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Constitutive Model for Shale Rock under High Strain Rates

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Abstract – Modeling the response of Shale rock towards impact loads imparted during hydraulic fracturing is essential in order to explain its strain rate sensitive behavior. In this paper, modified constitutive model, proposed by Zhu-Wang-Tang, is used to analyze the dynamic compressive properties of outcrop Shale rock of Khyber Pakhtunkhwa, Pakistan. The model comprises of multiple elastic and viscoelastic elements and is capable of predicting the dynamic behavior over a wide range of strain rates. Using the method of least squares, the model has been fitted to the experimental stress-strain curves obtained under the strain rates of the order of 10^2 s^{-1} and the parameters of the model are determined. It was found that the experimental and fitted curves were in good agreement.

Keywords – Impact loads; Hydraulic Fracturing; Dynamic Compressive Properties; Viscoelastic Elements

BACKGROUND

The development of shale gas and shale oil reservoirs with increased well production rates is mostly carried out by horizontal drilling and multi-stage fracturing. With the advent of hydraulic fracturing or fracking, though the flow rates of the wells have been significantly increased but its drawbacks such as large amount of water consumption, increased costs and induced seismicity in the ground still need to be addressed. The Exploration and Production (E&P) sector is now looking for the possible alternatives of Hydraulic Fracturing one of which is the dynamic fracturing of Shale by utilizing the kinetic energy of high shear strain rate called as the communition [1]. The objective of every fracking technique is to increase the productivity of the well by increasing the permeability of Shale formations in order to capture the entrapped oil and gas reserves. Whether it is hydraulic fracturing or waterless fracking, the first step in both the techniques is to stimulate the reservoir by inducing the cracks in the formation at the targeted depths. In hydraulic fracturing, this crack generation is carried out by the hydraulic pressure of the pumping fluid while in the propane fracking this is achieved by the dynamically induced stress wave generated by the vapor pressure of propane. Very little scientific evidence is available in the support of technical advantages of waterless fracking techniques in the shale formations mainly because of the fact that the mechanical response of shale towards the applied dynamic loads is not completely known. This arises the need of thorough research in the characterization of the mechanical properties of Shale at dynamic loads which will ultimately help the well planners and fracking engineers.

In order to enhance the efficiency of the fracking technique, the need for thorough investigation of rate dependent behavior of shale with kinetic approach is required [2], [3]. The loads during this process are of high magnitude and less time duration and can be attributed to a number of factors such as blast wave, stress wave and gas expansion. Since the fracking of shale formations involves well stimulation, the need for constitutive modelling at high strain rates is thus necessitated. A strain rate dependent model for shale by conducting the tri-axial compression tests under the strain rates of 10^{-6} to 10^{-3}/sec was presented by [4]. The model takes into consideration of the damage evolution of rock and employs the famous Lematire principle of strain equivalence in describing the micromechanical strain rate dependent properties. Moreover, the findings of [4] established an exponential relation between the Young's Modulus and the strain rate. A statistical distribution of strength based model for Shale was presented in [5] in which the main assumption was that apart from the viscoelastic nature, the strength of shale obeys a Weibull distribution. The model was implemented in the strain rate range of 100-200 s^{-1} with good agreement between the experimental and the predicted curves. In this research, the modified ZWT viscoelastic model parameters with damage incorporated have been determined for Shale rock by using the experimental dynamic stress-strain curves obtained by [6].

I. METHODOLOGY

1.1. Selection of the Constitutive Model

The constitutive relations of viscoelastic materials can be expressed by a non-linear relation proposed by Zhu-Wang-Tang, also known as the ZWT model. Initially, this model was developed for viscoelastic polymers [7] and has been used by numerous researchers in the constitutive modelling of the materials. Reference [8] explained the differences between the static and dynamic strength of polycarbonate samples in the strain rate range of 10^{-4} to 10^{3} s^{-1}. The ZWT model also finds applications in explaining the viscoelasticity of rocks as well over a wide range of strain rates. The parameters for anthracite in the strain rate range of 5-85 s^{-1} were determined in [9]. Reference [10] used this model for explaining the strain rate dependent properties of basalt fiber-reinforced concrete. Reference [11], [12] determined the parameters of damage based ZWT model for granite. The novelty of the present research lies in the fact that the dynamic response of shale at high strain rate of the order of 10^2 s^{-1} has been captured using the damage modified ZWT constitutive model.

