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Mixture Problem for Abstract Policalorical The Equations

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Abstract – In article the incorrect task for abstract polycaloric the equation is studied and a stability assessment according to Tikhonov is given.

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I. Introduction

The following is considered Problem. It is required to find the decision abstract policalorical the equations.

$$K_+^n u(t) \equiv \left(\frac{d}{dt} + A\right)^n u(t) = 0, \quad 0 < t < T, \tag{1}$$

Satisfying to following conditions:

$$\begin{cases} u\big|_{t=l} = u(l), \\ \dfrac{du}{dt}\Big|_{t=l} = u'(l), \\ \dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots \\ \dfrac{d^{(n-1)}u_{(t)}}{dt}\Big|_{t=l} = u_{(l)}^{(n-1)} \end{cases}$$

Where $u(t)$ - abstract function with values in Gilbert space $H.\ A$ - constant, positively defined, self-interfaced, linear, unlimited with everywhere dense range of definition. $\left(D\left(A^n\right), DCH\right)$ the operator operating from H in H, and $u(l),\ u'(l),\ u''(l),...u^{(n-1)}(l) \in H.$

The theorem 1. If $u_1,\ u_2,...u_n$ - the decision calorical the equations function $u = u_1 + (t-l)u_2 + (t-l)^2 u_3 + ...(t-l)^{n-1} u_n$ is the decision (1) and back, for everyone set abstract policalorical functions u are such functions $u_1, u_2, ...u_n$ that equality takes place

$$u(t) = \sum_{k=1}^{n} (t-l)^{k-1} u_k(t) \tag{3}$$

<u>The proof</u>. The theorem proof is spent by an induction method.

1. If $u_1, u_2, ...u_n$ - the decision kalorical the equations u there is a decision policalorical the equations.

$$K_+ u = K_+\left[u_1 + (t-l)u_2\right] = K_+ u_1 + u_2 + (t-l)\frac{du_2}{dt} + A(t-l) = u_2 =$$

$$= K_+ u_1 + u_2 + (t-l)\left(\frac{du_2}{dt} + A\right)u_2 = K_+ u_1 + u_2 + (t-l)K_+ u_2 = u_2,$$

As $K_+ u_1 = 0, \quad K_+ u_2 = 0$

Hence, applying $n-1$ time operator K_+, considering, that $K_+ u_1 = 0, \quad K_+ u_2 = 0, ... K_+ u_n = 0,$ we will receive

$u(l) = u_1$

$K_+ u = u_2$

$K_+^2 u = 2u_3$

$K_+^3 u = 6u_4$

....

....

$K_+^{n-1} u = (n-1)! u_n$

Let at $k = n$ it is true, т.е, equality $K_+^n u = \sum_{k=1}^{n} (k-1)!(t-l)^{k-1} u_k(t)$ takes place.

Let's prove, that the statement is true for $k = n+1$,

$$K_+^{n+1} u = K_+ K_+^n u = K_+\left[\sum_{k=1}^{n} (k-1)!(t-l)^{k-1} u_k(t)\right] = \sum_{k=1}^{n+1} (k-1)!(t-l)^{k-1} u_k(t).$$

So, we will receive equality

$$K_+^{n+1} u = \sum_{k=1}^{n+1} (k-1)!(t-l)^{k-1} u_k(t).$$

2. If u the decision policalorical the equations will be such kalorical functions $u_1, u_2, \dots u_n$ that $u = u_1 + (t-l)u_2 + (t-l)^2 u_3 + \dots + (t-l)^{n-1} u_n$.

For the proof of this statement it is enough to establish possibility of choice $u_1, u_2, \dots u_n$.

Let's put: $u_1 = u - (t-l)u_2 - (t-l)^2 u_3 - \dots - (t-l)^{n-1} u_n$,

$$u_2 = K_+ u,$$

$$u_3 = K_+^2 u,$$

....

....

$$u_n = K_+^{n-1} u.$$

$$K_+ u_1 = K_+ u - K_+ (t-l) u_2 - K_+ (t-l)^2 u_3 - \dots - K_+ (t-l)^{n-1} u_n =$$

$$= K_+ u - u_2 - (t-l) K_+ u_2 - 2K_+ (t-l) u_3 - (t-l)^2 K_+ u_3 - \dots$$

$$-\dots (n-1) u_n - (t-l)^{n-1} K_+ u_n = K_+ u - K_+ u - (t-l) K_+ u_2 -$$

$$-2(t-l) u_3 - (t-l)^2 K_+ u_3 - \dots - (t-l)^{n-1} K_+ u_n = 0,$$

From here $K_+ u_1 = 0$.

Hence, $K_+ u_2 = 0, \quad K_+ u_3 = 0, \dots K_+ u_n = 0.$

The theorem 1. It is completely proved.

By means of representation (3) decision of a problem (1) - (2) can be reduced to the decision following n problems:

$$\begin{cases} \dfrac{du_1}{dt} + Au_1 = 0, \\ u_1\big|_{t=l} = u_1(l), \end{cases} \quad \begin{cases} \dfrac{du_2}{dt} + Au_2 = 0, \quad \dots \\ u_2\big|_{t=l} = u_2(l), \quad \dots \end{cases} \quad \begin{cases} \dfrac{du_n}{dt} + Au_n = 0, \\ u_n\big|_{t=l} = u_n(l), \end{cases} \qquad (4)$$

Where

$$u_1(l) = u(l)$$
$$u_2(l) = u'(l) + Au(l)$$
$$u_3(l) = \frac{1}{2}\left(u''(l) - 2Au(l) + A^2u(l)\right)$$

....

