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# Joint Application Of Market And Income Approaches To New Machines And Equipment Valuation Under Uncertainty

Smolyak S.A.[1], Samatov A.S.[2], Rakhimova G.R.3, Odilova G. Sh.[3]

[1]Doctor of Central Economics and Mathematics Institute, RAS (Moscow).

[2]Candidate of economic sciences, associate professor of "Construction technology and its organization" department,

[3]Assistants of "Construction Technology and Organization" department of Samarkand State Architectural and Civil Engineering Institute.



**Abstract – A market, income, or cost approach is a term used to describe the methods used to value assets. Meanwhile, by combining multiple methodologies, correct valuation methods can be obtained. One of these ways is based on a formula given by D.S. Lvov at one point, which connects the market values of the assessed machine and its analogue while accounting for changes in machine operating characteristics and usable life. However, when calculating the formula, it is assumed that the machine's properties do not vary over time. For a case where the operating parameters of machines fluctuate throughout operation and the useful life is random, we suggest a generalization of this formula.**

**Keywords – Machinery, Equipment, Valuation, Benefits, Useful Life, Probability Distribution, Market Value, Cash Flow Discounting.**

## I. Introduction

Professional valuators value numerous objects in accordance with the International Valuation Standards (IVS), national valuation standards, and current legislation. Machinery and equipment are among the items valued (hereinafter referred to as "machinery"). The appraisal literature does not devote considerable emphasis to methods for determining the market value of machinery. Some of these are discussed in detail in (Asaul and etc., 2011; Fedotova, 2018). The majority of practical approaches, on the other hand, are founded on unduly rigid (and thus unrealistic) assumptions.

There are several sorts of value, the most important of which is market value (MV). The definition and comments on this subject in (IVS) span multiple pages. As a result, we will not provide such a definition, but we will note that the MV of an object on the valuation date reflects both the price of the object in a transaction taking place on that date between typical and economically rational market participants, as well as the benefits received by the object's owner from its future use.

This work focuses on the improvement of one of the commonly used methods for determining the MV of equipment in order to account for physical wear and tear as well as the uncertainty of terms of their useful life. (TUL).

Market value (MV) valuation methods are usually referred to as the cost, comparative or income approach. Most often, MV machines are valued using a comparative approach, using information on the market prices of their counterparts. In doing so, additive or multiplicative adjustments are made to the price of the analogue to take account of its differences from the machine being valued in one way or another. The income approach methods do not require information on the counterparts of the machine being valued. They use information about the economic benefits to the owner of the machine from its subsequent use.

Machinery produced under the same project is divided into new (not yet in service) and used machinery. The MV of used cars is usually estimated by properly adjusting the value of similar new cars (Smolyak, 2016; Fedotova, 2018). The methods of such adjustment are worth considering on their own, and we will not dwell on them. We will pay particular attention to the evaluation of new machines. In most cases, their MV are valued using a comparative approach based on manufacturers' or dealers' prices. However, it is often the case that the machines being valued are not available on the market at the valuation date. This is the situation discussed in this article.

It appears that there is an opportunity here to value MV machines using the ideas of the comparative and income approaches together. One method of this type is based on a formula proposed in (Lvov, 1969), which was widely used in Soviet times to assess the efficiency of new machinery and to set prices for new engineering products. In this article we first give the necessary definitions of the basic concepts, set out a brief derivation of the Lvov formula, and then show how it can be improved to more adequately take into account the specifics of the machine operating process.

## II. BASIC CONCEPTS. LVOV'S FORMULA

As a rule, the machines are used for their intended purpose, doing a specific job. This work has utility for market participants and therefore has a certain MV.The cost of some jobs (e.g. painting surfaces or transporting goods) can be estimated from market data, but many jobs are intermediate operations in the production process of the final product and it is difficult to estimate their MV.

The main characteristics of machine use in a given period are its productivity and operating costs (without depreciation and taxes). By the benefits of using a machine in a period, we mean the MV of the work it performs in that period, less the operating costs. Three circumstances need to be noted in relation to this definition.

1. The (IVS) does not explicitly define the concept of benefits, but our definition does not contradict other provisions of these standards.
2. The benefit indicator is similar in content to net income and EBITDA (earnings before interest, taxes, depreciation and amortization) used in business valuation.
3. The value of the benefits accruing from the machine over a period reflects both the MV of the machine's lease for that period. In the System of National Accounts (SNA 2008), which also estimates MV of capital assets using the comparative and income approaches, the benefits are treated as such, but referred to as services of [invested asset] capital.

