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A Case of a Particular Type of N-Soft Sets as a Generalization of Bijective Soft Sets

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Abstract – Previous studies introduced the concepts of bijective soft sets and bijective fuzzy soft sets as the particular types of soft sets and fuzzy soft sets, respectively, and their application in decision-making. In this article, it is found that the decision-making algorithm on the bijective soft set or the bijective fuzzy soft set can be applied to an N-soft set by providing adjustments to the operations on the N-soft set. This opens the opportunity to theoretically study a specific type of an N-soft set as a generalized form of a bijective soft set.

Keywords – ***Soft Set; Bijective Soft Set; Bijective Fuzzy Soft Set; N-Soft Set***

I. Introduction

The problem of uncertainty often occurs in various areas of life because it involves uncertain data. One example of this problem is the problem of decision making. In 1965, Prof. L.A. Zadeh [5] introduced the fuzzy set theory (FS) to solve decision-making problems on objects whose data contains an element of uncertainty associated with one particular parameter. In FS theory, the membership values of objects are studied, where the membership values are in the interval [0, 1]. However, FS theory has limitations, so that in 1999 Molodtsov [1] introduced soft set theory (SS) to deal with decision-making problems involving more than one parameter or attribute. SS theory is used for grouping objects against a certain parameter or attribute. If these objects meet or are related to a certain parameter or attribute, then they are given a value of 1, otherwise, they are given a value of 0. However, the fact is that in everyday life the problems encountered cannot always be evaluated with the numbers 0 and 1, or expressed in a real number between 0 and 1 but can also be evaluated by a value called a rank or a grade which is a whole number. So, in 2018, Fatimah et al. [2] generalized the SS theory called the N-soft set (NSS), in which the NSS examines ranked objects that meet a parameter or attribute.

In 2010, Gong et al. [3] introduced the bijective soft set (BSS) concept. BSS is a special type of SS where each object is only associated with one particular parameter or attribute of the many parameters in the SS. Furthermore, Gong et al. In 2013[4] also introduced a special type of fuzzy soft set (FSS), which is a combination of the concepts of SS and FS, called the bijective fuzzy soft set (BFSS). This BFSS is an amalgamation of the concepts of BSS and FSS. Inspired by the BFSS theory where the membership value of an object is not only 0 and 1, it is interesting to study an example of a case from NSS which, under certain conditions, can meet conditions such as in BSS or BFSS, so that a special type is also obtained from NSS. Of course, from the results of this study, it is hoped that mathematical theoretical studies will be developed, as is the case in [3] and [4], related to this special type of NSS.

II. Fuzzy Set, Soft Set, Fuzzy Soft Set, N-Soft Set, Bijective Soft set, and Bijective Fuzzy Soft Set

In this section, we will review some related definitions introduced in the previous article, which form the basis for the study in this article.

Definition 2.1. [5] *Let U be a set of objects. A fuzzy set (FS) A over U is defined as:*

$$A = \{(u, \mu(u)) |\ u \in U\},$$

where $\mu : U \to [0,1]$ *is the membership function, and* $\mu(u)$ *is the degree of membership (the membership value) of* $u \in U$ *in fuzzy set A.*

Definition 2.2. [1] *Let U is a set of objects, P(U) is the power set of U and E is a set of attributes. The pair* (F, E) *is called a soft set (SS) over U if and only if F is a function given by F: E→P (U), which can be expressed as a set of ordered pairs of the form*

$$(F, E) = \{(\varepsilon_i, F(\varepsilon_i)) | \varepsilon_i \in E, F(\varepsilon_i) \in P(U)\}.$$

A soft set can be expressed in the form of a representation table as in Table 1. In this case $\mu_{\varepsilon_j}(u_i) = 1$ if $u_i \in F(\varepsilon_j)$ and $\mu_{\varepsilon_j}(u_i) = 0$ if $u_i \notin F(\varepsilon_j)$. The SS (F, E) can be expressed as $(F, E) = \left\{\left(\varepsilon_j, \left\{\left(u_i, \mu_{\varepsilon_j}(u_i)\right) \middle| u_i \in U\right\}\right) | \varepsilon_j \in E\right\}$.

