

Asymmetric Laws Of Motion In The Cam Mechanisms

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Abstract – Various technological processes require the generation of asymmetric laws of motion. A family of asymmetric laws of motion for the synthesis of cam mechanisms was introduced to facilitate engineer-designers. These laws of motion depend on only one independent parameter, which can be determined by conditions related to the necessary asymmetry of the first input-output transfer function (analog of velocities) and the desired ratio of the extreme values of the second transfer function - analog of accelerations.

The study aims to derive a family of asymmetric laws of motion for the synthesis of cam mechanisms dependent on only one independent parameter, with the possibility of being determined by conditions, related to the required asymmetry of the first transfer function and the desired ratio of the extreme values of the second transfer function.

Among the asymmetric laws, the ordinary and the "smoothing" trapezoidal law takes the main place. Its disadvantages compensate for a different power, trigonometric and power-trigonometric laws of motion. But achieving pre-defined conditions requires high skill and time on the part of the engineer-designers. Therefore, asymmetric laws of motion are derived for the synthesis of cam mechanisms depending on only one independent parameter. This parameter can easily be determined by the conditions associated with the required asymmetry of the first transfer function and the desired ratio of the extreme values of the accelerations.

Keywords – *Cam Mechanisms, Laws Of Motion, Asymmetry.*

I. INTRODUCTION

Various technological processes require the generation of asymmetric laws of motion, for example in weaving machines (looms). It has been established that the law of motion of the shafts must be asymmetric [1]. The requirements for other technological processes are similar. For this purpose, it is necessary to generate laws of motion with different asymmetries and a ratio of the extreme values of the second transfer function - analog of accelerations.

A law used to design cam mechanisms for the formation shed for weaving machines is the asymmetric trapezoidal law for changing acceleration [2], which results in an intermittent rectangular change of the second accelerations (the so-called jerk) and increased vibrations and rattle of the mechanism. Better results are obtained with a modified trapezoidal law for changing acceleration [3], [4]. Modification by "smoothing" with parabolic arcs with common tangents in the transitions to the straight sections [5], [6], [7] derives a trapezoidal law for changing the second acceleration. Despite the good results obtained, a significant drawback of the trapezoidal functions of the acceleration is that they are defined individually by intervals, the

number of intervals increases from 5 to 9 when "smoothing" the upper trapezoid bases and at 11 intervals to overall "smoothing" the trapezoids.

Power, trigonometric and power-trigonometric polynomials provide good possibilities for generating asymmetric laws of motion of the cam mechanisms [8], [9], [10], [11], [12]. The synthesis disadvantage is related to the difficulty in determining the coefficients of the polynomials to satisfy the conditions for the asymmetry of the transfer functions and the corresponding ratios of the extreme values of the second transfer function.

The purpose of the study is to derive a family of asymmetric laws of motion for the synthesis of cam mechanisms dependent on only one independent parameter, with the possibility of being determined by conditions, related to the required asymmetry of the first transfer function and the desired ratio of the extreme values of the second transfer function. Such a family of laws of motion would facilitate the synthesis of cam mechanisms, since it is relatively easy to draw up a desired asymmetric law of motion.

II. A FAMILY OF A ONE-PARAMETER FAMILY OF LAWS OF MOTION

For a family of asymmetric laws of motion, the *displacement function* $\Delta\psi = \Delta\psi(\varphi)$ will be determined, of the oscillating follower (roller or flat-face) of the cam mechanism and its derivatives of the first and second transfer functions - $\psi' = \psi'(\varphi)$ and $\psi'' = \psi''(\varphi)$, where the input coordinate $\varphi \in [0, \Phi]$ is the rotation of the cam within a predetermined phase angle Φ and the output coordinate ψ is the rotation of the follower angular motion (swing) α . Similarly, cases where the input-output movements are different can be considered.

