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Find A General Solution Of An Equation Of The Hyperbolic Type With A Second-Order Singular Coefficient And Solve The Cauchy Problem Posed For This Equation

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Abstract – This paper presents a general solution of a hyperbolic type equation with a second-order singular coefficient and a solution to the Cauchy problem posed for this equation.

Keywords – Singular, coefficient, characteristic equation, canonical equation, Cauchy problem.

Consider an equation of the hyperbolic type with a second-order singular coefficient,

$$-(-y)^m u_{xx} + u_{yy} + \frac{\beta_0}{y} u_y = 0 \qquad (1)$$

$$m > 0\ ,\quad y < 0\ ,\ -\frac{m}{2} \le \beta_0 < 1$$

To make the equation look canonical, we find its characteristics,

$$-(-y)^m dy^2 + dx^2 = 0$$

$$\left(dx - (-y)^{\frac{m}{2}} dy\right) \cdot \left(dx + (-y)^{\frac{m}{2}} dy\right) = 0$$

$$\xi = x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}} \quad \text{ва} \quad \eta = x - \frac{2}{m+2} \cdot (-y)^{\frac{m+2}{2}} \qquad (2)$$

New ξ and η We calculate the products involved in equation (1) by entering the variables:

$$u_{xx}=v_{\xi\xi}+2v_{\xi\eta}+v_{\eta\eta}$$

$$u_y=-(-y)^{\frac{m}{2}}v_\xi+(-y)^{\frac{m}{2}}v_\eta$$

$$u_{yy}=(-y)^m v_{\xi\xi}+(-y)^m v_{\eta\eta}-2v_{\xi\eta}(-y)^m+\frac{m}{2}(-y)^{\frac{m-2}{2}}v_\xi-\frac{m}{2}(-y)^{\frac{m-2}{2}}v_\eta=0$$

Substituting the findings into Equation (1), we obtain the following canonical equation.

$$v_{\xi\eta}-(-y)^{-\frac{m+2}{2}}\left(\frac{m+2\beta_0}{8}\right)v_\xi+(-y)^{-\frac{m+2}{2}}\left(\frac{m+2\beta_0}{8}\right)v_\eta=0 \qquad [1] \quad (3)$$

In Equation (3) we make the following notation,

$$a=\frac{m+2\beta_0}{2(m+2)}\ ,\ 0<a<1 \quad \text{ва} \quad \xi-\eta=\frac{4}{m+2}(-y)^{\frac{m+2}{2}}$$

After the notation, the appearance of the equation becomes as follows: Euler-Darbu.

$$v_{\xi\eta}-\frac{a}{\xi-\eta}v_\xi+\frac{a}{\xi-\eta}v_\eta=0 \qquad (4)$$

$$u_{xy}-(-y)^{-\frac{m+2}{2}}\left(\frac{m+2\beta_0}{8}\right)u_x+(-y)^{-\frac{m+2}{2}}\left(\frac{m+2\beta_0}{8}\right)u_y=0 \qquad (5)$$

$$u_1(x,y)=\int_x^y \Phi(\xi)(\xi-x)^{-\alpha}\cdot(y-\xi)^{-\beta}\,d\xi \qquad (6)$$

$$u_2(x,y)=(y-x)^{1-2a}\int_x^y \Psi(\xi)(\xi-x)^{a-1}(y-\xi)^{a-1}\,d\xi \qquad (7)$$

$\Phi(\xi)$ and $\Psi(\xi)$ - is an arbitrary continuous function that satisfies equation (5).

$\xi=x(1-t)+yt$ If we make the substitution, the general solution of equation (4) can be found in the following form:

$$v(\xi,\eta)=(\eta-\xi)^{1-2a}\int_0^1 \Phi\left[\xi+t(\eta-\xi)\right]\ t^{-a}\cdot(1-t)^{-a}\,dt+\int_0^1 \Psi\left[\xi+(\eta-\xi)t\right]\cdot t^{a-1}\cdot(1-t)^{a-1}\,dt \quad (8)$$

(8)- In equation (8) ξ and η Substituting Equation (2) for

$$u(x,y)=\left[-\frac{4}{m+2}(-y)^{\frac{m+2}{2}}\right]^{1-2a}\int_0^1 \Phi\left[x+(1-2t)\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right]\cdot t^{-a}(1-t)^{-a}\,dt+$$
$$+\int_0^1 \Psi\left[x+(1-2t)\frac{2}{m+2}(-y)^{\frac{m+2}{2}}\right]t^{a-1}(1-t)^{a-1}\,dt \qquad (9)$$

(9) - equation (1) - general solution of equation.

Cauchy issue.

Such a function $u(x,y)\in C(\overline{D})\cap C^2(D)$ as, Equation (1) and the following

$$\lim_{y\to -0}u(x,y)=\tau(x) \qquad (10)$$

$$\lim_{y\to -0}(-y)^{\beta_0}u_y(x,y)=\nu(x) \qquad (11)$$

Find a solution that satisfies the conditions. Here $\tau(x)$ and $\nu(x)$ - given features,

$\tau(x) \in C[-1;1] \cap C^2(-1;1)$

$\nu(x) \in C^2(-1;1)$ belongs to the class.

$\nu(x)$ function $x \to \pm 1$ can suddenly achieve a small specificity.

Substituting Equation (9) into Condition (10), we obtain

$$\Psi(x) \cdot \int_0^1 t^{a-1} \cdot (1-t)^{a-1} \, dt = \tau(x) \quad (12)$$

$$\psi(x) = \frac{\Gamma(2a) \cdot \tau(x)}{\Gamma^2(a)} \quad (13)$$

If we set equation (9) to condition (11),

$$\Phi(x) = \frac{\left[\frac{4}{m+2}\right]^{\frac{2\beta_0-2}{m+2}} \Gamma(2-2a) \cdot \nu(x)}{(\beta_0 - 1) \cdot \Gamma^2(1-a)} \quad (14)$$

we have equality.

If we reduce the findings to equation (9), we get the solution of the Cauchy problem for equation (1),

$$u(x,y) = 2 \frac{1}{\beta_0 - 1} \frac{\Gamma(2-2a)}{\Gamma^2(1-a)} (-y)^{1-\beta_0}$$

$$\int_0^1 \nu\left[x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}}(2t-1)\right] \cdot t^{-a}(1-t)^{-a} \, dt + \quad (15)$$

$$+ \frac{\Gamma(2a)}{\Gamma^2(a)} \cdot \int_0^1 \tau\left[x + \frac{2}{m+2}(-y)^{\frac{m+2}{2}}(2t-1)\right] \cdot t^{a-1} \cdot (1-t)^{a-1}.$$

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