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Subspace and Product Topology of Intuitionistic Fuzzy Soft Set

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Abstract – In this paper, we construct an intuitionistic fuzzy soft subspaces for an intuitionistic fuzzy soft topology and intuitionistic fuzzy soft product topology. Furthermore, we introduce some concept of intuitionistic fuzzy soft subspace and product topology like basis of subspace and product topology of intuitionistic fuzzy soft set and establish some important theorem.

Keywords – Intuitionistic fuzzy soft set, Intuitionistic fuzzy soft topology, Intuitionistic fuzzy soft subspace, Intuitionistic fuzzy soft product.

I. Introduction

The concept of a fuzzy set was introduced by Prof. L.A. Zadeh [7] University of California, USA, in 1965 as a potensial tool for handling imprecision and uncertainties, for the example like beautiful women, delicious food, etc.

A more general concept of the fuzzy set is an intuitionistic fuzzy set that was introduced by Atanassov [1] in 1980. In addition to the degree of membership there is also a degree of non-membership in this set. In 1990, D. Molodtsov [2] was introduced a new concept about soft set theory. Then, in 2001, Maji and friends [3] were developed the theory of intuitionistic fuzzy soft set by combining the intuitionistic fuzzy set theory and soft set theory. In the development of set theory there is a term topology. Concepts, properties, and operations contained in the topological space of the original set can also be applied within the intuitionistic fuzzy soft topological space. In recent years, many researchers are interested in studying the concept of topology on fuzzy soft set and intuitionistic fuzzy soft set. Such as Roy and Samanta [4] in 2012 put forward some ideas about fuzzy soft topological space, like definition of basis and subbasis on fuzzy soft set with some theorems. Then , in 2013 Zhaowen and Rongchen [8] learn about topological structures of intuitionistic fuzzy soft set that contains the definition of intuitionistic fuzzy soft basis, interior and closure. In 2015, Shuker [6] studied the theory of closed set on intuitionistic fuzzy soft topological space. Some types of topological structures are subspace topology and product topology. The concepts subspace topology and product topology of the original set can also be applied to intuitionistic fuzzy soft subspace and product topology.

In this paper, we try to define some definitions on intuitionistic fuzzy soft set. Then we define subspace topology and product topology on intuitionistic fuzzy soft set, basis for subspace and product topology of intuitionistic fuzzy soft set, and also here we established some important theorems related to this topic.

II. Preliminaries

Let U is an initial universe set and E is the set of parameters. Let P^U denote the power set of U.

Definition 2.1.([7]). A fuzzy set L in U is defined by:

$$L = \{(x, \mu_L(x)) | x \in U\}$$

where $\mu_L: U \to [0,1]$ is membership function and $\mu_L(x)$ represents the degree of membership of $x \in U$. The collection of all fuzzy sets on U is denoted by I^U.

In general, for each $x \in A$, $l(x)$ is a fuzzy set of U, and can symbolize the membership function of $l(x)$ by $l_x : U \to [0,1]$.

Definition 2.2. ([1]). An intuitionistic fuzzy set F in U is defined by

$$F = \{(x, \mu_F(x), \nu_F(x)) | x \in U\}$$

where

$$\mu_F: U \to [0,1]$$
$$\nu_F: U \to [0,1]$$

with the condition

$$0 \leq \mu_F(x) + \nu_F(x) \leq 1, \ \forall x \in U.$$

The number $\mu_F(x)$ and $\nu_F(x)$ symbolized the degree of membership and non-membership of the element $x \in U$ to F. The collection of all intuitionistic fuzzy sets on U is symbolized by IF^U.

Definition 2.3.([1]). Let $F, G \in IF^U$, with

$$F = \{(x, \mu_F(x), \nu_F(x)) | x \in U\}$$

and

$$G = \{(x, \mu_G(x), \nu_G(x)) | x \in U\}$$

then

(1) F is said subset of G if and only if $\mu_F(x) \leq \mu_G(x)$ and $\nu_F(x) \geq \nu_G(x)$ for all $x \in U$.
(2) $F \wedge G = \{(x, \min\{\mu_F(x), \mu_G(x)\}, \max\{\nu_F(x), \nu_G(x)\}) | x \in U\}$.
(3) $F \vee G = \{(x, \max\{\mu_F(x), \mu_G(x)\}, \min\{\nu_F(x), \nu_G(x)\}) | x \in U\}$.

Definition 2.4.([1]). A set F where $F = \{(x, 0,1) | x \in U\}$ is called intuitionistic fuzzy empty set. We symbolized by $\tilde{0}$.