Asymmetry is an important factor in choosing a law of motion. It is found at odd derivatives $u'(\xi)$, $u'''(\xi)$, ..., of the normalized function $u(\xi) \in [0, 1]$; $\xi \in [0, 1]$, where $u(\xi) = \psi(\varphi) / \alpha$; $\xi = \varphi / \Phi$. It is evaluated by the asymmetry coefficient.

$$(1) \quad a = 2\xi^* - 1,$$

where ξ^* is the value of the argument ξ , at which function $u'(\xi)$ has maximum, respectively the derivative $u''(\xi)$ is zeroed.

When setting the value of a as a condition of synthesis, from (1) is determined

$$(2) \quad \xi^* = 0.5(1 + a).$$

A **family of one-parameter laws of motion** can be determined by the first transfer function of the mechanisms with elliptical gears [13] (Figure 1):

$$(3) \quad i_{2,1} \equiv \frac{\omega_2}{\omega_1} = \frac{d\varphi_2}{d\varphi_1} = \frac{1 - \varepsilon^2}{1 + \varepsilon^2 - 2\varepsilon \cos \varphi_1} \quad \left(\begin{array}{l} 0 < \varepsilon < 1, \\ \varphi_1 \in (0, \pi) \end{array} \right),$$

in which the angles of rotation of the gears 1 and 2, are indicated by φ_1 and φ_2 , respectively.

Function (3) can be modified and converted into a *second transfer function* suitable for the synthesis of cam mechanisms generating asymmetric motion laws dependent on one independent parameter (ε).

The changes are as follows:

- from function (3) is develop the mean $(i_{2,1})_{av} = 1$ of the $i_{2,1}$;
- the angle φ_1 is replaced by $\varphi_1 = (2\pi\varphi / \Phi_1) - \varphi_1^*$ where $\varphi \in (0, \Phi_1)$ is the angle of rotation of the cam, Φ_1 is the cam angle of the follower rise, $\varphi_1^* = \arccos(\varepsilon)$ is the angle at which $i_{2,1} = (i_{2,1})_{av} = 1$;
- a factor f is introduced, which is determined by the condition that at $\varphi = \Phi_1$ the output angle $\Delta\psi$ is equal to the angular motion (swing) $\alpha = \Delta\psi_{\max}$ of the follower.

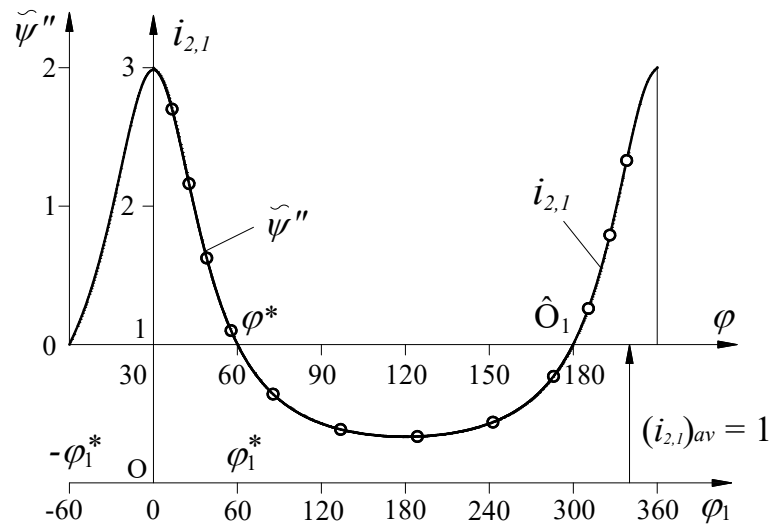


Fig. 1. Graphs of: first transfer function $i_{2,1}(\varphi_1)$ of an elliptical gear mechanism; second transfer function $\psi''(\varphi)$ of the cam mechanism at $\Phi_1 = \pi$ and $\varepsilon = 0.5$

These changes for the **second transfer function** of the cam mechanism (Figure 2) determined:

$$(4) \quad \psi''(\varphi) \equiv \frac{d^2\psi}{d\varphi^2} = f \cdot \tilde{\psi}''(\varphi),$$

where

$$(5) \quad \tilde{\psi}'' = \frac{1 - \varepsilon^2}{1 + \varepsilon^2 - 2\varepsilon \cos\left((2\pi\varphi / \Phi_1) - \varphi_1^*\right)} - 1.$$