1.2. Constitutive Equations of the Model

Figure 1 presents the rheological form of the ZWT model. The model consists of a non-linear spring with Elastic constant (E_o) and two Maxwell elements connected in parallel. The first Maxwell element has a high relaxation time (θ_1) of the order of 10^2 seconds while the second Maxwell element has relaxation time (θ_2) of the order of 10^{-6} seconds. The first Maxwell element represents the viscoelastic response at low strain rates while the second element represents the viscoelastic response at high strain rates.

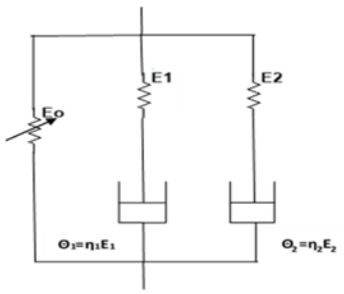

Figure 1: Zhu Wang Tang (ZWT)Viscoelastic Model

Equation 1 is the generalized solution of the ZWT model with stress as a function of strain (ε) and strain rate ($\acute{\varepsilon}$).

$$\sigma = (E_0\varepsilon + \phi\varepsilon^2 + \xi\varepsilon^3) + E_1\int_0^t \acute{\varepsilon}(t)\exp(-\frac{t-\tau}{\theta_1})\,d\tau + E_2\int_0^t \acute{\varepsilon}(t)\exp(-\frac{t-\tau}{\theta_2})\,d\tau \tag{1}$$

The model can be simplified for dynamic experiments on rocks as presented in [13]. The non-linearity in the elastic portion of stress strain curve of rocks arises from the compaction of rock due to porosity and other factors [14]. In high strain rate testing, since the time of loading is very short, so this non-linearity is not manifested and the non-linear spring can be treated as Hook's spring. The high strain rate testing of materials is carried out at very short duration of time i.e. of the order of few microseconds. Hence, it is reasonable to assume that dashpot of the first Maxwell element does not get sufficient time to relax during such a short duration of time. Equation 1 can be written as:

$$\sigma = (E_0 + E_1)\varepsilon + E_2 \int_0^t \acute{\varepsilon}(t) \exp(-\frac{t-\tau}{\theta_2})\, d\tau \tag{2}$$

or

$$\sigma = E_{eff}\varepsilon + E_2 \int_0^t \acute{\varepsilon}(t) \exp(-\frac{t-\tau}{\theta_2})\, d\tau \tag{3}$$

The constitutive modelling for rocks must take into account the evolution of micro cracks. The strength at any damaged state of the rock is dependent on a scalar damage variable (D). The cumulative effect of distributed micro-cracks and flaws in the rock leads to reduction of the stress and the resulting stress can be expressed as:

$$\sigma_{D=}(1 - D)\, \sigma \tag{4}$$

Where σ_D is the damaged stress and σ is the stress from ZWT model without damage.

Considering the damage variable as a function of strain [13], D can be expressed as:

$$D = Z\varepsilon \tag{5}$$

Substituting (3) and (5) in (4) gives:

$$\sigma(\varepsilon, \acute{\varepsilon}, Z) = (1 - Z\varepsilon)\left\{E_{eff}\, \varepsilon + E_2 \int_0^t \acute{\varepsilon}(t) \exp\,(-\frac{t-\tau}{\theta_2})\, d\tau\right\} \tag{6}$$

Equation 6 can be reduced as:

$$\sigma(\varepsilon, \acute{\varepsilon}, Z) = (1 - Z\varepsilon)\left\{E_{eff}\, \varepsilon + E_2\acute{\varepsilon}\theta_2\left[1 - \exp(\frac{-\varepsilon}{\acute{\varepsilon}\theta_2})\right]\right\} \tag{7}$$

Equation 7 is known as the damage modified ZWT constitutive model [13]. This model was fitted to the results of dynamic compression experiments conducted by [6] on outcrop shale samples of Sheikh Badin Station, Khyber Pakhtunkhwa, Pakistan using Split Hopkinson Pressure Bar Apparatus. For details regarding the apparatus and experimental procedure, readers are advised to refer to [6]. The parameters of the model for Shale were obtained through the method of least squares.

II. Results And Discussion

Figure 2 and 3 show the experimental and fitted dynamic true stress strain curve for shale obtained at 406.8 s^{-1} and 630.4 s^{-1}. Good correlation between the empirical and the fitted curves demonstrates the ability of the model to characterize the behavior of shale under dynamic compression. The parameter E_{eff} for both the strain rates was found to be 0.77 GPa while the elastic constant E_2 was found to decrease from 1.99 GPa to 0.78 GPa. This is in accordance with the variation in the peak stress from 39.31 GPa to 38.36 GPa. The mean relaxation time (θ_2) for these strain rates was found to be 6.18 μs. A higher damage evolution parameter for strain rate of 630.4 s^{-1} is responsible for enhanced softening behavior beyond the peak stress.

