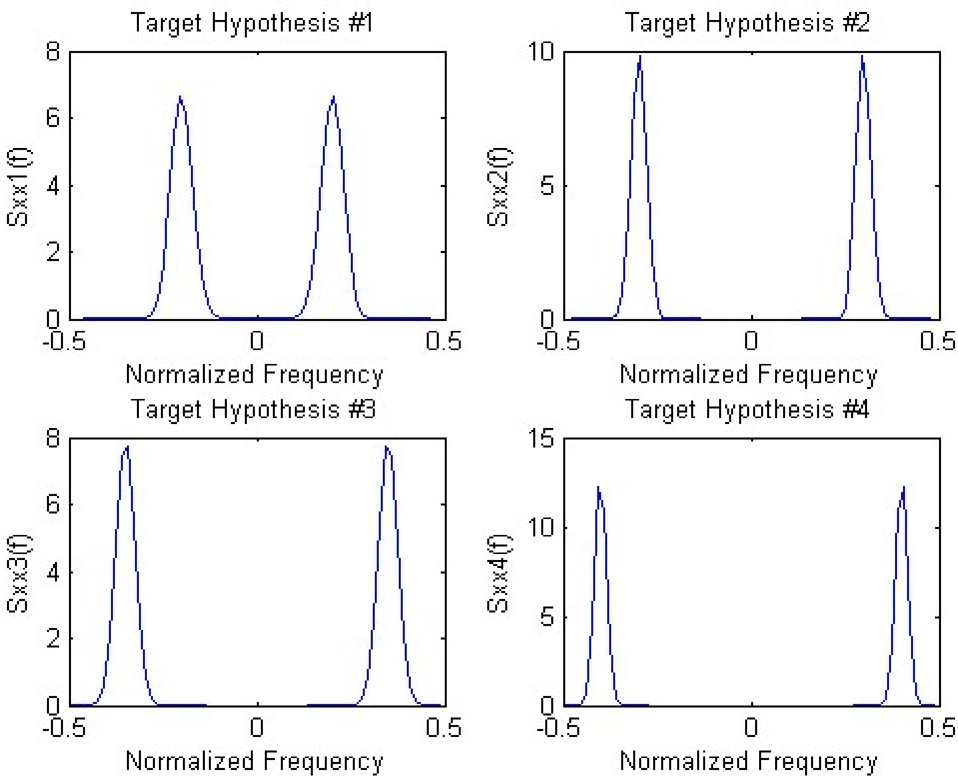


Fig 4: The four proposed PSVs for the target hypotheses

By putting the four PSVs on one plot, we can see that none of the target spectra completely overlap any of the other target spectra. All target hypotheses have different amplitudes and bandwidths in their PSVs.

Next, a particular target realization from target hypothesis H_1 is generated. The PSV of this target realization is illustrated in Figure 5. Because the experiment only sets the number of samples to 64, the PSV of any stochastic target realization is not very smooth in nature. However, the generated target still possesses the general properties of the target hypothesis proposed in Figure 4 in terms of total energy and band-pass frequencies. The spectra presented in Figure 5 conform with the specified PSVs of the target hypotheses.