The function ψ'' is zeroed at:

$$(6) \quad \varphi = 0; \quad \varphi = \varphi^* = (\Phi_1 / \pi) \cdot \arccos(\varepsilon); \quad \varphi = \Phi_1.$$

The extremums

$$(7) \quad \begin{cases} \psi''_{\max} = f \left((1 + \varepsilon) / (1 - \varepsilon) - 1 \right), \\ \psi''_{\min} = f \left((1 - \varepsilon) / (1 + \varepsilon) - 1 \right) \end{cases}$$

of the function ψ'' are determined respectively at angles

$$(8) \quad \begin{cases} \varphi_{(\max)} = \varphi^* / 2 = 0.5 \pi^{-1} \Phi_1 \arccos(\varepsilon); \\ \varphi_{(\min)} = (\varphi^* + \Phi_1) / 2 = 0.5 \Phi_1 \left(1 + \pi^{-1} \arccos(\varepsilon) \right), \end{cases}$$

determined from zeroing of the **third transfer function**:

$$(9) \quad \psi''' = \frac{4\pi f \varepsilon (\varepsilon^2 - 1) \sin\left((2\pi\varphi / \Phi_1) - \varphi_1^*\right)}{\Phi_1 \left[1 + \varepsilon^2 - 2\varepsilon \cos\left((2\pi\varphi / \Phi_1) - \varphi_1^*\right)\right]^2}.$$

The only independent parameter is the $\varepsilon \in (0, 1)$, on which the extreme ratio depends

$$(10) \quad p = \psi''_{\max} / \psi''_{\min} = -(1 + \varepsilon) / (1 - \varepsilon)$$

and asymmetry coefficient

$$(11) \quad a = 2\xi^* - 1 = (2\varphi^* / \Phi_1) - 1 = 2 \cdot \pi^{-1} \arccos(\varepsilon) - 1.$$

As the value of the parameter ε ($\varepsilon = 0.3; 0.4; 0.5$) increases, the asymmetry coefficient also increases ($a \approx 0.194; 0.292; 0.333$), with no asymmetry for the interval $a \in (0, 1)$ at $a = 0$ and $p = \psi''_{\max} / \psi''_{\min} = -1$, and at $a = 1$, respectively $\varepsilon = 1$ practically unacceptable results are calculated - $\varphi^* = 0$ and $p = \psi''_{\max} / \psi''_{\min} = -\infty$.

Graphs of $\psi''(\varphi)$ at $f = 0.16$ and $\Phi_1 = \pi$ for three values of $\varepsilon = 0.3; 0.4; 0.5$ are given in Figure 2.

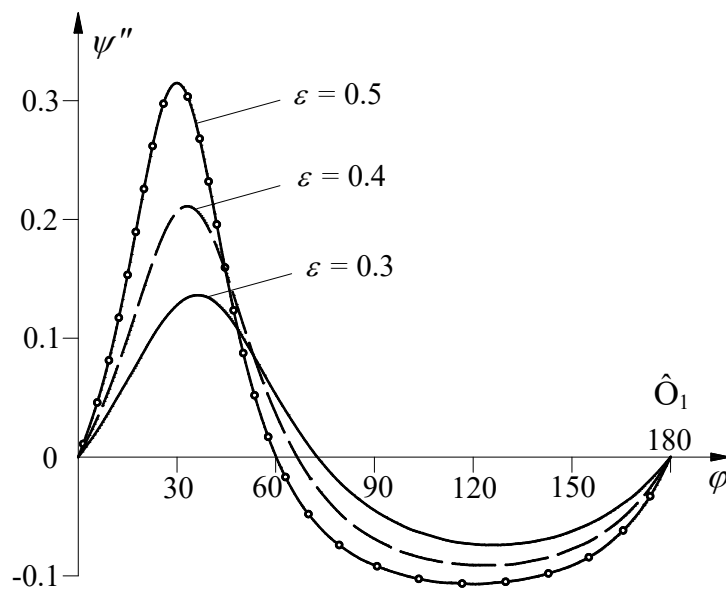


Fig. 2. Graphs of $\psi''(\varphi)$ at $f = 0.16$ and $\Phi_1 = \pi$ for three values of $\varepsilon = 0.3; 0.4; 0.5$

- There are two main tasks in determining the second transfer function by the set points of the angular motion $\alpha = \Delta\psi_{\max}$ of the oscillating follower (roller or flat-face) and phase angle.