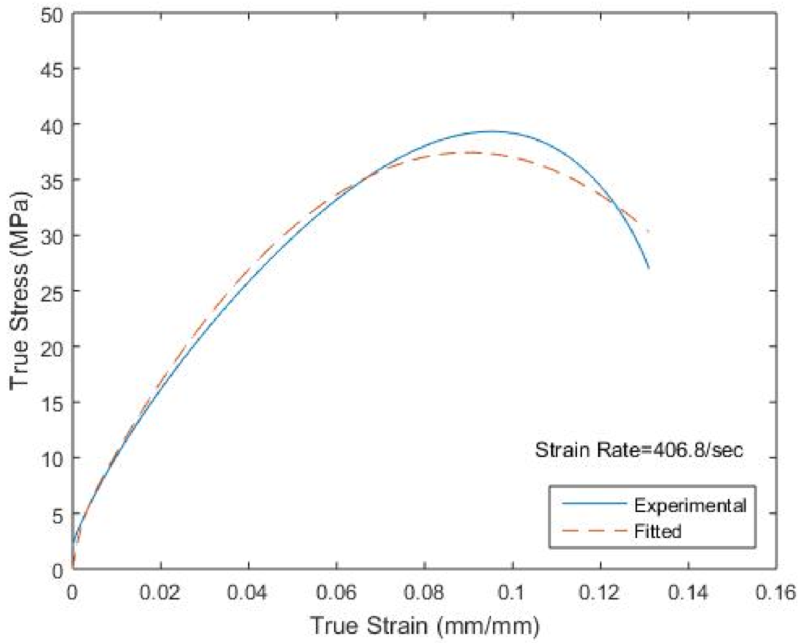

Figure 2: True stress-strain curve for Shale obtained at 406.8 s^{-1}

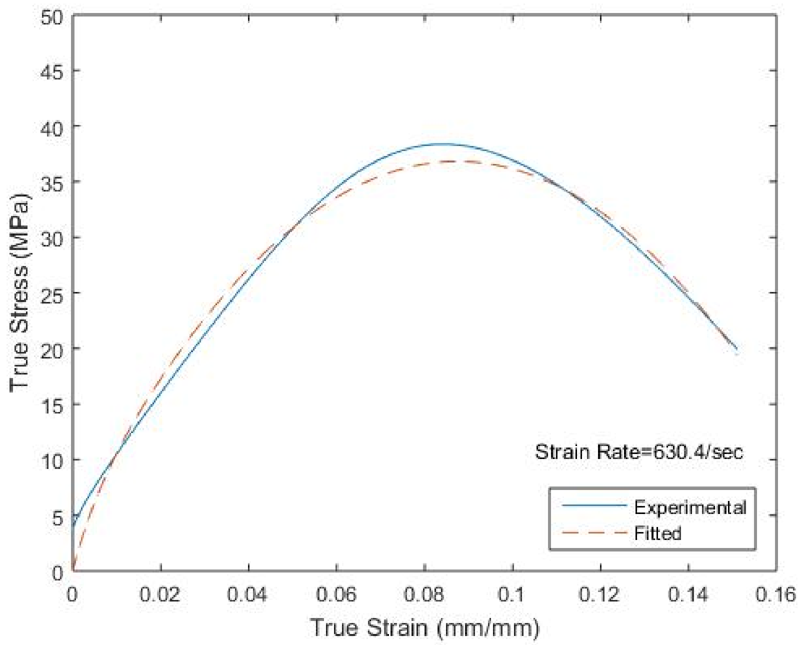

Figure 3: True stress-strain curve for Shale obtained at 630.4 s^{-1}

The parameters for Shale in the damage modified ZWT model under high strain rates are shown in table 1.

Table 1: Parameters of Damage Modified ZWT Model for Shale Rock

S.No.	Strain Rate (s^{-1})	E_{eff} (GPa)	E_2 (GPa)	θ_2 (μs)	Damage Evolution Factor (Z)	Coefficient of Determination (R^2)
1	406.8	0.77	1.99	4.06	5.43	0.98
2	630.4	0.77	0.781	8.35	5.55	0.98

III. Conclusions

A four parameter, modified ZWT model with damage variable incorporated, has been fitted to the experimental dynamic stress-strain curves for Shale rock samples tested at two different strain rates. It was found that the elastic constant E_2 mainly controls the value of peak stresses in shale while the damage evolution parameter (Z) controls the post peak response of shale. A good agreement was found between the experimental and fitted stress strain curves.

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