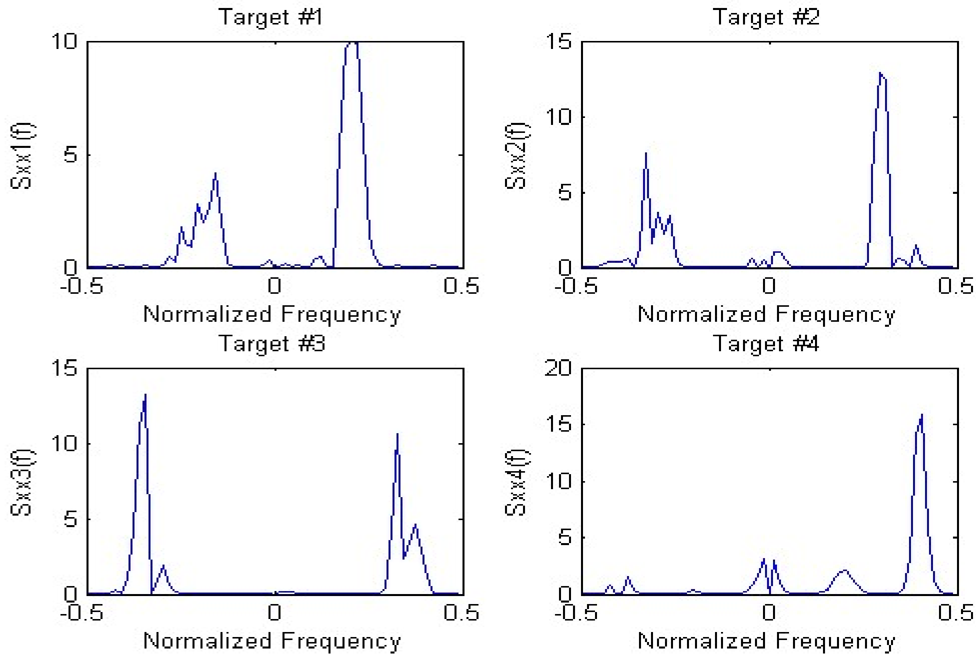


Fig5: The spectra of one realization of targets from the first target hypothesis set based on four proposed PSVs and four different non-Gaussian PDFs

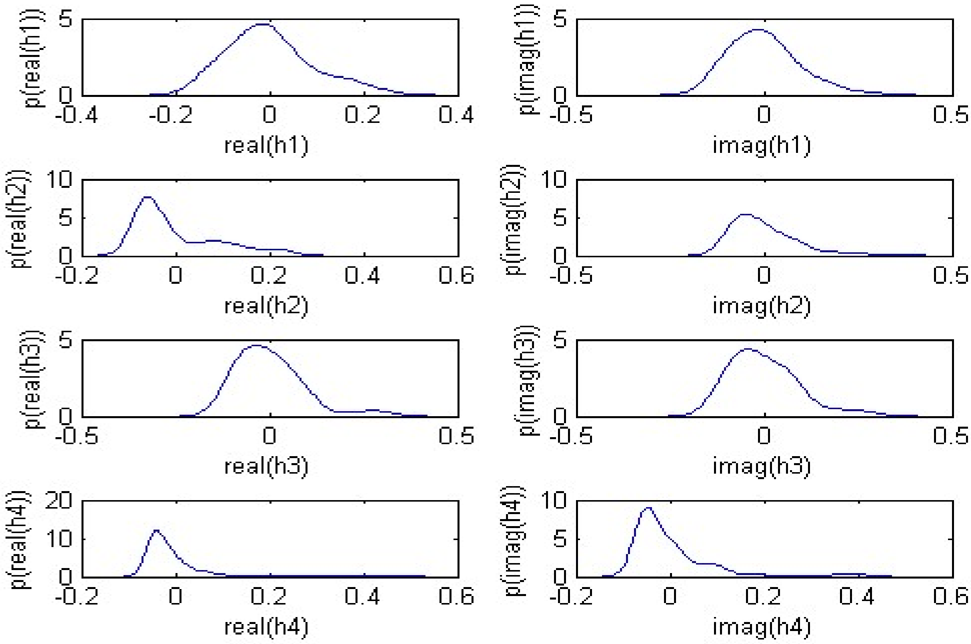


Fig6: PDF estimates the four target realization

The PDF estimates of one set of target realizations based on the proposed PDFs and PSVs are illustrated in Figure 6. The PDFs of both the real and imaginary values in the first target have a rough looking Rayleigh distribution with parameter $\sigma = 10$; the PDFs of both the real and imaginary values in the second target have a rough looking Exponential distribution with parameter $\mu = 2$; the PDFs of both the real and imaginary values in the third target have a rough looking Gamma distribution with parameter $k = 2$ and $\theta = 2$; the PDFs of both the real and imaginary values in the fourth target have a rough looking Log-Normal distribution with parameter $\mu = 0$ and $\sigma = 1$. These PDF estimates show that these target realizations all approximate the proposed non-Gaussian PDFs.