Let us also assume that at the end of its useful life (UL) the machine is disposed of, yielding zero benefits. This assumption is justified because there is usually little difference between costs and revenues associated with disposal.

Taking into account that machines do not necessarily have a whole number of years of UL and that their characteristics change continuously, the process of using the machine will be considered in continuous time. In this case, it is convenient to use operating cost and benefit intensity indicators that reflect, respectively, the costs and benefits of using the machine for a small unit of time.

In the situation examined by D.S. Lvov, no inflation is assumed, time is measured in years and fractions of a year, and there is no influence of random factors. The useful life $T$ years, productivity $W$ and operating cost intensity (without depreciation and taxes) $Z$ are considered to be known about the machine.

Denote the market value per unit ofworkdonebythemachineby$p$. Its benefit intensity (D) can then be defined as the value of the work done per unit of time, less the associated transaction costs. This leads to the following formula:

$$D=pW-Z. \quad (1)$$

The relationship between the value of an asset and the benefits from its use is established through the principle of expectation of benefits. It is mentioned in (IVS) among the core principles, and its essence is explained in a number of provisions in the standard describing evaluation methods. However, there is no specific formulation of the principle in (IVS). In (Smolyak, 2016), this principle is stated in the following form, also applicable under uncertainty: *the value of the asset at the measurement date equals the expected discounted value of the benefits arising from its subsequent economic use in the forecast period (including the value of the asset at the end of the period)*.The forecast period can be selected at any time.

Since when designing a machine one tries to select economically rational values for its characteristics and typical machine owners use it for its intended purpose, we will assume that the above machine characteristics correspond to its economically rational use. In this section, the benefits of the machine are deterministic, so it follows from the expectation of benefits principle that the value of the new machine V equals the sum of the discounted (at the market pre-tax[1] continuous rate r, 1/year) benefits from its use:

$$V = \int_0^T De^{-rt}dt = D\frac{1-e^{-rT}}{r} = \left(pW - Z\right)m_c\left(T;r\right), \tag{2}$$

where$m_c(T;r)$ – the constant-income multiplier over time, defined by the formula:

$$m_c\left(T;r\right) = \int_0^T e^{-rt}dt = \frac{1-e^{-rT}}{r}. \tag{3}$$

Note that valuers usually determine market pre-tax discount rates ($E$) as a percentage per annum. The same rate for continuous time is calculated according to the formula: $r = \ln\left(1 + E/100\right)$.

If the cost of work $p$ is known, the machine could be valued using the formula. (2). But, as noted, this is not always possible. It is for such situations that Lvov's formula has been proposed. In essence, it is based on a comparative approach to valuing MVs. It is assumed that a similar (doing the same job) new machine can be found on the market, of which not only its performance is known $W_a$, operating cost intensity $Z_a$ and UL$T_a$, but alsoMV$V_a$. For this machine, formula (2) is written as follows:

$$V_a = \left(pW_a - Z_a\right)m\left(T_a;r\right). \tag{4}$$

From this equation you can find the unknown MV of the unit of work:

$$p = \frac{V_a + Z_a m\left(T_a;r\right)}{W_a m\left(T_a;r\right)}. \tag{5}$$

It is easy to see that the resulting $p$-value is equal to the ratio of the total discounted lump-sum and running costs of the work over the lifetime of the analogue to the total discounted workload, which is fully consistent with the requirements of the cost approach to valuation (IVS; Fedotova, 2018). In other words, the unit cost of the work when the comparative and income approaches are applied together is exactly the same as the unit discounted cost of producing that work using a peer machine.

Substituting the value of $p$ from (4) into formula (1), we obtain:

$$V = V_a \cdot \frac{W}{W_a} \cdot \frac{m\left(T;r\right)}{m\left(T_a;r\right)} + \left[Z_a\frac{W}{W_a} - Z\right] \cdot m\left(T;r\right). \tag{6}$$

It was this formula (in a somewhat different form and for discrete time) that was proposed by D.S. Lvov in his doctoral dissertation and book (Lvov, 1969), and was then widely used in the SU to assess the efficiency of new machinery (Methodology, 1977) and to set prices for new industrial products. In (Asaul, 2011) and a number of other sources, the corresponding valuation method is referred to as the Equivalent Analogue Method.

As can be seen from formula (6), the value of the machine can be found by adjusting the value of the analogue to take account of its differences in productivity, operating costs and UL, which corresponds to the comparative approach to valuation.

