

# *Computational Algorithm for Solving of Problem the Two-Phase Filtration*

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**Abstract – This paper presents the problem of one-dimensional two-phase liquid filtration. Mathematical support for the problem of liquid filtration is given and a numerical solution method is developed. An algorithm for calculating the problem using the finite difference method was developed and tested by means of test functions. The software was developed based on the created algorithm. In the developed software, computational experiments were performed using accurate mining data. The results obtained were analyzed.**

**Keywords – Computational Algorithm, for Solving of Problem, Two-Phase Filtration**

## I. INTRODUCTION

To fully determine the physical or mechanical meanings, it is necessary to solve the equation describing the process in a three-dimensional formulation. However, in most cases, a one-dimensional problem is considered for the process under study due to the cumbersomeness of the computational algorithm and software, or even the unreliability of the input data in three-dimensional space. For physical meanings, sometimes, it is also sufficient to consider the one-dimensional case; actually, an  $X_i$  input parameter greatly influences the  $Y_i$  resulting value or, on the contrary, it hardly affects the value. Then, in the first case, we will consider a multidimensional problem, and it will be sufficient to consider a one-dimensional problem in the second case. Multidimensional problems shall be solved as a system of one-dimensional problems. Therefore, the numerical solution of one-dimensional problems does not lose its relevance.

In [1], the problems of the modeling, monitoring, control and management of technological processes associated with the movement of structured heterogeneous fluids with complex (non-equilibrium and nonlinear) characteristics are considered. It is shown that when describing such fluids, it is necessary to use the representation of the theory of self-organization, reflecting the most common properties of complex natural objects. Due to the lack of reliable theoretical prerequisites, models of complex systems are normally of identification character. In this regard, part of the book is devoted to the consideration of methods and examples of solving inverse problems of oilfield mechanics.

The work [3] is devoted to one of the approaches in the numerical simulation of filtration processes of a two-phase incompressible fluid. Buckley - Leverett problem is considered with solutions of a shock wave type. Within the mixed finite element method upwind algorithm for solution to equation for saturation is proposed, which gives solutions without numerical oscillations at the discontinuity. Special attention is paid to a new approach to solving the degenerate Neumann problem in a mixed formulation for the pressure and total flow velocity. The work is accompanied by an illustration of the results of numerical experiments for 3D problem.

[4] considers a numerical solution of a two-dimensional gas filtration problem for a model problem, the area of which has been taken as a rectangle, and the coefficients of the equation have been taken as constant, i.e. the reservoir has been considered homogeneous. However, when solving practical problems occurring in a reservoir, the type of the area of real fields is arbitrary, and geological and geophysical data shall be obtained using the core analysis at some given points of the reservoir. Consequently, it is necessary to determine the values of these parameters at all points of the area under consideration by a method.

In [5], the fundamentals of mechanics of fluid, gas and multiphase media are laid down on the basis of the basic concepts of continuum medium mechanics. The derivation of conservation laws in the integral and differential form is given, the elements of hydrostatics are presented, various types of flow of ideal and viscous liquids, the basic concepts of turbulence, the theory of dimensions and similarity are considered.

In [6], the basic principles and methodological approaches to the selection of technologies and development systems are outlined in accordance with the potential of an oil reservoir production facility. The issues of the theory of creating environmentally friendly intrastratal processes associated with changes in the phase state of fluids and the physicochemical properties of the reservoirs are highlighted in order to intensify oil production and withdrawal from the subsoil.

Under consideration is some mathematical model of two-phase filtration of incompressible fluids that accounts for the capillary pressure. For this model, we construct a monotone balance difference scheme, propose an efficient implementation, and test it in computational trials including those at the cluster MVS-1000[7].

## II. THE STATEMENT OF THE PROBLEM

In most oil or gas fields, the filtration process is multiphase. This requires more efficient solution methods as compared with single-phase flow, since a system of coupled nonlinear partial differential equations is considered [8–10].

It is required to determine the  $U(x, t)$  vector function in the  $(0 \leq x \leq 1, t > 0)$  domain, from the system of differential equations of parabolic type:

$$\begin{aligned} \frac{\partial}{\partial x} \left( K(x, t) A(U) \frac{\partial U(x, t)}{\partial x} \right) &= M(x, t) \frac{\partial U(x, t)}{\partial t} + F(x, t), \quad x \in (0, 1), \quad t > 0 \\ S_1 + S_2 &= 1, \\ U_k(S) &= U_1(x, t) - U_2(x, t), \end{aligned} \quad (1)$$

Satisfying the initial

$$U(x, 0) = U^0(x) . \quad (2)$$

and boundary conditions

$$\begin{aligned} \lambda_1 K(x, t) A(U) \frac{\partial U(x, t)}{\partial x} + \theta_1 U(x, t) &= \gamma_1(t), \quad x = 0 \\ \lambda_2 K(x, t) A(U) \frac{\partial U(x, t)}{\partial x} + \theta_2 U(x, t) &= \gamma_2(t), \quad x = 1 \end{aligned} \quad (3)$$

describing the process of two-phase filtration with the  $(S_1, S_2)$  saturation and the  $(U_1, U_2)$  pressure subject to the  $(U_k)$  capillary forces.