....

$$u_n(l) = \sum_{k=1}^{n}\sum_{i=0}^{n-1} C_{k-1}^{i} A^{k-i} u^{i}(l)$$
$$C_{k-1} = \frac{1}{(k-1)!}$$

These problems in classical sense are incorrect. Them we will investigate on a conditional correctness on Tikhonov [1]. Problem Koshi for the equation (1) is considered and investigated in [2]. The multidot problem for the equation (1) is investigated in [3].

Let's prove the theorem characterising an estimation of stability of the decision of problems:

$$\begin{cases} \dfrac{du_1}{dt} + Au_1 = 0, & (5) \\ u_1\big|_{t=l} = u(l), & (6) \end{cases}$$

Problem (5) $<$ (6) at $0 < t < l$ uncorrect in classical sense, at $l < t < T$ in classical sense are correct.

The theorem 2. For any decision of a problem (5) - (6) the inequality is fair

$$\|u_1(t)\| \le \begin{cases} \|u(l)\|^{\frac{t}{l}} \|u(0)\|^{1-\frac{t}{l}}, & 0 \le t < l, \\ \|u(l)\|, & l \le t < T \end{cases} \qquad (7)$$

<u>The proof</u>. We will consider function [1].

$$\varphi(t) = \|u_1(t)\|^2 = (u_1, u_1)$$

Differentiating it, we will receive

$$\varphi'(t) = 2(u_1', u_1) = 2(Au_1, u_1).$$
$$\varphi''(t) = 2(u_1', u_1) + 2(u_1, u_1'') = 2(Au_1, Au_1) + 2(u_1, A^2u_1).$$

As operator A self-interfaced $(A = A^*)$ $(u_1, A^2u_1) = (Au_1, Au_1)$ and, means,

$$\varphi''(t) = 4(Au_1, Au_1)$$

Now we will consider function

$$\psi(t)=\ln\varphi(t)$$

Differentiating it, we will receive

$$\psi''(t)=\frac{1}{\varphi^2(t)}\cdot\left[\psi''(t)\cdot\psi'^2(t)\right]=\frac{4}{\varphi^2_{(t)}}\left[(Au_1,Au_1)(u_1,u_1)-(Au_1,u_1)^2\right]\geq 0 \quad . (8)$$

Owing to a known inequality Bunjakovsky the inequality (8) will mark, that function $\psi(t)$ is turned bend upwards, whence follows, that function $\psi(t)$ on piece $[0,l]$ does not surpass the linear function accepting on the ends of a piece the same values, as $\psi(t)$.

From (8) follows $$\psi(t)\leq\frac{l-t}{l}\cdot\psi(0)+\frac{t}{l}\psi(l) \quad (9)$$

Potential an inequality (9), we will receive

$$\varphi(t)\leq\left[\varphi(0)\right]^{1-\frac{t}{l}}\left[\varphi(l)\right]^{\frac{t}{l}}$$

Whence

$$\|u_1(t)\|\leq\|u(l)\|^{\frac{t}{l}}\|u(0)\|^{1-\frac{t}{l}}$$

Now we will consider problems

$$\begin{cases}\dfrac{du_2}{dt}+Au_2=0, & (10)\\ u_2\big|_{t=l}=u_2(l), & (11)\end{cases}$$

Where

$$u_2(l)=u'(l)+Au(l) \text{, as } u(t)=u_1+(t-l)u_2.$$

$$u'(t)=u_1'(t)+u_2(t)+(t-l)u_2'(t)$$

$$u_2(t)=u'(t)-u_1'(t)-(t-l)u_2(t), \quad u_1'(t)=-Au(t)$$

At

$$t=l, \quad u_2(l)=u'(l)-u_1'(l)=u'(l)+Au(l)$$

Problems (10) - (11) at $0<t<l$ it is incorrect in classical sense, at $l<t<T$ in classical sense are correct.

Similarly investigating problems (10) - (11) on a conditional correctness on Tikhonov as problems (5) - (6), we will receive stability estimations.

$$\|u_2(t)\|\leq\begin{cases}\left(\|u'(l)\|+\|Au(l)\|\right)^{\frac{t}{l}}\cdot\|K_+u(o)\|^{1-\frac{t}{l}}, & 0<t\leq l,\\ \|u'(l)\|+\|Au(l)\|, & l\leq t\leq T.\end{cases}$$

Hence,

$$\|u_3(t)\| \le \begin{cases} \left(\|u''(l)\| + \|Au(l)\| + \|A^2u(l)\|\right)^{\frac{t}{l}} \cdot \|K_+^2 u(0)\|,^{1-\frac{t}{l}} & 0 < t \le l \\ \|u''(l) + \|\|Au(l) + \|\|A^2u(l)\|, & l \le t \le T \end{cases}$$

Repeating this procedure n of times, we will receive the following theorem.

The theorem 3. For any decision of a problem (1) - (2) it is fair

$$\|u(t)\|_H \le \sum_{k=1}^{n}\sum_{i=1}^{k} C_{k-1} \begin{cases} \left(\|u^{(i)}(l)\| \cdot \|A^{k-i}\|\right)^{\frac{t}{l}} \cdot \|K_+^{i-1}u(0)\|^{1-\frac{t}{l}}, & 0 \le t \le l, \\ \|u(l)\|, & l \le t \le T \end{cases} \tag{12}$$

Let's notice, that from an inequality (12) uniqueness of the decision of a problem (1) - (2) and a conditional correctness of this problem in a class follows

$$\left\{u : \|K_+^{n-1}u(0)\| \le M\right\}.$$

This towers is proved by a method of logarithmic camber $[1]$.

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