Table 1: Representation of a soft set

(F, E)	ε_1	ε_2	…	ε_i	…	ε_n
u_1	$\mu_{\varepsilon_1}(u_1)$	$\mu_{\varepsilon_2}(u_1)$	…	$\mu_{\varepsilon_i}(u_1)$	…	$\mu_{\varepsilon_n}(u_1)$
u_2	$\mu_{\varepsilon_1}(u_2)$	$\mu_{\varepsilon_2}(u_2)$	…	$\mu_{\varepsilon_i}(u_2)$	…	$\mu_{\varepsilon_n}(u_2)$
:						
u_j	$\mu_{\varepsilon_1}(u_j)$	$\mu_{\varepsilon_2}(u_j)$	…	$\mu_{\varepsilon_i}(u_j)$	…	$\mu_{\varepsilon_n}(u_j)$
:						
u_m	$\mu_{\varepsilon_1}(u_m)$	$\mu_{\varepsilon_2}(u_m)$	…	$\mu_{\varepsilon_i}(u_m)$	…	$\mu_{\varepsilon_n}(u_m)$

Definition 2.3.[1] *Let* (F, A) *and* (G, B) *be two SS. The operation AND,* (F, A) *AND* (G, B) *denoted by* $(F, A) \wedge (G, B)$ *is defined as* $(H, A \times B)$ *where* $H(a, b) = F(a) \cap G(b)$.

Definition 2.4.[3] *Let E be a set of attributes. Let* (F, E) *be an SS over U where F is the mapping given by* $F: E \to P(U)$. *The pair* (F, E) *is called a bijective soft set (BSS) over U if*

1. $\bigcup_{e \in E} F(e) = U$,
2. *For any two attributes,* $e_i, e_j \in E, e_i \neq e_j$, *then* $F(e_i) \cap F(e_j) = \emptyset$.

Definition 2.5.[6] *Let* (F, A) *and* (G, B) *be two soft sets over U. A pair* (F, A) *is called a soft subset of* (G, B) *denoted by* $(F, A) \subseteq (G, B)$ *if*

1. $A \subset B$,
2. $\forall e \in E, F(e) \subseteq G(e)$.

Definition 2.6.[3] *Let U be a set of objects,* $X \subseteq U$. *Let* (F, E) *be a bijective soft set over U. The restricted operation AND between* (F, E) *and X denoted by* $(F, E) \wedge_R X$ *is defined as*

$\bigcup_{e \in E}\{F(e) | F(e) \subseteq X\}$ *for each* $e \in E$.

Definition 2.7.[3] *Let* (F, E) *and* (D, C) *be two BSSs over U, where* $E \cap C = \emptyset$. (F, E) *is said to depend on* (D, C) *with a degree* κ, $0 \leq \kappa \leq 1$, *which is denoted by* $\gamma((F, E), (D, C))$ *if*

$$\kappa = \frac{|\cup_{e \in C}((F,E) \wedge_R D(e))|}{|U|}, \tag{1}$$

where $|\bullet|$ *is the cardinality of a set.*

If $\kappa = 1$ *then (F, E) is said to be fully dependent on* (D, C).

If $\kappa = 0$ *then (F, E) is said to be completely independent of* (D, C).