Definition 2.5.([1]). A set F where $F = \{(x, 1,0) | x \in U\}$ is called intuitionistic fuzzy universe set. We symbolized by $\tilde{1}$.

Definition 2.6.([2]). A pair (F, A) is said a soft set in U where F is a mapping given by $F: A \to P(U)$ and $A \subseteq E$. where $P(U)$ is the power set of U.

Next, we will give the definition of fuzzy soft set by combining fuzzy set theory and soft set theory. Also definition of intuitionistic fuzzy soft set by combining the concept of intuitionistic fuzzy set and soft set.

Definition 2.7.([3]). A pair (F, A) is said fuzzy soft set over U, if F is a mapping given by $F: A \to I^U$. We symbolized (F, A) by F_A.

Definition 2.8.([6]). A pair (f, A) is said an intuitionistic fuzzy soft set over U, where f is mapping given by $f : A \to IF^U$. We symbolized (f, A) by f_A.

The collection of all intuitionistic fuzzy soft sets over U is symbolized by IFS^U.

Definition 2.9.([6]). Let (f, A) and (g, B) be two intuitionistic fuzzy soft sets over U, and $A, B \subseteq E$.

(1) The intersection of $(f, A) \sqcap (g, B) = (h, C)$ where $C = A \cap B$ and $\forall e \in C$, $h(e) = f(e) \wedge g(e)$. We write $f_A \sqcap g_B = h_C$.

(2) The union of $(f,A) \sqcup (g,B) = (h,C)$ where $C = A \cup B$ and $\forall e \in C$,

$$h(e) = \begin{cases} f(e), & e \in A - B \\ g(e), & e \in B - A \\ f(e) \vee g(e), & e \in A \cap B \end{cases}$$

we write $f_A \sqcup g_B = h_C$.

(3) f_A is called intuitionistic fuzzy soft subset of g_B, if $A \subseteq B$ and $f(e)$ is intuitionistic fuzzy subset of $g(e)$ for any $e \in A$. We write $f_A \sqsubseteq g_B$.

(4) f_A and g_B are called equal, if $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$. We write $f_A = g_B$.

Definition 2.10.([6]). Let $f_E \in IFS^U$.

(1) f_E is said absolute intuitionistic fuzzy soft over U, if $f(e) = \tilde{1}$ for any $e \in E$. We symbolized it by I_E.

(2) f_E is said null intuitionistic fuzzy soft over U, if $f(e) = \tilde{0}$ for any $e \in E$. We symbolized it by Φ_E.

Example 2.11. Let $U = \{x_1, x_2, x_3, x_4\}$ be a universe consisting of four gown as possible alternatives, and let $A = \{e_1, e_2, e_3\} \subseteq E$, where e_1, e_2 and e_3 represent the parameters "beautiful","expensive", and "stylish", respectively. Consider a soft set f_A which describes the "attractiveness of the gowns" that Miss X is going to buy. Let f_A defined as follows

$$f(e_1) = \{(x_1, 0.7,0.2), (x_2, 0.3,0.6), (x_3, 0.4,0.5), (x_4, 0.8,0.2)\}$$
$$f(e_2) = \{(x_1, 0.7,0.3), (x_2, 0.5,0.3), (x_3, 0.5,0.5), (x_4, 0.9,0.1)\}$$
$$f(e_3) = \{(x_1, 0.8,0.2), (x_2, 0.2,0.7), (x_3, 0.6,0.3), (x_4, 0.9,0.1)\}.$$

Thus

$$\begin{aligned} f_A = \{&(e_1, \{(x_1, 0.7,0.2), (x_2, 0.3,0.6), (x_3, 0.4,0.5), (x_4, 0.8,0.2)\}), \\ &(e_2, \{(x_1, 0.7,0.3), (x_2, 0.5,0.3), (x_3, 0.5,0.5), (x_4, 0.9,0.1)\}), \\ &(e_3, \{(x_1, 0.8,0.2), (x_2, 0.2,0.7), (x_3, 0.6,0.3), (x_4, 0.9,0.1)\})\}. \end{aligned}$$

Definition 2.12.([8]). Let IFS^U is the family of all intuitionistic fuzzy soft sets over U, and E is a parameter set. Let $\tau \sqsubseteq IFS^U$. Then τ is said an intuitionistic fuzzy soft topology on U if it satisfies the following three conditions:

(i) $\Phi_E, I_E \in \tau$.

(ii) $f_E, g_E \in \tau$ implies $f_E \sqcap g_E \in \tau$.

(iii) $\{(f_\alpha)_E : \alpha \in \Gamma\} \sqsubseteq \tau$ implies $\bigsqcup\{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau$.