### III. SOLUTION METHOD OF THE PROBLEM

To solve the problem, let us construct an algorithm using the stream sweep method [11,12].

here

$A, U, F, \gamma_1, \gamma_2$  - vectors of the following kind

$$A = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}, \quad U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}, \quad F = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} \gamma_{11} \\ \gamma_{12} \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} \gamma_{21} \\ \gamma_{22} \end{pmatrix},$$

$K, M, \lambda_1, \lambda_2, \theta_1, \theta_2$  - matrices

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}, \quad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \quad \lambda_1 = \begin{pmatrix} \lambda_{11}^{(1)} & \lambda_{12}^{(1)} \\ \lambda_{21}^{(1)} & \lambda_{22}^{(1)} \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} \lambda_{11}^{(2)} & \lambda_{12}^{(2)} \\ \lambda_{21}^{(2)} & \lambda_{22}^{(2)} \end{pmatrix},$$

$$\theta_1 = \begin{pmatrix} \theta_{11}^{(1)} & \theta_{12}^{(1)} \\ \theta_{21}^{(1)} & \theta_{22}^{(1)} \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} \theta_{11}^{(2)} & \theta_{12}^{(2)} \\ \theta_{21}^{(2)} & \theta_{22}^{(2)} \end{pmatrix}$$

Let us cover the area under consideration with a uniform grid.

$$\overline{\omega}_{h\tau} = \{x_i = ih, \quad i = \overline{0, N}, \quad h = 1/N, \quad t_j = j\tau, \quad j = \overline{0, N_1}, \quad \tau = T/N_1\}.$$

According to the stream sweep method, the relationship between the  $U_i$  and  $W_{i-1/2}$  difference functions is expressed as

$$U_i = \frac{\tau}{h} \alpha_i W_{i-1/2} + \beta_i$$

We shall present the final form of the algorithm in order to avoid personal computations and with the value  $A = 1$  :

$$\lambda_2 \neq 0: \quad \alpha_N = -\frac{h}{\tau} (\lambda_2^{-1} \theta_2 + 0.5 \frac{h}{\tau} M_N)^{-1},$$

$$\beta_N = (\lambda_2^{-1} \theta_2 + 0.5 \frac{h}{\tau} M_N)^{-1} (\lambda_2^{-1} \gamma_2 - 0.5 \frac{h}{\tau} \Phi_N),$$

$$\lambda_2 = 0: \quad \alpha_N = 0, \quad \beta_N = \gamma_2$$

$$\alpha_i = -\left[ \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} + M_i \right]^{-1},$$

$$\beta_i = \left[ \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} + M_i \right]^{-1} \left[ \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} \beta_{i+1} - \Phi_i \right],$$

$$i = N-1, \dots, 0$$

$$\lambda_1 \neq 0: \quad U_0 = \left[ \frac{h}{\tau} \left( \frac{h^2}{\tau} - K_{1/2} \alpha_1 \right)^{-1} K_{1/2} + 0.5 \frac{h}{\tau} M_0 - \lambda_1^{-1} \theta_1 \right]^{-1} \times \\ \times \left[ \frac{h}{\tau} \left( \frac{h^2}{\tau} - K_{1/2} \alpha_1 \right)^{-1} K_{1/2} \beta_1 - 0.5 \frac{h}{\tau} \Phi_0 - \lambda_1^{-1} \gamma_1 \right];$$

$$\lambda_1 = 0: \quad U_0 = \gamma_1$$

$$\lambda_1 \neq 0: \quad W_{1/2} = \left( 0.5 \frac{h}{\tau} M_0 - \lambda_1^{-1} \theta_1 \right) U_0 + \left( 0.5 \frac{h}{\tau} \Phi_0 + \lambda_1^{-1} \gamma_1 \right)$$

$$\lambda_1 = 0: \quad W_{1/2} = \gamma_1$$

$$U_{i+1} = -\alpha_{i+1} \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} U_i + \beta_{i+1} \left[ 1 + \alpha_{i+1} \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} \right], \\ i = 0, 1, \dots, N-1$$

$$W_{i+1/2} = \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} \left[ \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} + M_i \right]^{-1} W_{i-1/2} + \\ + \frac{h}{\tau} \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} \times \\ \times \left[ \beta_{i+1} + \left[ \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} + M_i \right]^{-1} \left[ \left( \frac{h^2}{\tau} - K_{i+1/2} \alpha_{i+1} \right)^{-1} K_{i+1/2} \beta_{i+1} - \Phi_i \right] \right] \\ i = 1, 2, \dots, N-1$$