Definition 2.8.[3] *Let U be a set of objects. Let* (F, E) *be a set of n BSSs* (F_i, E_i) *over U which is symbolized by* $\wedge_{i=1}^{n}(F_i, E_i)$ *for* $i, j = 1, \dots, n$ *where* $\forall i \neq j, E_i \cap E_j = \emptyset$. *Let* (G, B) *is a BSS over U, with* $B \cap E_i = \emptyset$. $((F, E), (G, B), U)$ *is called a BSS decision system over U, with* (F, E) *and* (G, B) *respectively called the condition and decision of the BSS decision system over U.*

Definition 2.9.[3] *Let* $((F, E), (G, B), U)$ *be a BSS decision system over U, where* $(F, E) = \wedge_{i=1}^{n}(F_i, E_i)$ *and* (F_i, E_i) *is a BSS. The dependence between* $(F_1, E_1) \wedge (F_2, E_2) \wedge \dots \wedge (F_n, E_n)$ *and* (G, B) *is called the BSS decision system dependency* $\big((F, E), (G, B), U\big)$ *denoted by* $\gamma(\wedge_{i=1}^{n}(F_i, E_i), (G, B))$ *and defined as*

$$k = \frac{\left|\cup_{b \in B}(\wedge_{i=1}^{n}(F_i, E_i) \wedge_R G(e))\right|}{|U|}. \tag{2}$$

Definition 2.10. [3] *Let* $\big((F, E), (G, B), U\big)$ *be a BSS decision system over U where* (F, E) *is the set of n BSS* (F_i, E_i) *over U which is symbolized by* $\wedge_{i=1}^{n}(F_i, E_i)$. $\gamma\big((F, E), (G, B)\big)$ *is a dependency* $(F_1, E_1) \wedge (F_2, E_2) \wedge \dots \wedge (F_n, E_n)$ *on* (G, B). *If*

$$\gamma(\wedge_{i=1}^{m}(F_i, E_i), (G, B)) = \gamma(\wedge_{i=1}^{n}(F_i, E_i), (G, B)), \tag{3}$$

then $\wedge_{i=1}^{m}(F_i, E_i)$ *is called the reduction of the BSS decision system* $\big((F, E), (G, B), U\big)$.

In the following, an algorithm for decision-making problems will be presented as an application of the BSS concept [3].

Algorithm:

1. Define BSS decision system $(\wedge_{i=1}^{n}(F_i, E_i), (F_{n+1}, E_{n+1}), U)$.
2. Calculate the dependency of the BSS decision system $(\wedge_{i=1}^{n}(F_i, E_i), (F_{n+1}, E_{n+1}), U)$ based on Definition 2.9.
3. Based on Definition 2.10, determine the reduction of the BSS decision system $(\wedge_{i=1}^{n}(F_i, E_i), (F_{n+1}, E_{n+1}), U)$.
4. Based on the reduction of the BSS decision system $(\wedge_{i=1}^{n}(F_i, E_i), (F_{n+1}, E_{n+1}), U)$, a decision rule is obtained.

Theorem 2.11.[3] *Let* (F, A) *and* (G, B) *be two BSS.* $(F, A) \wedge (G, B) = (H, A \times B)$ *is also a BSS.*

Definition 2.12.[6] *Let U be a set of objects, E is a set of attributes, and* $A \subseteq E$. *A fuzzy soft set (FSS)* G_A *over U is a set by defining a function* g_A, *which can be represented in the set*

$$G_A = \{(a, g_A(a)) | a \in A, g_A(a) \in I^U\},$$

where $g_A : A \to I^U$ *and* I^U *is a collection of FSs over U. In this case,*

$$g_A(a) = \{(u, \mu_a(u)) | u \in U\},$$

with $\mu_a : U \to [0,1]$. *Then, it can be expressed that* $G_A = \{(a, \{(u, \mu_a(u)) | u \in U\}) | a \in A\}$.

With this definition, an FSS can be expressed in the form of a representation table as in Table 2. In this case $\mu_{a_j}(u_i) \in [0,1]$ is the degree of membership of the u_i associated with the parameter a_j.