τ is said to be topology for an intuitionistic fuzzy soft over U and (U, E, τ) is said an intuitionistic fuzzy soft topological space in U. Each element of τ is said an intuitionistic fuzzy soft open sets in U.

Example 2.13. Let $U = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. Let $f_E, g_E \in IFS^U$ where

$$f(e_1) = \{(x_1, 0.7,0.2), (x_2, 0.2,0.6)\},$$
$$f(e_2) = \{(x_1, 0.7,0.1), (x_2, 0.5,0.3)\};$$
$$g(e_1) = \{(x_1, 0.6,0.2), (x_2, 0.3,0.5)\},$$
$$g(e_2) = \{(x_1, 0.7,0.2), (x_2, 0.4,0.3)\};$$

Let $h_E = f_E \sqcup g_E$, where

$$h(e_1) = \{(x_1, 0.7,0.1), (x_2, 0.3,0.5)\},$$
$$h(e_2) = \{(x_1, 0.7,0.2), (x_2, 0.5,0.3)\};$$

and $l_E = f_E \sqcap g_E$, where

$$l(e_1) = \{(x_1, 0.6,0.2), (x_2, 0.2,0.6)\},$$
$$l(e_2) = \{(x_1, 0.7,0.2), (x_2, 0.4,0.3)\}.$$

Thus, $\tau = \{\Phi_E, I_E, f_E, g_E, h_E, l_E\}$ is a topology for an intuitionistic fuzzy soft on U.

Definition 2.14. ([5])

(1) Let τ is the family of all intuitionistic fuzzy soft sets that can be defined in U. τ is said discrete topology for an intuitionistic fuzzy soft set on U, and (U, E, τ) is called an intuitionistic fuzzy soft discrete space over U.

(2) $\tau = \{\Phi_E, I_E\}$ is said the intuitionistic fuzzy soft indiscrete topology on U, and (U, E, τ) is called an intuitionistic fuzzy soft indiscrete space over U.

Definition 2.15. ([8]) Let (U, E, τ) is an intuitionistic fuzzy soft topological space and $\beta \sqsubseteq \tau$. β is a basis for τ, if for each $g_E \in \tau$, there is $\beta' \sqsubseteq \beta$ such that $g_E = \sqcup\beta'$.

Example 2.16. Let τ is the intuitionistic fuzzy soft topology in Example 2.13. Then, β that defined by $\beta = \{\Phi_E, I_E, f_E, g_E, l_E\}$ is a basis for τ. This statement will be shown using Definition 2.15. Note that $\tau = \{\Phi_E, I_E, f_E, g_E, h_E, l_E\}$ is an intuitionistic fuzzy soft topology on U. Then, there is $\beta' = \{\Phi_E\} \sqsubseteq \beta$ such that $\sqcup\beta' = \Phi_E \in \tau$. Next, there is $\beta' = \{\Phi_E, f_E\} \sqsubseteq \beta$ such that $\sqcup\beta' = f_E \in \tau$. There is $\beta' = \{\Phi_E, g_E\} \sqsubseteq \beta$ such that $\sqcup\beta' = g_E \in \tau$. And then, there is $\beta' = \{\Phi_E, f_E, g_E\} \sqsubseteq \beta$ such that $\sqcup\beta' = h_E \in \tau$. Furthermore, there exists $\beta' = \{\Phi_E, l_E\} \sqsubseteq \beta$ such that $\sqcup\beta' = l_E \in \tau$, and there exists $\beta' = \{\Phi_E, I_E, f_E, g_E, l_E\} \sqsubseteq \beta$ such that $\sqcup\beta' = I_E \in \tau$. Thus, $\beta = \{\Phi_E, I_E, f_E, g_E, l_E\}$ is a basis for τ.

III. Subspace and Product Topology of Intuitionistic Fuzzy Soft Set

Let U is an initial universe set, E is the set of parameters, and IFS^U is a collection of all intuitionistic fuzzy soft sets of U.

Theorem 3.1. Let (U, E, τ_1) and (U, E, τ_2) are two intuitionistic fuzzy soft topologies over U. Denote the intersection of τ_1 and τ_2 as $\tau_1 \sqcap \tau_2 = \{f_E | f_E \in \tau_1, f_E \in \tau_2\}$. Then $\tau_1 \sqcap \tau_2$ is also topology for it on U.