[1] We do not include income tax as an operating cost. The benefit indicator then becomes 'pre-tax'. In this case, according to valuation standards, a pre-tax discount rate must also be used.

Note that formula (6) is also applicable to second-hand machines, but it is difficult to obtain information on UL, productivity and operating costs for such machines.

Let us point out two significant disadvantages of Lvov's formula.

1. There is no such thing as a machine whose technical and economic performance does not change during operation. It is true that sometimes examples of such valuations are given in the valuation literature for illustrative purposes (referred to in the English language literature as "one-hoss shay").
2. 2. The UL of the machine is assumed to be known. However, identical machines produced at the same time and operating under the same conditions are not retired at the same time. In reliability theory, the service lives of machines (which may differ from the UL) are treated as random.

Next, we will try to address these shortcomings, using the same technique as in (Smolyak, 2018), to make appropriate changes to the Lvov formula.

## III. Consideration of the dynamics of the machine's technical and economic performance

Above, the technical and economic performance of the machine was considered to be independent of its age. In this section we will consider such a correlation. Generally, with increasing age, there is a general tendency for the machine's productivity to decrease and its operating costs to increase.Deviations from this trend are usually caused by factors that are difficult to take into account in a practical evaluation. Nevertheless, by analyzing the performance of machines of different ages, it is possible to identify a general trend for a number of machines. In (Smolyak, 2016), information is provided on the age dependence of these indicators for some types of machines, derived from a small number of available publications on the subject. It turns out that, in most cases, the decline in productivity and the increase in operating costs with age follows a linear pattern. Accordingly, by virtue of formula (1), the intensity of the machine's benefits should also decrease linearly with age.It remains to be seen that the typical market participant who owns a car will use it as long as it brings positive benefits. Thus, the machine will reach the end of its life when its Benefit Intensity (BI) has not turned to zero. Such a term in the valuation standards (IVS), is referred to in International Financial Reporting Standards (IFRS 16) and valuation literature as economic life (or economic life).

Consider a machine with an initial BI of *D* and a service life of *T* years under rational operation. In such a case, by virtue of the assumption made, the BI will decrease at a rate of *D/T* during its lifetime and will therefore be *D(1-t/T)* after t years of operation.In this case, the sum of the discounted benefits of the machine would be

$$\int_0^T D\left(1-\frac{t}{T}\right)e^{-rt}dt = Dm_l\left(T,r\right), \quad (7)$$

where$m_l(T; r)$ – the multiplier of income decreasing linearly to zero, defined by the formula:

$$m_l\left(T,r\right)=\frac{rT+e^{-rT}-1}{r^2T}. \quad (8)$$

As you can see, formula (7) differs from (2) only in that it includes a different multiplier $m_l$ instead of $m_c$. All subsequent calculations in section 2 remain unchanged, and we obtain the same formula (5) for the value of the machine being valued, only with a multiplier of $m_l$.But even after this replacement, formula (5) will not adequately reflect the actual processes of machine operation. The point is that all previous reasoning has assumed that all new machines (as well as their new counterparts) have the same UL. However, this assumption is not borne out in practice.In the next section we will try to address this shortcoming.

## IV. Consideration of the probabilistic nature of useful life

In reliability theory, it is assumed that the operation of a machine ends at the end of its useful life when the machine reaches a certain technical limit state (so the useful life is generally longer than the UL). In addition, it is taken into account here that machine operating processes are of a probabilistic nature, so that the useful lives of machines will also be random.There are many publications on the mathematical modelling of machine operating processes and the determination of average values and coefficients of variation of machine life, e.g, (Bekker, 1991; Erumban, 2008; Linetal., 2014; Wangetal., 2011). Some experts have carried out accelerated reliability tests on machines, or collected data on their useful life failures, approximating the data by a normal or other known distribution. The national accounting systems of various countries use and publish average values and probability distributions of the UL of capital assets, but they refer to excessively large groups of assets (e.g. machines used in the electricity sector). It seems that the coefficients of variation of useful lives found by these methods can also be used in relation to useful lives (UL).

As you can see, the available information is not sufficient to take into account the probabilistic nature of a particular machine in the practical evaluation of its UL. Therefore, methods have been proposed to approximate the mean and coefficient of variation of the UL, based on the information available to the assessors. For example, guidelines for determining the UL of machines on the basis of their depreciation, standard or assigned periods are given in a handbook (Leifer, 2019), and recommended lifetime variation coefficient values in (RD 26-01-143-83, Tables 1-2 of Annex 3; Ostreikovsky, 2003). In (Smolyak, 2021) it was proposed to divide machines into three classes according to the values of the variation coefficient of UL and indicated the attributes by which assessors can expertly assign a particular machine to one or another class.