Problem 1. Determine the value of the parameter ε from the appropriately chosen extreme ratio

$p = \psi''_{\max} / \psi''_{\min} = (\varepsilon + 1) / (\varepsilon - 1)$, from which:

$$(12) \quad \varepsilon = (p + 1) / (p - 1).$$

For example, at $p = -2; -3; -4$ are determined corresponding values of $\varepsilon = 1/3; 0.5; 0.6$.

Problem 2. Determine the value of the parameter ε from the appropriately chosen asymmetry coefficient

$$a = 1 - (2\varphi^* / \Phi_1) = 1 - 2 \cdot \pi^{-1} \arccos(\varepsilon),$$

from where determined:

$$(13) \quad \varepsilon = \cos(0.5\pi(1-a)).$$

For example, at $a = 1/6; 1/3; 1/2$ are determined corresponding values of $\varepsilon = 0.2588; 0.5; 0.7071$.

Graphs of the functions (12) and (13) are presented in Figure 3. These graphs can be used for parameters quick evaluation of the second transfer function. If necessary, this assessment may be analytically specified. For example, given a coefficient of asymmetry $a = 0.3$ from (13), $\varepsilon = 0.454$ is obtained, at this value of (9) the extreme ratio $p = -2.663$ is determined, as shown in Figure 4. Conversely, for a given ratio p , the values of ε and of the coefficient a successively are determined.

The first transfer function can be determined analytically in the quadratures of equations (4) and (5):

$$(14) \quad \psi'(\varphi) = f \cdot \tilde{\psi}'(\varphi),$$

where

$$(15)$$

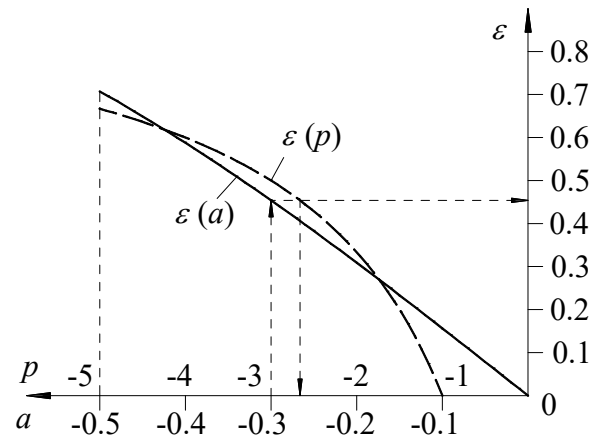


Fig. 3. Graphs of $\varepsilon(p)$ and $\varepsilon(a)$, determined corresponding from equations (8) and (3)

$$\begin{aligned} \tilde{\psi}'(\varphi) &= \int_0^\varphi \tilde{\psi}''(\varphi) d\varphi + C = \\ &= 2 \operatorname{arctg} \left[\frac{1+\varepsilon}{1-\varepsilon} \operatorname{tg} \left(\frac{\pi\varphi}{\Phi_1} - \frac{\varphi_1^*}{2} \right) \right] - \frac{2\pi\varphi}{\Phi_1} + \varphi_1^* + C. \end{aligned}$$

The constant C is derived from the initial condition $\tilde{\psi}' = 0$ at $\varphi = 0$:

$$(16) \quad C = -2 \operatorname{arctg} \left[\frac{1+\varepsilon}{1-\varepsilon} \operatorname{tg} \left(-\frac{\varphi_1^*}{2} \right) \right] - \varphi_1^*.$$