Proof. Let τ_1 and τ_2 be two intuitionistic fuzzy soft topology on U. Obviously $\Phi_E, I_E \in \tau_1 \sqcap \tau_2$. Let $f_E, g_E \in \tau_1 \sqcap \tau_2$, then $f_E, g_E \in \tau_1$ and $f_E, g_E \in \tau_2$. It is mean $f_E \sqcap g_E \in \tau_1$ and $f_E \sqcap g_E \in \tau_2$. Hence $f_E \sqcap g_E \in \tau_1 \sqcap \tau_2$. Let $\{(f_\alpha)_E : \alpha \in \Gamma\} \sqsubseteq \tau_1 \sqcap \tau_2$, then $(f_\alpha)_E \in \tau_1$ and $(f_\alpha)_E \in \tau_2$ for any $\alpha \in \Gamma$. Since τ_1 and τ_2 are two intuitionistic fuzzy soft topologies on U, then $\sqcup\{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau_1$ and $\sqcup\{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau_2$. Therefore, $\sqcup\{(f_\alpha)_E : \alpha \in \Gamma\} \in \tau_1 \sqcap \tau_2$.■

Definition 3.2. Let (U, E, τ) is an intuitionisticfuzzy soft topological space, Y is a subset of U and h_E is an intuitionistic fuzzy soft set on Y such that for every $e \in E$,

$$\mu_{h_E}(x) = \begin{cases} 1, & x \in Y \\ 0, & x \notin Y \end{cases}$$

$$\nu_{h_E}(x) = \begin{cases} 0, & x \in Y \\ 1, & x \notin Y. \end{cases}$$

Let $\tau_Y = \{h_E \sqcap g_B : g_B \in \tau, B \subseteq E\}$. Then τ_Y is said a subspace topology of intuitionistic fuzzy soft set over Y.

Theorem 3.3. Let (U, E, τ) is an intuitionistic fuzzy soft topological space, Y is a subset of U and h_E is an intuitionistic fuzzy soft set over Y such that for every $e \in E$,

$$\mu_{h_E}(x) = \begin{cases} 1, & x \in Y \\ 0, & x \notin Y \end{cases}$$

$$\nu_{h_E}(x) = \begin{cases} 0, & x \in Y \\ 1, & x \notin Y. \end{cases}$$

If β is a basis of intuitionistic fuzzy soft topology on U, then collection $\beta_Y = \{h_E \sqcap f_A : f_A \in \beta, A \in E\}$ is a basis for intuitionistic fuzzy soft subspace topology on Y.

Proof. Since $f_A \in \beta$ then $f_A \in \tau$. Let β is a basis for τ, then for each $f_A \in \tau$ there is $\beta' \sqsubseteq \beta$ such that $f_A = \sqcup\beta'$. Since $h_E \in IFS^Y$ and $f_A \in \tau$, then $(h_E \sqcap f_A) \in \tau_Y$. Because

$$h_E \sqcap f_A = h_E \sqcap (\sqcup\beta') = \sqcup\beta'.$$

Then for each $(h_E \sqcap f_A) \in \tau_Y$ there is $\beta' \sqsubseteq \beta_Y \sqsubseteq \beta$ such that $h_E \sqcap f_A = \sqcup \beta'$. Hence, β_Y is a basis of intuitionistic fuzzy soft subspace topology on Y. ■

Definition 3.4. Let (U, E, τ) is an intuitionistic fuzzy soft topological space on U with topology τ and (Y, E, τ_Y) is an intuitionistic fuzzy soft topological space on Y with topology τ_Y, and $Y \subseteq U$. We say that intuitionistic fuzzy soft set h_E is open in (Y, E, τ_Y) if it belongs to τ_Y.

Theorem 3.5. Let (U, E, τ) is an intuitionistic fuzzy soft topological space on U and (Y, E, τ_Y) is an intuitionistic fuzzy soft topological space on Y. Let h_E is an intuitionistic fuzzy soft set on Y. If j_E is open in (Y, E, τ_Y) and h_E is open in (U, E, τ), then j_E is open in (U, E, τ).

Proof. Since j_E is open in (Y, E, τ_Y), then $j_E \in \tau_Y$ and h_E is open in (U, E, τ), then $h_E \in \tau$. Since τ_Y is a subspace topology on Y, then $j_E = h_E \sqcap g_B$ where $g_B \in \tau$ and $B \subseteq E$. Since $h_E \in \tau$ and $g_B \in \tau$, then $j_E \in \tau$. Thus, j_E is open in (U, E, τ). ■