Table 2: Representation of a fuzzy soft set

G_A	a_1	a_2	□□□	a_i	□□□	a_n
u_1	$\mu_{a_1}(u_1)$	$\mu_{a_2}(u_1)$	□□□	$\mu_{a_i}(u_1)$	□□□	$\mu_{a_n}(u_1)$
u_2	$\mu_{a_1}(u_2)$	$\mu_{a_2}(u_2)$	□□□	$\mu_{a_i}(u_2)$	□□□	$\mu_{a_n}(u_2)$
□						
u_j	$\mu_{a_1}(u_j)$	$\mu_{a_2}(u_j)$	□□□	$\mu_{a_i}(u_j)$	□□□	$\mu_{a_n}(u_j)$
□						
u_m	$\mu_{a_1}(u_m)$	$\mu_{a_2}(u_m)$	□□□	$\mu_{a_i}(u_m)$	□□□	$\mu_{a_n}(u_m)$

Definition 2.13. [6] *Let F_A and G_B are 2 FSSs over U. Operation AND, F_A AND G_B which is notated with $F_A \wedge G_B$ is defined as* $(H, A \times B)$ where $H(a,b) = \{(c, h_C(c)) | c \in C, h_C(c) \in I^U\}$, *with $C = A \times B$, $h_C(c) = \{(u, \mu_c(u)) | u \in U\}$. In this case* $\mu_c(u) = \min(\mu_a(u), \mu_b(u))$, *$a \in A$ and $b \in B$.*

Definition 2.14.[4] *Given an FSS*

$$F_A = \{(a, \{(u, \mu_a(u)) | u \in U\}) | a \in A\},$$

over U. Let $\lambda \in [0,1]$. *A λ - level soft set of FSS F_A is an SS (G, A) with*

$$(G, A) = \left\{\left(a, \left\{\left(u, \xi_{u_\lambda}(a)\right) \middle| u \in U\right\}\right) | a \in A\right\},$$

where $\xi_{u_\lambda}(a) := \begin{cases} 1, & if\ \mu_a(u) \geq \lambda, \\ 0, & others. \end{cases}$

In this case, $\xi_{u_\lambda}(a)$ is called a characteristic function $\lambda -$level soft set of the FSS F_A.

Definition 2.15.[4] *Let A be a set of parameters. Let (G, A) be a λ - level soft set of FSS F_A over U. F_A is called $\lambda -$level bijective fuzzy soft set if (G, A) is a BSS.*

To be more concise, a $\lambda -$*level bijective fuzzy soft set will be called a bijective fuzzy soft set (BFSS) when λ is clear.*

Definition 2.16. [4] *Let U be a set of objects and let I^U be a collection of FSs over U. Let G_B and F_E be two $\lambda -$level bijective fuzzy soft sets over U, and H_E is a combination of $\lambda -$level bijective fuzzy soft set of F_E (see Definition 2.18). The restricted operation AND between H_E and $g_B(b)$ which is denoted by $H_E \wedge_R g_B(b)$ defined as*

$\bigcup_{e \in E}\{h_E(e) | h_E(e) \subset g_B(b)\}$ *for some $b \in B$,*

with $h_E(e) = \left\{\left(u, \mu_e(u). \xi_{u_\lambda}(e)\right) | u \in U\right\}$. *In this case,* $h_E(e) \subset g_B(b)$ means $h_E(e)$ is a fuzzy subset of $g_B(b)$.

Definition 2.17. [4] *Suppose F_A and G_B are two $\lambda -$level bijective fuzzy soft set over U, with $A \cap B = \emptyset$. Let* $J_C = \bigcup_{b \in B}(F_A \wedge_R g_B(b))$. *$F_A$ is said to depend on G_B with a degree β, $0 < \beta < 1$, denoted by $\gamma(F_A, G_B)$ if*

$$\beta = \frac{T_C}{T_A} \qquad (4)$$

where $T_C = \sum_{u \in U} \sum_{c \in C} \mu_c(u)$ and $T_A = \sum_{u \in U} \sum_{a \in A} \mu_a(u)$ for $(u, \mu_c(u)) \in j_C(c)$ and $(u, \mu_a(u)) \in f_A(a)$.

If $\beta = 1$ then F_A is said to be fully dependent on G_B. If $\beta = 0$ then F_A is said to be independent of G_B.