The time waveforms of one set of target realizations are illustrated in Figure 7. From the time waveforms, we can see the correlation in the data samples, which also indicate that the data samples in these targets are not independent due to the ZMNL transformation process.

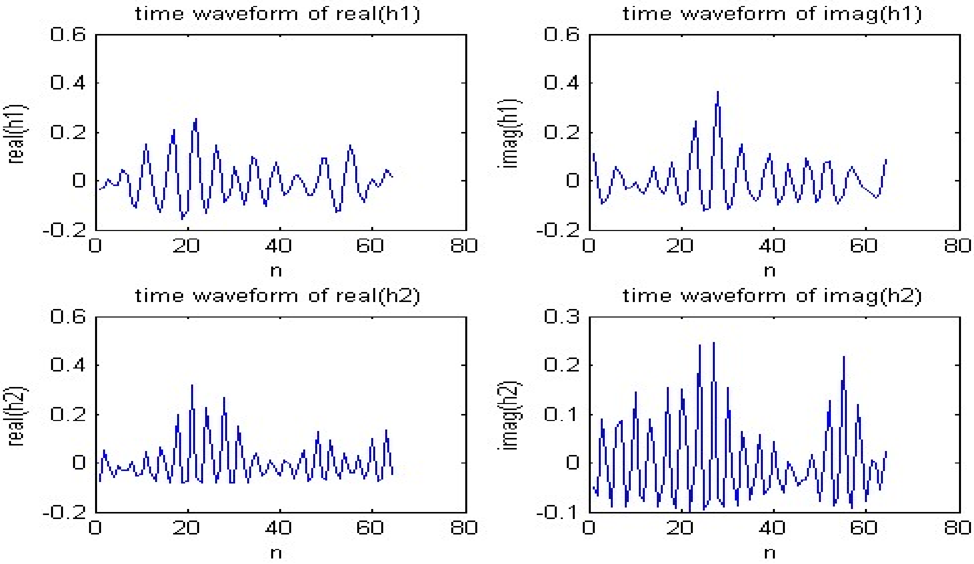


Fig7: Time waveforms of two target hypothesis

The normal distribution plots of the first two targets from one set of target realizations are presented in Figure 8. From these normal distribution plots, we can see that the data samples do not align to the red diagonal line in the normal distribution plot, which confirms that first two target realizations are not Gaussian distributed.

The normal distribution plots of the last two targets from one set of target realizations are presented in Figure 9. From these normal distribution plots, we can see that the data samples do not align to the red diagonal line in the normal distribution plot, which confirms that last two target realizations are not Gaussian distributed.

