

Construction Of The Shadows Of Polyhedra

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Abstract – This article discusses the use of multiparameter functions for construction of shadows and automated imaging with the help of software Auto CAD.

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The rays of sunlight can be considered parallel and rectilinear to a large extent.

It is known [1] that falling on objects around us, sunlight makes them visible. Thanks to chiaroscuro that regenerating on the surface of illuminated objects, their shape and plasticity are revealed.

The surface of an object, when illuminated from one side, does not receive light from the other side. The unlit part of the surface is called its own shadow. If an object is located in front of the illuminated surface of another object, then it blocks part of it from the rays of light. This part is called the falling shadow from the object.

Let's consider the geometric essence of construction of shadows.

In fig. 1 a ball illuminated by parallel rays and a shadow falling from it onto a plane is shown. The side of the ball facing the plane is in its own shadow. The difference between its own shadow and the falling one is that the first is formed on the surface directed from the light; the second is on the surface facing the light.

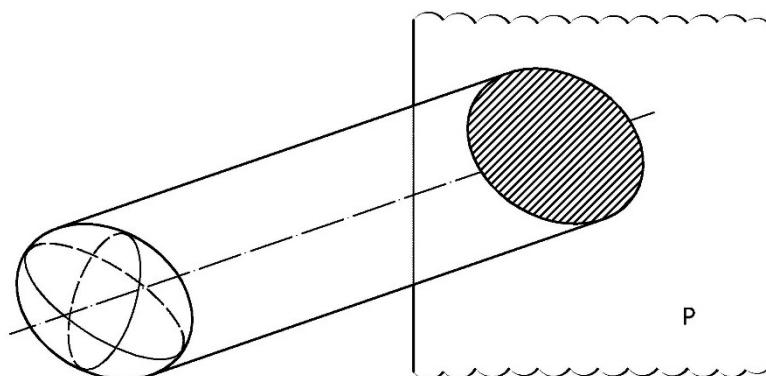


Figure: 1. Falling shadow of the ball on the plane

Depicting the facades of buildings in orthogonal projections, an architect seeks to reveal their plasticity in reality; buildings are illuminated in different directions of sunlight. Constructed direction of light has the advantages over the variable that the eye, getting used to it, more easily captures the relief, performed with shadows with the familiar directional light, and is faster oriented in the drawing.

The constant direction of light adopted for architectural images is determined by the position of the vertical (frontal) and horizontal projection the ray of the light, which with the axis of projections OX is equal to 45° (Fig. 2)

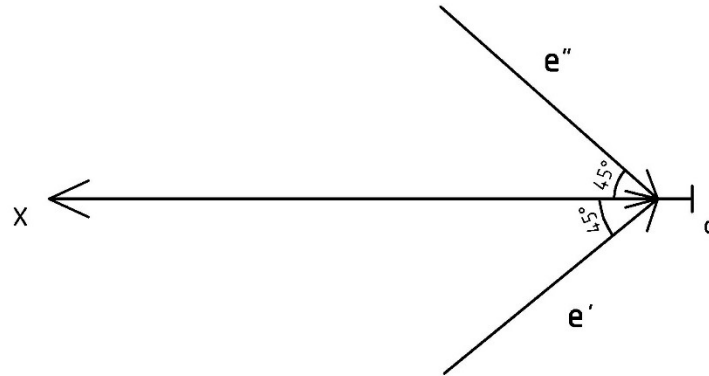


Figure: 2. Direction of light

This direction of light corresponds to the position of the diagonal of the cube, the edge of which is vertical (frontal), horizontal and profile - the projecting lines. A cube built on a segment of light ray, as on a diagonal, we will agree to call light.

In fig. 3 a light cube, depicted on the frontal plane of the projections by a square equal to its face is shown: the projection of the light beam clipping, which is the basis for the formation of the light cube, serves as the diagonal of this square. From the geometric properties of the cube, we establish the following dependence: the edge of the cube is the diagonal of its side (base), corresponding to the projection of the light ray and the diagonal in space, i.e. the ray itself, relate to each other as $1 : \sqrt{2} : \sqrt{3}$

These elements are located in the frontal projection plane passing through the diagonal of the cube. They form a right-angled triangle, which we will agree to call light.