Definition 3.6. Let IFS^{U_1} and IFS^{U_2} are two intuitionistic fuzzy soft spaces. Let $f_{E_1} \in IFS^{U_1}$ and $g_{E_2} \in IFS^{U_2}$. The cartesian product of f_{E_1} and g_{E_2}, denoted by $f_{E_1} \otimes g_{E_2}$ is an intuitionistic fuzzy soft set over $U_1 \times U_2$ with regards to parameter set $E_1 \times E_2$ defined as below:

$$f_{E_1} \otimes g_{E_2} : E_1 \times E_2 \longrightarrow IF^{U_1} \times IF^{U_2}$$
$$(e_1, e_2) \mapsto f(e_1) \times g(e_2)$$

such that $f(e_1) \times g(e_2)$ is the intuitionistic fuzzy product of intuitionistic fuzzy sets $f(e_1)$ and $g(e_2)$ where

$$\mu_{f(e_1) \times g(e_2)} : U_1 \times U_2 \longrightarrow [0,1]$$
$$(x_1, x_2) \mapsto \min\{\mu_{f_{E_1}}, \mu_{g_{E_2}}\}$$
$$\nu_{f(e_1) \times g(e_2)} : U_1 \times U_2 \longrightarrow [0,1]$$
$$(x_1, x_2) \mapsto \max\{\nu_{f_{E_1}}, \nu_{g_{E_2}}\}.$$

Definition 3.7. Let IFS^{U_1} and IFS^{U_2} are two intuitionistic fuzzy soft spaces. Then

$$IFS^{U_1} \otimes IFS^{U_2} = \{f_{E_1} \otimes g_{E_2} : f_{E_1} \in IFS^{U_1}, g_{E_2} \in IFS^{U_2}\}$$

is called the intuitionistic fuzzy soft Cartesian product of intuitionistic fuzzy soft spaces IFS^{U_1} and IFS^{U_2}.

Definition 3.8. Let (U_1, E_1, τ_1) and (U_2, E_2, τ_2) are two intuitionistic fuzzy soft topological spaces. The topology $\tau^{\otimes}$ generated by $\beta = \{f_{E_1} \otimes g_{E_2} : f_{E_1} \in \tau_1, g_{E_2} \in \tau_2\}$ is called intuitionistic fuzzy soft product topology over $U_1 \times U_2$. We denote this new topology by $(U, E, \tau^{\otimes})$ where $U = U_1 \times U_2$ and $E = E_1 \times E_2$.

Theorem 3.9. Let (U_1, E_1, τ_1) and (U_2, E_2, τ_2) are two intuitionistic fuzzy soft topological spaces. If β is a basis for intuitionistic fuzzy soft topology on U_1 and γ is a basis of intuitionistic fuzzy soft topology on U_2, then the collection

$$\delta = \{f_{E_1} \otimes g_{E_2} : f_{E_1} \in \beta, g_{E_2} \in \gamma\}$$

is a basis of intuitionistic fuzzy soft topology on $U_1 \times U_2$.

Proof. Since $f_{E_1} \in \beta$, then $f_{E_1} \in \tau_1$ and since $g_{E_2} \in \gamma$, then $g_{E_2} \in \tau_2$. Let β is a basis for τ_1, then for each $f_{E_1} \in \tau_1$ there is $\beta' \sqsubseteq \beta$ such that $f_{E_1} = \sqcup \beta'$. Let γ is a basis for τ_2, then for each $g_{E_2} \in \tau_2$ there is $\gamma' \sqsubseteq \gamma$ such that $g_{E_2} = \sqcup \gamma'$. Since

$$f_{E_1} \otimes g_{E_2} = \sqcup \beta' \otimes \sqcup \gamma' = \sqcup (\beta' \otimes \gamma')$$

then there is $(\beta' \otimes \gamma') \sqsubseteq (\beta \otimes \gamma) = \delta$ such that $f_{E_1} \otimes g_{E_2} = \sqcup (\beta' \otimes \gamma')$ for each $f_{E_1} \in \tau_1$ and $g_{E_2} \in \tau_2$. Hence, δ is a basis for intuitionistic fuzzy soft topology on $U_1 \times U_2$.

IV. CONCLUSION

In this research, we have studied about intuitionistic fuzzy soft set, some relations and operations of intuitionistic fuzzy soft set, intuitionistic fuzzy soft topology, intuitionistic fuzzy soft subspace topology, basis for intuitionistic fuzzy soft subspace topology, intuitionistic fuzzy soft product topology, and basis for intuitionistic fuzzy soft product topology. An intuitionistic fuzzy soft subspace topology is a collection of intersection of two intuitionistic fuzzy soft sets with one of them is an element of intuitionistic fuzzy soft topology. Product topology of two intuitionistic fuzzy soft topological spaces is topology generated by basis, where members of basis in form product of member two topology on intuitionistic fuzzy soft sets.

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