Definition 2.18 [4] *Let F_A be a λ -level bijective fuzzy soft set over U with*

$$F_A = \{(a, \{(u, \mu_a(u)) | u \in U\}) | a \in A\}.$$

Let (G, A) is a λ −level bijective fuzzy soft set F_A with

$$(G, A) = \left\{\left(a, \left\{\left(u, \xi_{u_\lambda}(a)\right) \middle| u \in U\right\}\right) | a \in A\right\}.$$

The combination of λ −level bijective fuzzy soft set of F_A denoted by H_A is defined as:

$$H_A = \{(a, H(a)) | a \in A\},$$

with $H(a) = \left\{\left(u, \mu_a(u). \xi_{u_\lambda}(e)\right) | u \in U\right\}$.

From Definition 2.18 above it is clear that H_A is an FSS.

Definition 2.19.[4] *Let (F_E, G_B, U) be a λ – level bijective fuzzy soft set decision system over U, where F_E is a set of λ- level bijective fuzzy soft sets $F_{i_{E_i}}$ over U symbolized with $\wedge_{i=1}^{n} F_{i_{E_i}}$ for $i = 1, \dots, n$. Let $I_D = F_{1_{E_1}} \wedge F_{2_{E_2}} \wedge \dots \wedge F_{n_{E_n}}$ and let H_D be the combination λ - level bijective fuzzy soft set of I_D. Let $J_E = \cup_{b \in B}(H_D \wedge_R g_B(b))$. The dependency between $I_D = F_{1_{E_1}} \wedge F_{2_{E_2}} \wedge \dots \wedge F_{n_{E_n}}$ and G_B is called λ - level bijective fuzzy soft set decision system dependency (F_E, G_B, U) which is notated with $\gamma(I_D, G_B)$ and defined as*

$$\kappa_\lambda = \frac{T_E}{T_D}, \tag{5}$$

where $T_E = \sum_{u \in U} \sum_{e \in E} \mu_e(u)$ *and* $T_D = \sum_{u \in U} \sum_{d \in D} \mu_d(u)$ *for* $(u, \mu_e(u)) \in j_E(e)$, *and* $(u, \mu_d(u)) \in h_D(d)$.

Definition 2.20.[4] *Let (F_E, G_B, U) be a decision system of λ - level bijective fuzzy soft set where F_E is a set of λ-level bijective fuzzy soft sets $F_{i_{E_i}}$ over U symbolized by $\wedge_{i=1}^{n} F_{i_{E_i}}$ for $i = 1, \dots, n$. $\gamma(\wedge_{i=1}^{n} F_{i_{E_i}}, G_B)$*

is dependency $F_{1_{E_1}} \wedge \dots \wedge F_{n_{E_n}}$ of G_B. If

$$\gamma(\wedge_{i=1}^{m} F_{i_{E_i}}, G_B) = \gamma(\wedge_{i=1}^{n} F_{i_{E_i}}, G_B), \tag{6}$$

then $\wedge_{i=1}^{m} F_{i_{E_i}}$ *is called reduction of decision system of λ − level bijective soft set (F_E, G_B, U).*

In the following, an algorithm for decision-making problems will be given as an application of the BFSS concept [4].

Algorithms:

1. Define λ-level bijective fuzzy soft set decision system $(\wedge_{i=1}^{n} F_{i_{E_i}}, F_{n+1_{E_{n+1}}}, U)$.
2. Calculate the dependency of the bijective fuzzy soft set *decision* system $(\wedge_{i=1}^{n} F_{i_{E_i}}, F_{n+1_{E_{n+1}}}, U)$ based on Definition 2.19.
3. Based on Definition 2.20, determine the reduction of the bijective fuzzy soft set decision system $(\wedge_{i=1}^{n} F_{i_{E_i}}, F_{n+1_{E_{n+1}}}, U)$.
4. Based on the reduction of the bijective fuzzy soft set decision system $(\wedge_{i=1}^{n} F_{i_{E_i}}, F_{n+1_{E_{n+1}}}, U)$, it is obtained a decision rule.