In this paper, it is assumed that the ULs of machines have a gamma distribution, as is the case, for example, in the German System of National Accounts.The gamma distribution is concentrated on the positive side of the numerical axis and is defined by two parameters□and a scale parameter *L*. The density of this distribution is: $p\left(x\right)=x^{\alpha-1}\frac{e^{-x/L}}{L^{\alpha}\Gamma\left(\alpha\right)}$, where□ – Euler gamma function. A random variable with this distribution has a mean*L*□ andvariation coefficient □$^{-1/2}$.

It is assumed that the information available allows the evaluator to estimate the average value (S) and the coefficient of variation of the UL of the machine being evaluated($v$). In this case, the parameters of the gamma distribution are $L = Sv^2$and□ = $1/v^2$.

The probabilistic nature of a machine's UL can be taken into account as follows. If the UL of machine T were a deterministic quantity, its MV, by virtue of formula (7), would be equal to $Dm_l\left(T,r\right)$, where*D*-the intensity of the benefits the machine brings at the beginning of its operation.However, in the case of a random UL, the valuation standards require that the MV of a machine be determined by the expected value of the total discounted benefits of its operation. It can be approximated by taking the mathematical expectation from a random variable $Dm_l\left(T,r\right)$. This leads to the following expression for the MV of the machine:

$$V=\int_0^{\infty}D\frac{rT+e^{-rT}-1}{r^2T}T^{\alpha-1}\frac{e^{-T/L}}{L^{\alpha}\Gamma\left(\alpha\right)}dT=\frac{D}{r^2}\left[r-\frac{1-\left(1+rL\right)^{1-\alpha}}{\left(\alpha-1\right)L}\right].$$

If you put it here $L = Sv^2$, □ = $1/v^2$и$D=pW-Z$from formula (1), this expression will take the form similar to (2):

$$V=Dm_{lg}\left(S;v;r\right)=\left(pW-Z\right)m_{lg}\left(S;v;r\right), \tag{9}$$

where$m^*(S;v;r)$ – the linearly decreasing income multiplier corresponding to the gamma distribution of the UL, defined by the formula:

$$m_{lg}\left(S;v;r\right)=\frac{1}{r^2}\left[r-\frac{1-\left(1+rSv^2\right)^{1-v^{-2}}}{\left(1-v^2\right)S}\right]. \tag{10}$$

Further reasoning proceeds in the same way as in the derivation of Lvov's formula (5). This results in the following formula, similar to (5), but taking into account the dynamics of its main technical and economic characteristics and the probabilistic nature of its useful life:

$$V = V_a \cdot \frac{W}{W_a} \cdot \frac{m_{lg}\left(S;v;r\right)}{m_{lg}\left(S;v;r\right)} + \left[ Z_a \frac{W}{W_a} - Z \right] \cdot m_{lg}\left(S;v;r\right). \tag{11}$$

In contrast to Lvov's "original" formula, instead of the given UL of the machine and the analogue, (11) includes the average values and variation coefficients of these terms.

As you can see, even with the random machine UL, the formula is the same, only the expression for the income multiplier has changed.

Figure 1 shows the multiplier dependencies$m_{lg}(S; v; r)$ from$S$for$r = 0.1$anddifferent$v$, in fig. 2. $m_{lg}(S; v; r)$ from$r$fordifferent$S$and$v$.