The **displacement function** $\Delta\psi(\varphi)$ of the follower is derived after integrating $\psi'(\varphi)$:

$$(17) \quad \Delta\psi(\varphi) = f \cdot \Delta\tilde{\psi} = f \cdot \left(\int_0^\varphi \tilde{\psi}'(\varphi) d\varphi + C_0 \right).$$

The constant C_0 is derived from the initial condition $\Delta\psi = 0$ at $\varphi = 0$. The multiplier f is determined by the condition $\Delta\psi(\varphi) = \alpha$ at $\varphi = \Phi_1$:

$$(18) \quad f = \alpha \left(\int_{\varphi=0}^{\Phi_1} \tilde{\psi}'(\varphi) d\varphi + C_0 \right)^{-1}.$$

By determining the multiplier f , the functions are finally derived:

$$(19) \quad \Delta\psi(\varphi) = f \cdot \Delta\tilde{\psi}; \quad \psi'(\varphi) = f \cdot \tilde{\psi}'(\varphi); \quad \psi''(\varphi) = f \cdot \tilde{\psi}''(\varphi).$$

The integrals $\int_0^{\varphi} \tilde{\psi}'(\varphi) d\varphi + C$ and $\int_{\varphi=0}^{\Phi_1} \tilde{\psi}'(\varphi) d\varphi + C_0$ corresponding from (14) and (15) are determined numerically by Simpson's method.

The graphs of the functions $\Delta\psi(\varphi)$, $\psi'(\varphi)$ and $\psi''(\varphi)$ determined at a given angular motion $\alpha = \pi/6$, phase angle $\Phi_1 = \pi$, and asymmetry coefficient $a = 1/3$ at which of the equation (13) are derived $\varepsilon = 0.5$, are presented in Figure 4. From the equation (10) are determined $p = -3$, respectively $\psi''_{\max} = 3|\psi''_{\min}|$.

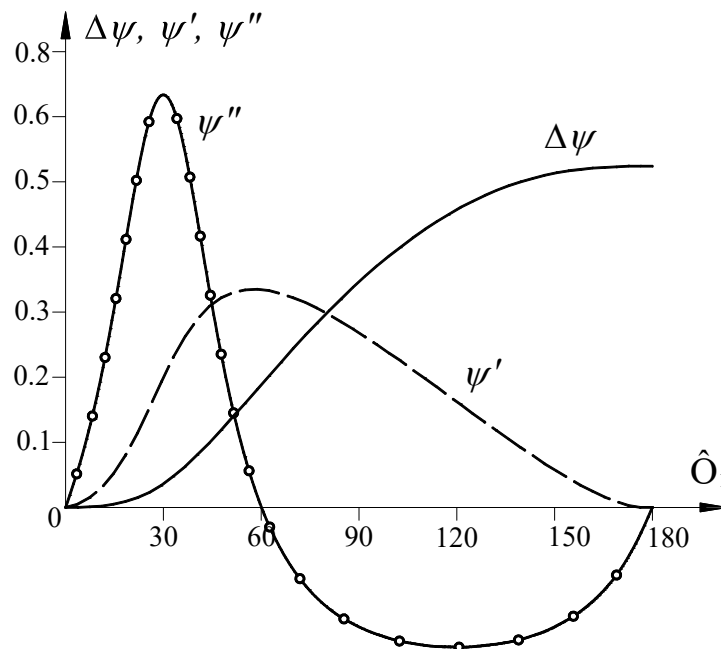


Fig. 4. Graphs of the functions $\Delta\psi$, ψ' and ψ'' when given:

$$\alpha = \pi/6, \quad \Phi_1 = \pi \text{ u } a = 1/3$$

A detailed solution to the question of the laws of motion and synthesis of cam mechanisms was made by Galabov, Roussev, and Paleva-Kadiyska in [14].

III. CONCLUSION

Among the asymmetric laws, the ordinary and the "smoothing" trapezoidal law takes the main place. Its disadvantages compensate for a different power, trigonometric and power-trigonometric laws of motion. But achieving pre-defined conditions requires high skill and time on the part of the engineer-designers. Therefore, asymmetric laws of motion are derived for the synthesis of cam mechanisms depending on only one independent parameter. This parameter can easily be

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