Definition 2.21. [2] *Let U be a set of objects and E is a set of attributes and, $A \subseteq E$. Let $R = \{0,1,2, \dots, N-1\}$ is a set of grades or rating where $N \in \{2,3,4, \dots\}$. An (F,A,N) is called an N-soft set (NSS) over U, if $F: A \to 2^{U \times R}$, with the property that for each $a_i \in A$, there is exactly one $(u_j, r_{a_i}(u_j)) \in U \times R$ such that $\left(u_j, r_{a_i}(u_j)\right) \in F(a_i), u_j \in U, r_{a_i}(u_j) \in R$.*

The following is the representation table of an N-soft set, as in Table 3.

Table 3: Representation of an n- soft set (F, A, N)

(F, A, N)	a_1	a_2	…	a_q
u_1	$r_{a_1}(u_1)$	$r_{a_2}(u_1)$	…	$r_{a_q}(u_1)$
u_2	$r_{a_1}(u_2)$	$r_{a_2}(u_2)$	…	$r_{a_q}(u_2)$
⋮				
u_p	$r_{a_1}(u_p)$	$r_{a_2}(u_p)$	…	$r_{a_q}(u_p)$

Definition 2.22. *Let (F, A, N_1) and (G, B, N_2) be two NSSs. Operation AND, (F, A, N_1) AND (G, B, N_2) which is notated with $(F, A, N_1) \wedge (G, B, N_2)$ is defined as $(H, A \times B, min(N_1, N_2))$ where for each $c = (a, b) \in A \times B$ and $u \in U$ applies,*

$$\left(u, r_c(u)\right) \in H(c) \Leftrightarrow r_c(u) := min\left(r_a(u), r_b(u)\right),$$

for $\left(u, r_c(u)\right) \in F(a)$, and $\left(u, r_b(u)\right) \in G(b)$.

III. Result and Discussion

In this section, one case will be given that can be represented as a form of NSS. Next, referring to the construction of BSS and BFSS and related decision-making algorithms, from this case, we will analyze whether an NSS can be constructed into a special type so that the decision-making process, as in the case of BSS or BFSS, can also be applied to NSS.

In this case, the authors do not explain in detail the technicalities and accuracy in obtaining data because this, of course, requires separate and in-depth research involving experts in the field under study. Therefore, the data, in this case, is only illustrative.

Case. A government agency/institution conducts research on villages to find out what attributes affect the education index of a village. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the collection of villages in an area. Let $A = A_1 \cup A_2 \cup A_3 \cup A_4$ is a set which is a combination of four sets of attributes, namely A_1, A_2, A_3 and A_4. A_1 represents the set of attributes in the form of the level of health services at $a_1 =$ clinics, $a_2 =$ puskesmas and $a_3 =$ hospitals. A_2 states the set of attributes in the form of the level of ease of access to education to $b_1 =$ SD, $b_2 =$ SMP and $b_3 =$ SMA. A_3 states the set of attributes in the form of categorizing the village economy into c_1 = family economy, c_2 = village budget and c_3 = village fund allocation. A_4 represents the set of attributes d_1 = high, d_2 = low, which categorizes the education index in the village. Furthermore, a ranking will be given to each village for each attribute in A_1, A_2 and A_3 based on the assessment predicate defined by the expert with the following conditions:

1. ”5” means very good predicate.
2. ”4” means good predicate.
3. ”3” means fairly good predicate.
4. ”2” means bad predicate.
5. ”1” means very bad predicate.

In relation to the set A_4, a rating is given to each village for each attribute in the A_4 based on the assessment predicate defined by the expert with the following provisions:

a) The village education index (u) is said to be high if the village education index is more than 0.5 by category.