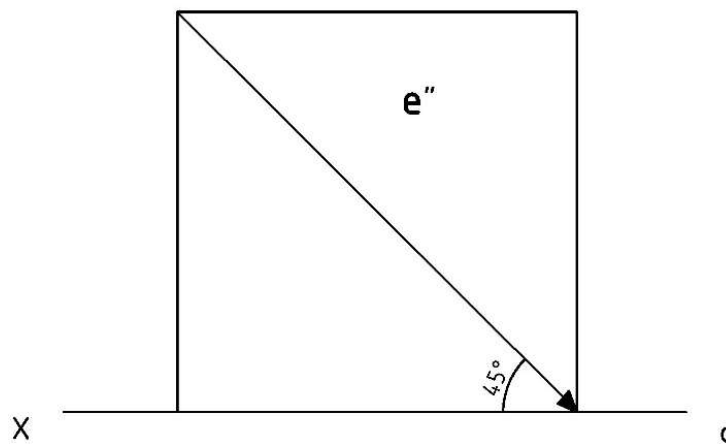


Figure: 3. Projection of the light beam

Bringing the plane of the light triangle to the frontal position, we get its true size Fig. 4

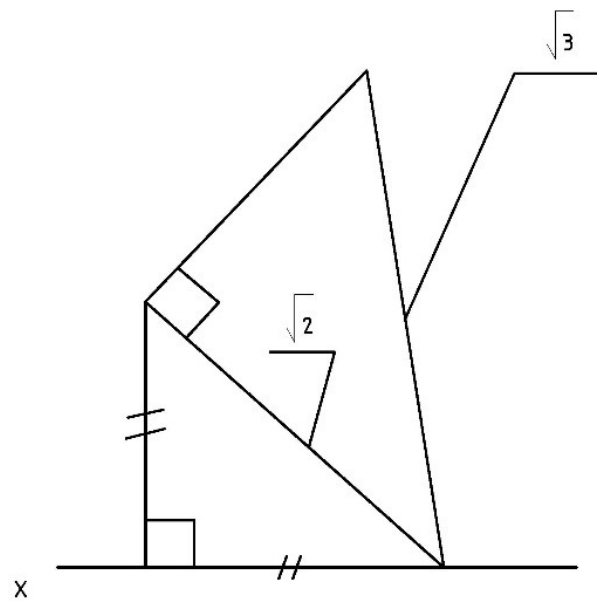


Figure: 4. Construction of the natural size of the light triangle

The direction of light accepted for architectural images is determined by the position of the frontal and horizontal projections of the light beam, which with the axis of their projections make up angles equal to α and β (Fig-5).

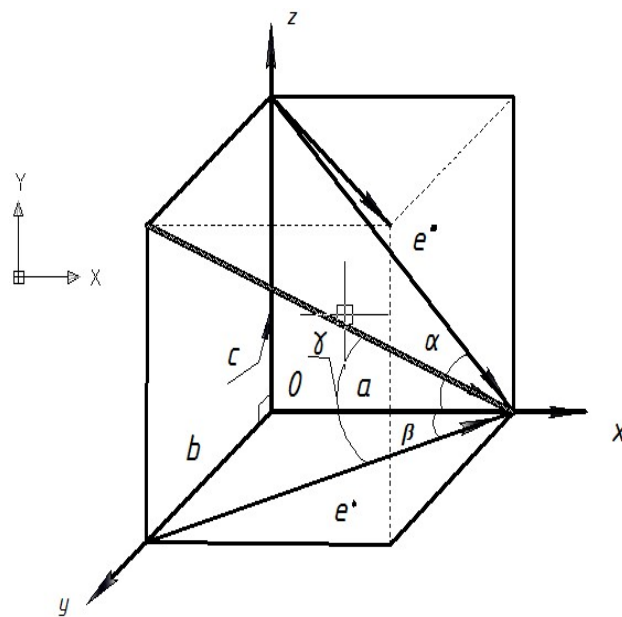


Figure: 5. Light parallelepiped

This direction of light corresponds to the position of the diagonal of a parallelepiped with dimensions $(a \times b \times c)$, the edges of which are frontal, horizontal and profile projecting lines. A parallelepiped built into segments, a , b , c of a light ray, as on a diagonal, we will agree to call light.

In fig. 6 a light rectangle depicted on the frontal plane of projections by a rectangle equal to its edge is shown;

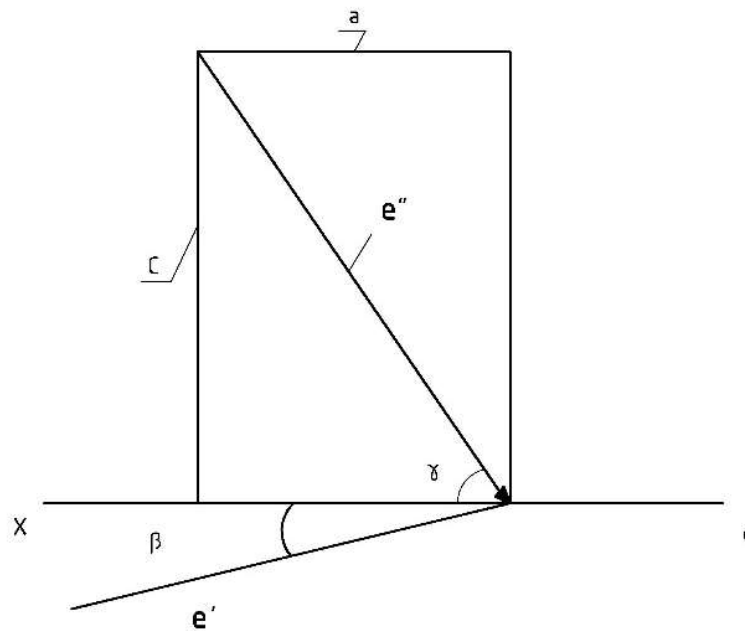


Figure: 6. General position of projection of the light beam

Below the analytical relationships of the edge of the light parallelepiped are presented