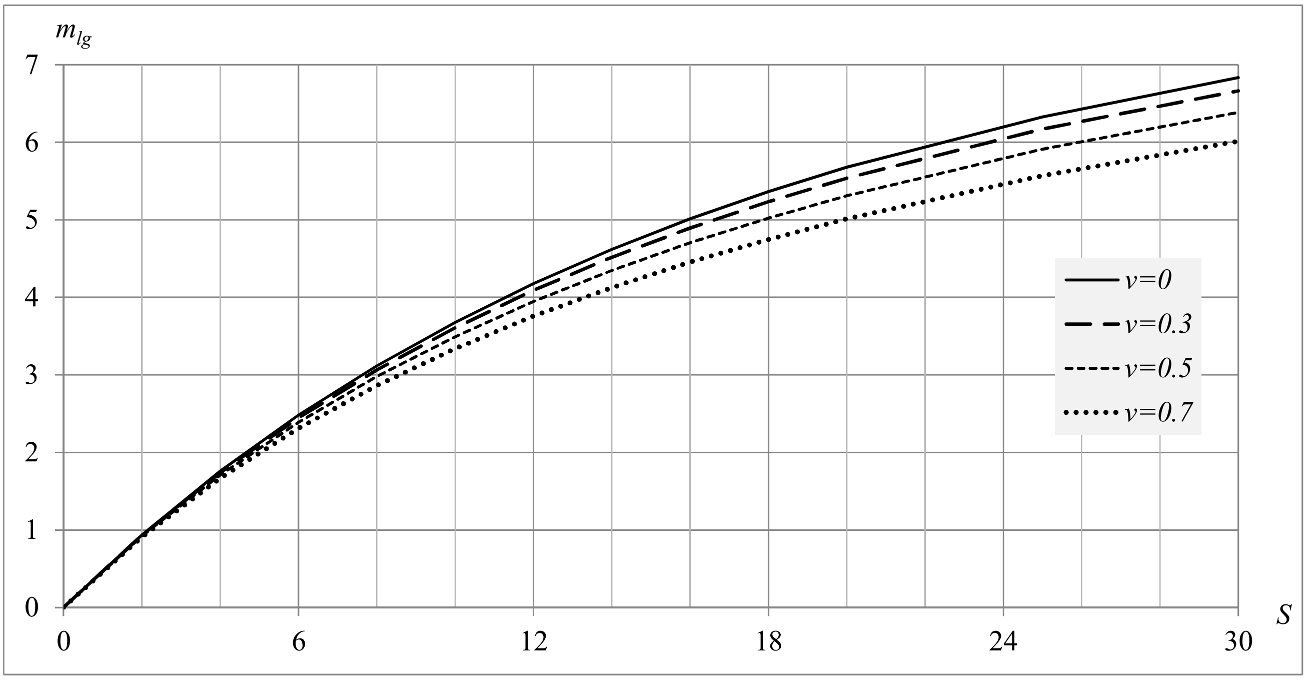


Рис. 1. Зависимости мультипликатора $m_{lg}$ от среднего срока службы ($S$) для $r = 0.1$ и разных $v$.