1. ”5” for index $0.9 < x \leq 1.0$.
2. ”4” for index $0.8 < x \leq 0.9$.
3. ”3” for index $0.7 < x \leq 0.8$.

4. ”2” for index $0.6 < x \leq 0.7$.
5. ”1” for index $0.5 < x \leq 0.6$.

b) The village education index (u) is said to be low if the village education index is less than 0.5 with the category.

1. ”5” for index $0.0 < x \leq 0.1$.
2. ”4” for index $0.1 < x \leq 0.2$.
3. ”3” for index $0.2 < x \leq 0.3$.
4. ”2” for index $0.3 < x \leq 0.4$.
5. ”1” for index $0.4 < x \leq 0.5$.

The definition of the assessment standard is fully determined by the expert. Based on the assessment results from the expert, for example, the grade or rating is obtained from each object for each set of, and it is presented as an N-soft set in Table 4, Table 5, Table 6, and Table 7, respectively.

Table 4: $(F_1, A_1, 6)$

$(F_1, A_1, 6)$	a_1	a_2	a_3
u_1	2	5	2
u_2	1	2	3
u_3	5	2	2
u_4	2	1	3
u_5	2	4	2
u_6	2	2	3

Table 5: $(F_2, A_2, 6)$

$(F_2, A_2, 6)$	b_1	b_2	b_3
u_1	3	2	1
u_2	2	2	5
u_3	2	4	2
u_4	2	3	2
u_5	2	2	4
u_6	3	2	2

Table 6: $(F_3, A_3, 6)$

$(F_3, A_3, 6)$	c_1	c_2	c_3
u_1	3	1	2
u_2	1	4	2
u_3	3	2	2
u_4	2	1	5
u_5	4	2	2
u_6	3	1	2

Table 7: $(F_4, A_4, 5)$

$(F_4, A_4, 5)$	d_1	d_2
u_1	0	3
u_2	4	0
u_3	0	4
u_4	4	0
u_5	3	0
u_6	0	3

Furthermore, if Table 4, Table 5 and Table 6 only consider grades that are more than or equal to 3, we will get Table 8, Table 9 and Table 10. If we observe the distribution of grades in these tables, it can be seen that each object is only associated with one particular attribute, because there is only one grade in each row that is more than zero, and there is no object that is not associated with an attribute/column (at least related to one object). The same thing also happened in Table 7.

On the other hand, if we pay attention to Definition 2.4 regarding BSS and Definition 2.15 regarding BFSS, it will be seen that the distribution of data in Table 8, Table 9 and Table 10 will be "similar" to the distribution of data in BSS and BFSS, but in the case of NSS. This becomes interesting because, with these conditions, there will be opportunities to expand or generalize the study of BSS and BFSS in the context of NSS. So that further, we can do a study in this case, what if the decision-making algorithms that exist in the previous BSS and BFSS concepts, we apply to this case, but adapted to the NSS definition. To realize this step, consider the following calculation process.

Table 8: $(F_1, A_1, 6)$

$(F_1, A_1, 6)$	a_1	a_2	a_3
u_1	0	5	0
u_2	0	0	3
u_3	5	0	0
u_4	0	0	3
u_5	0	4	0
u_6	0	0	3

Table 9: $(F_2, A_2, 6)$

$(F_2, A_2, 6)$	b_1	b_2	b_3
u_1	3	0	0
u_2	0	0	5
u_3	0	4	0
u_4	0	3	0
u_5	0	0	4
u_6	3	0	0

Table 10: $(F_3, A_3, 6)$

$(F_3, A_3, 6)$	c_1	c_2	c_3
u_1	3	0	0
u_2	0	4	0
u_3	3	0	0
u_4	0	0	5
u_5	4	0	0
u_6	3	0	0

Table 11 : $(F_4, A_4, 5)$

$(F_4, A_4, 5)$	d_1	d_2
u_1	0	3
u_2	4	0
u_3	0	4
u_4	4	0
u_5	3	0
u_6	0	3

First of all, construction $(\wedge_{i=1}^{3}(F_i, A_i, N), (F_4, A_4, 5), U)$ as a decision system on the N-soft set where $\wedge_{i=1}^{3}(F_i, A_i, N)$ is called the condition of the 6-soft set decision system and $(F_4, A_4, 5)$ is called the decision of 6-soft set decision system.