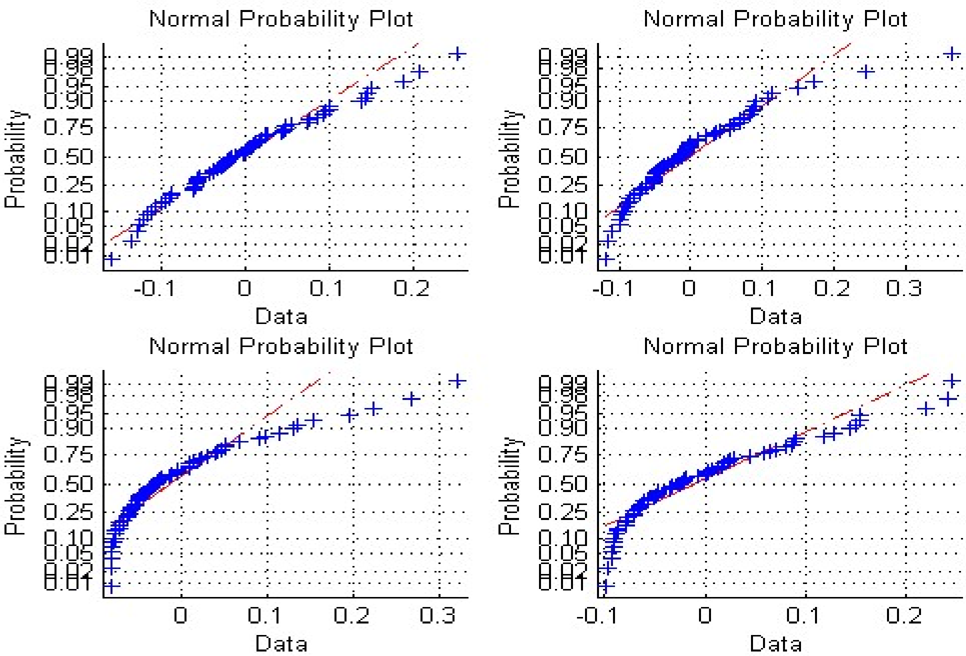


Fig8: The normal distribution plots of the first two targets from one set of target realizations

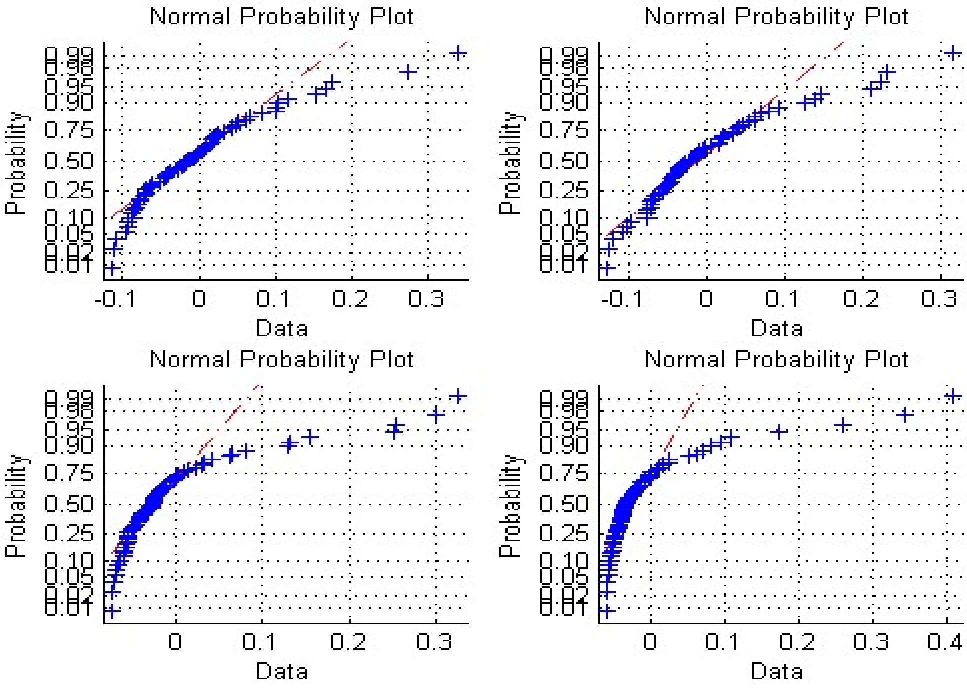


Fig9: The normal distribution plots of the last two targets from one set of target realizations

VII. Conclusion

Gaussian targets, the Bayesian mathematics of the GCCR MAP algorithm are simplified as closed-form expressions for the posterior density, and other terms can be derived with some effort. For arbitrary non-Gaussian targets, the basic Bayesian mathematics must be extended appropriately for the non-Gaussian case, then implemented numerically using concepts from statistical sampling theory. This is the primary contribution of this thesis. One key result of this article is the recognition that the Bayesian mathematics can be extended to the non-Gaussian case if the integrals that evaluate the expected value operation are replaced by online numerical ensemble average operations.

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