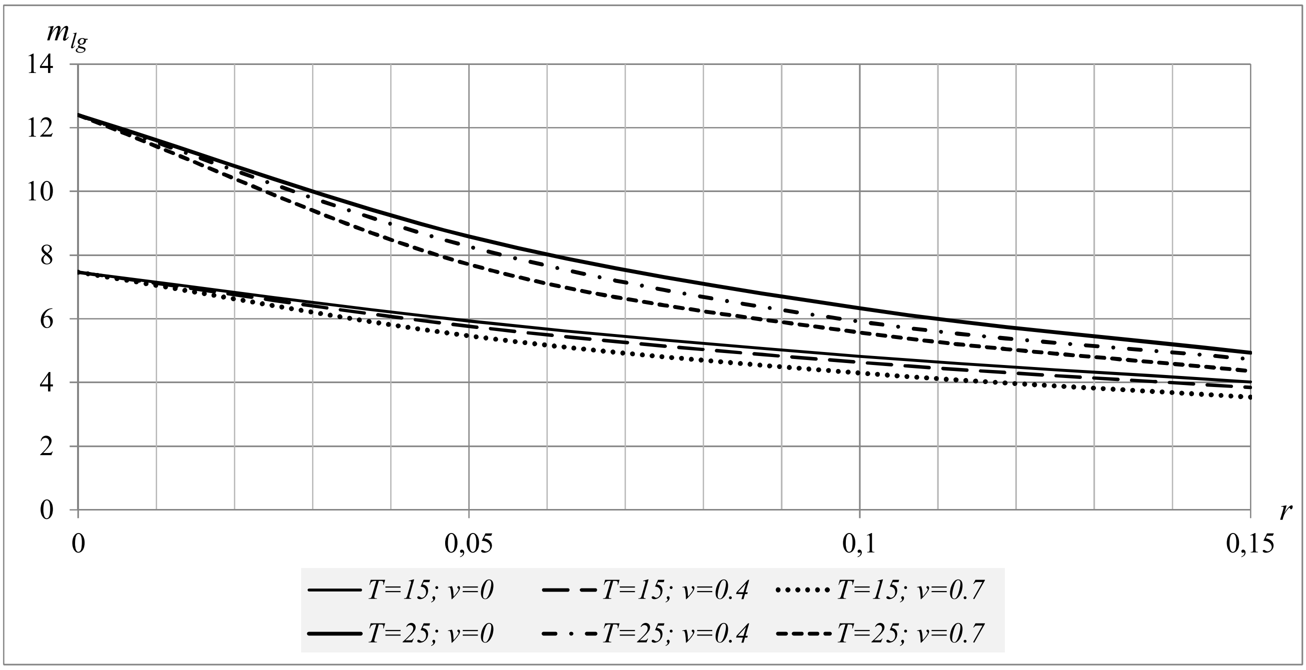


Fig. 2. Dependencies of the multiplier $m_{lg}$ on the discount rate ($r$) for different average values ($S$) and coefficients of variation ($v$) of lifetime.

**Example**. The productivity of the machine is 17% higher than its counterpart. The cost of the counterpart is 10 million roubles. The operating costs of the machine and its counterpart are 86and 79 million roubles, respectively, and their standard useful lives are 8 and 11 years. The machine and its counterpart are both general-purpose series equipment. According to (Leifer, 2019), the average UL of such machines will be 1.84 times the normative lifetime. In this case, the average UL of the machine and the analogue are 14.72 and 20.24 years, respectively.The coefficient of variation of the service life for both machines, taking into account the recommendations (Ostreikovsky, 2003), is taken as 0.4, the discount rate as 0.10. In this case$m_{lg}$(14.72; 0.4; 0.1)=4.577,$m_{lg}$(20.24; 0.4; 0.1)=5.467, and the MV of the machine will be:

$$V = 10 \cdot 1.17 \cdot \frac{4.577}{5.467} + \left[79 \cdot 1.17 - 86\right] \cdot 4.577 = 39.2 \text{ млн. руб.}$$

Note that a calculation using the 'normal' Lvov formula, i.e. using formula (2) and the average service life of both machines would give one and a half times the cost: $V$=59.9million roubles. ■

In this and the previous sections, it has been assumed that the main characteristics of the machines deteriorate with increasing age. Meanwhile, a number of authors believe that with proper maintenance they can be kept constant. Examples include equipment such as aircraft engines that provide constant performance over a set number of operating hours. In the meantime, the probability of failure, and therefore the expected damage from possible failure, increases with time. In order to account for this correctly, operating costs need to include expected damage from possible failure, and this increases with age/maintenance.

## V. Impact of more realistic assumptions

In the reasoning above, a number of not very realistic assumptions have been made in order to simplify. They can be freed from them by applying the general technique detailed in (Smolyak, 2016).

***Accounting for inflation and the risk of accidents***. In deriving formula (11), it is assumed that there is no inflation and that the machine is not subject to accidents that take it out of service (in reliability theory these are called resource failures). It turns out that to take these factors into account, the discount rate $r$ in the calculation formulas must be determined differently: $r = r_0 + r_f - i$, where$r_0$ – pre-tax nominal risk free rate, $r_f$ – the likelihood of a machine crash during the year, $i$ – the rate of increase in the price of a given type of machine.

***Accounting for utilisation value***. Up to now, the utilisation value of cars has been considered zero. In fact, for some types of machine it can be relatively large (up to 15% of its MV). It turns out that in this situation Lvov's formula and the above generalisations of it .

When taking this circumstance into account approximately, the "depreciated value" (the difference between the market value and the salvage value) should be used instead of the MV of the machines in Lvov's formula and the generalisations mentioned above. Inparticular, formula (11) willlooklikethis:

$$V = \left(V_a - U_a\right) \cdot \frac{W}{W_a} \cdot \frac{m_{lg}\left(S;v;r\right)}{m_{lg}\left(S;v;r\right)} + \left[ Z_a \frac{W}{W_a} - Z \right] \cdot m_{lg}\left(S;v;r\right) + U,$$

where$U$and $U_a$ – the scrappage values of the machine and its counterpart.

***Other life time distributions***. In deriving formula (11), it was assumed that the life time of the machine has a gamma distribution. However, this distribution may be different. For example, many experts believe that machine life has a Weibull distribution. In such cases, the formula for the multiplier $m_{lg}$ will change. However, we have checked how different this multiplier would be if it were defined for the gamma and Weibull distributions (where $m_{lg}$ turns out to be much more complex). In the typical ranges of variation of $S$, $v$ and $r$ parameters for real machines, the difference is found to be insignificant.

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