Next, in a manner similar to Definition 2.9 for BSS or Definition 2.19 for BFSS, calculate dependency $\wedge_{i=1}^{3}(F_i, A_i, N)$ with $(F_4, A_4, 5)$, which is symbolized by $\gamma(\wedge_{i=1}^{3}(F_i, A_i, N), (F_4, A_4, 5))$, using the AND operation on NSS. The results can be seen in the Table 12.

Table 12: Dependency $\wedge_{i=1}^{3}(F_i, A_i, N)$ with $(F_4, A_4, 5)$

$\gamma(\wedge_{i=1}^{3}(F_i, A_i, N), (F_4, A_4, 5))$	α
$\gamma((F_1, A_1, 6), (F_4, A_4, 5))$	0
$\gamma((F_2, A_2, 6), (F_4, A_4, 5))$	0.2727
$\gamma((F_3, A_3, 6), (F_4, A_4, 5))$	0.1904
$\gamma((F_1, A_1, 6) \wedge (F_2, A_2, 6), (F_4, A_4, 5))$	0.8
$\gamma((F_1, A_1, 6) \wedge (F_3, A_3, 6), (F_4, A_4, 5))$	0.7058
$\gamma((F_2, A_2, 6) \wedge (F_3, A_3, 6), (F_4, A_4, 5))$	1
$\gamma((F_1, A_1, 6) \wedge (F_2, A_2, 6) \wedge (F_3, A_3, 6), (F_4, A_4, 5))$	1

From Table 12, it is obtained, $\gamma((F_2, A_2, 6) \wedge (F_3, A_3, 6), (F_4, A_4, 5))$= $\gamma((F_1, A_1, 6) \wedge (F_2, A_2, 6) \wedge (F_3, A_3, 6), (F_4, A_4, 5)) = 1$. If we apply Definition 2.10 or Definition 2.20 in the context of NSS, we say that $(F_2, A_2, 6)$ and $(F_3, A_3, 6)$ are reductions of the N-soft set decision system $\gamma(\wedge_{i=1}^{3}(F_i, A_i, N), (F_4, A_4, 5))$. This means that the attributes that affect the village education index are the accessibility and the village economic level.

Furthermore, based on the distribution of the data (note the nonzero grade and the corresponding attributes) in Table 9, Table 10 and Table 11, it is obtained that the attributes that affect the decision attributes in A_4 are the attributes in A_2 and A_3. So that from the pattern of data distribution, a description of the conditions of each village is obtained, with the following details:

1. If the accessibility to SD (Elementary School) is fairly good and the family's economy is fairly good, the education index is low.
2. If the accessibility to SMA (Senior High School) is very good and the village budget is good, then the higher education index is high.
3. If the accessibility to SMP (Junior High School) is good and the family economy is fairly good, the education index is low.
4. If the accessibility to SMP (Junior High School) is fairly good and the allocation of village funds is very good, then the education index is high.
5. If the accessibility to SMA (Senior high school) is good and the family economy is good, then the education index is high.
6. If the accessibility to SD (Elementary School) is fairly good and the family economy is fairly good, the education index is low.

From this illustration of the case on the NSS, it is possible that the concept of BSS or BFSS can be applied and developed in the NSS which is a generalization of the SS as described by Fatimah et al. [2]. Therefore, from the study results, in this case, there will be opportunities for mathematical theoretical studies about NSS related to the general concepts of BSS and BFSS in articles [3] and [4].

IV. Conclusion

This study confirmed that a decision-making algorithm that applies the bijective soft set concept or the bijective fuzzy soft set concept could be applied to the N-soft set concept by adjusting the operations on the N-soft set.

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