#### IV. DISCUSSION OF RESULTS

Let us check the reliability of the obtained algorithm on a test case, which has an exact solution with the following given values of coefficients and parameters:

$$U = \begin{pmatrix} (x^2 - 2x^3 + x^4)t \\ (2x^2 - 4x^3 + 2x^4)t \end{pmatrix}, \quad \lambda_1 = \lambda_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \theta_1 = \theta_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \gamma_1 = \gamma_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$K = \begin{pmatrix} 0,5 & 0 \\ 0, & 0,5 \end{pmatrix}, \quad M = \begin{pmatrix} 2x + \frac{5}{2}x^4 & x + 2x^4 \\ x + 2x^4 & 2x + \frac{5}{2}x^4 \end{pmatrix},$$

$$F = \begin{cases} (1 - 6x + 6x^2)t - 4x^3 + 8x^4 - 4x^5 - 6.5x^6 + 13x^7 - 6.5x^8 \\ 2(1 - 6x + 6x^2)t - 5x^3 + 10x^4 - 5x^5 - 7x^6 + 14x^7 - 7x^8 \end{cases}$$

The figure shows a graph of the change in the exact and approximate value of the  $U(x, t)$  function ((a continuous line of the value of the exact solution and an approximate value with the  $U_1(x, t)$  rhombus-like point, with the  $U_2(x, t)$  rectangle-like point). The obtained results show that the developed algorithm can be used to solve the problem of two-phase filtration.

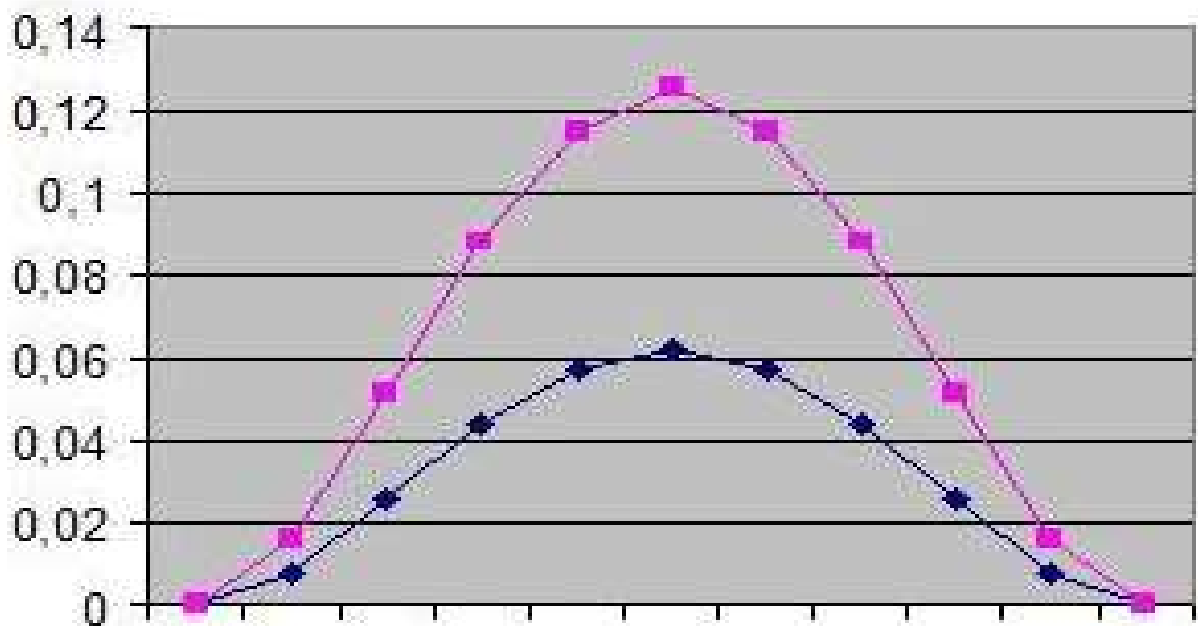


Fig.1. A graph of the exact and approximate value of the solution.

## V. CONCLUSION

From the above results we can say that the developed algorithm is applicable for solving the problem of filtering in a porous medium. The symmetry of the results obtained was tested on a test example and in a model problem. From the obtained results we can say that the developed algorithm and software can be used to determine the performance of real fields.

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