$$\begin{aligned} |\ell''| &= \sqrt{a^2 + c^2} \\ |\ell'| &= \sqrt{a^2 + b^2} \\ |\vec{\ell}| &= \sqrt{a^2 + b^2 + c^2} \\ |\vec{\ell}| &= \sqrt{(\ell')^2 + c^2} = \sqrt{(\ell'')^2 + b^2} \end{aligned} \quad (1)$$

$$\sin \alpha = \frac{c}{\ell''}; \ell'' = \frac{c}{\sin \alpha} \quad (1) \text{ здесь } 0 < \alpha < 90^\circ \quad 0 < \beta < 90^\circ$$

$$\cos \alpha = \frac{a}{\ell'} \rightarrow \ell' = \frac{a}{\cos \alpha} \quad (2)$$

$$(1), (2) \Rightarrow \frac{c}{\sin \alpha} = \frac{a}{\cos \alpha} \Rightarrow c = \frac{\sin \alpha}{\cos \alpha} = \operatorname{tg} \alpha$$

$$\text{So } \ell'' = (c^2 + a^2 \operatorname{tg}^2 \alpha)^{0.5} \quad \frac{c}{a} = \operatorname{tg} \alpha \rightarrow c = a \operatorname{tg} \alpha; \quad (3)$$

$$\text{We analytically define } b = a \operatorname{tg} \beta; \quad (4)$$

Calculations of the vertical edge of parallelepiped (3), (4) $\rightarrow c / (\operatorname{tg} \alpha) = b / (\operatorname{tg} \beta)$;

$$c = \frac{b \operatorname{tg} \alpha}{\operatorname{tg} \beta}; \quad (5) \quad \text{or} \quad \operatorname{tg} \alpha = \frac{c \operatorname{tg} \beta}{b}; \quad (6)$$

$$|\vec{\ell}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{a^2 \operatorname{tg}^2 \alpha + a^2 \operatorname{tg}^2 \beta + a^2} = a \sqrt{1 + \operatorname{tg}^2 \alpha + \operatorname{tg}^2 \beta};$$

formula of calculating the diagonal of the light parallelepiped.

$$\text{Thus, } |\vec{\ell}| = \sqrt{1 + \operatorname{tg}^2 \alpha + \operatorname{tg}^2 \beta} \quad (7)$$

Let a straight prism be given, standing on the plane H . It is required to construct its own and falling shadows on the projection plane H and V (Fig. 7). Let's analyze the illumination of the sides. For a given direction of light flux, the top and left front sides will be illuminated. The rest of the sides (including the bottom one) are in shadow. To build a falling shadow of a volumetric body, it is necessary to identify the contour of its own shadow, which in this case will be a spatial polyline. The elements of this line are the edges of the prism located at the boundaries of the illuminated and unlit planes. In the same figure an isometric image of a closed contour of its own shadow, from which is constructed a falling shadow is presented.

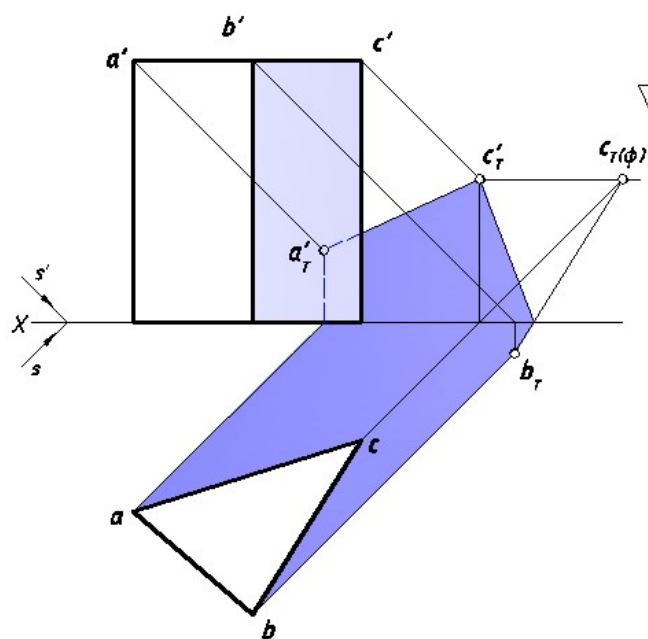


Figure: 7. Construction of own and falling shadows of the prism

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