

Construction Of A Mathematical Model Of The Geometric Nonlinear Problem Of A Vibrating Beam

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Abstract – Based on the variation principle of Hamilton-Ostrogradsky which system of differential equations of geometric nonlinear proportional equations of the oscillating girder and its corresponding initial and boundary processes were introduced.

Keywords – Hamilton - Ostrogradsky principle of variation, kinetic energy, potential energy, external force, Guck's law, voltage, volumetric force vector, surface force vector, geometric line, moving beam, initial condition, boundary condition, system of differential equations.

I. INTRODUCTION

Much attention is paid to the use of automated systems and their use in physical and mechanical properties in design and engineering work. Therefore, the implementation of design work on the basis of modern computer technology is developing rapidly.

One of the main tasks in our country is the creation of software packages based on the use of mathematical modeling processes in design work, the development of convenient algorithms for their automation.

The Action Strategy for 2017-2021 identifies key issues, including "... issues of accelerated introduction of information and communication technologies in the field of road transport, engineering and communications and social infrastructure ...". To solve such problems, the design process involves the development of various mathematical models, convenient and precise algorithms, the creation of specific software for them.

II. MATERIALS AND METHODS

The problem of creating a mathematical model of the geometric nonlinear problem of the oscillating beam, i.e. the development of an equilibrium equation involving a definite process, serves as an answer to the above question.

In the 70s of the last century, academician VK Kabulov developed mathematical models of the mechanics of many solids, ie mathematical models of the laws of displacement, stress and strain as a result of vibration of the body [2].

The problem of vibration of a flat rod as a result of stress and its mathematical model, taking into account the kinetic and potential energy, is given in the analytical formula [5], and the model for determining the tension of a rod with a right rectangular

cross section is given in [6]. To do this, the system of differential equations on the axes OX and OU and the corresponding boundary conditions are described.

In general, great attention is paid to the automation of design and engineering issues using modern computers using physical and mechanical properties.

Many existing large deflection problems are limited to seeing geometric nonlinear issues. The essence of the problem we are considering is that we consider the oscillation of the problem without a geometric line, taking into account the cross section of the beam..

III. THE MAIN PART

Based on the Hamilton-Ostrogradsky variational principle, we derive a mathematical model of a vibrating beam, that is, a differential equation with a special product without a geometric line and its boundary, initial conditions [2-4]. The Hamilton-Ostrogradsky principle of variation is expressed by the following formula [2]:

$$\delta \int_t (K - \Pi + A) dt = 0 \quad (1)$$

Here

$$\delta K = \int_v \rho \delta \left(\frac{1}{2} \vec{V}^2 \right) dV - \text{variation of kinetic energy} \quad (2)$$

ρ – density;

$$\vec{V} = \frac{\partial \vec{u}}{\partial t} - \text{velocity vector}$$

$\vec{u}$ – migration vector

According to the principle of action, potential energy is:

$$\delta \Pi = \int_v \sigma_{ij} \delta \varepsilon_{ij} dV \quad (3)$$

Here σ_{ij} – voltage tensor

ε_{ij} - voltage tensor;

$$\delta A = \int_v \vec{Q} \delta \vec{U} dV + \int_s \vec{N} \delta \vec{U} ds - \quad (4)$$

External power variation

Here

$\vec{Q}$ – volumetric force vector; $\vec{N}$ – surface force vector;

Based on the hypothesis of plane sections, the components of the displacement vector at arbitrary points of the beam are represented by the following formula:

$$u_1 = u - z \frac{\partial w}{\partial x}, \quad u_2 = V = 0, \quad u_3 = w \quad (5)$$

Here

U, V, W – the beam represents the midlines of the displacement vector.

The relationship between voltage and voltage (Guk's law) for a beam is as follows:

$$\left. \begin{aligned} \sigma_{11} &= E * \varepsilon_{11} \\ \sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{31} &= 0 \end{aligned} \right\}, \quad (6)$$

Here

$$\varepsilon_{11} = \frac{\partial U}{\partial x} - z \frac{\partial^2 W}{\partial x^2} + \frac{\sigma_1}{2} \left(\frac{\partial U}{\partial x} - z \frac{\partial^2 W}{\partial x^2} \right)^2 \quad \sigma_1 - \text{an arbitrary constant number.}$$

The external power variation (4) is as follows:

$$\begin{aligned} \int_t \delta A dt &= \int_t \int_x \left\{ [Q_1 + q_1^+ + q_1^-] \delta U + \left[\frac{\partial M}{\partial x} \bar{Q}_1 + Q_3 + \frac{\partial M_{q_1^+}}{\partial x} - \frac{\partial M_{q_1^-}}{\partial x} + q_3^+ + q_3^- \right] \delta W \right\} dx dt + \\ &+ \int_t \left\{ P_{\bar{N}_1} \delta U - M_{\bar{N}_1} \delta \left(\frac{\partial W}{\partial x} \right) + \left(P_{\bar{N}_3} - M_{\bar{Q}_1} - M_{q_1^+} + M_{q_1^-} \right) \delta W \right\} dt \Big|_x, \end{aligned} \quad (7)$$

Here

$$M_{\bar{Q}_k} = \int_y \int_z z \bar{Q}_k dz dy, \quad M_{N_k} = \int_y \int_z z \bar{N}_k dz dy;$$

$$P_{\bar{N}_k} = \int_y \int_z \bar{N}_k dz dy, \quad Q_k = \int_y \int_z \bar{Q}_k dz dy;$$

$$q_k^{+(-)} = q_k^{+(-)}; \quad M_{q_1^+} = q_1^+ * h_1; \quad M_{q_1^-} = q_1^- * h_2; \quad k = 1, 3.$$

Substituting (5), (7) -formulas into (4), (2), (3) -formulas with (5), (6) formulas and putting them in formula (1) we derive the equilibrium equation.

U, W – An equilibrium equation is constructed based on the initial and boundary conditions to determine Thus the system of nonlinear differential equations of the equation of motion of the beam and the boundary and initial conditions of the natural state arise:

$$\left. \begin{aligned} &-F(x) \frac{\partial^2 W}{\partial t^2} - \alpha_1 S(x) \frac{\partial^3 U}{\partial x \partial t^2} + \alpha_2 J(x) \frac{\partial^4 W}{\partial x^2 \partial t^2} + \\ &+ \frac{\partial^2}{\partial x^2} \left[\left(\alpha_3 + \sigma_1 \alpha_4 \frac{\partial U}{\partial x} \right) M(\sigma_{xx}) - \sigma_1 \alpha_5 \frac{\partial^2 W}{\partial x^2} B(\sigma_{xx}) \right] + \\ &+ \alpha_3 \frac{\partial}{\partial x} \left[\frac{\partial M}{\partial x} P(\sigma_{xx}) \right] + F_1(x, t) = 0 \\ &-F(x) \frac{\partial^2 U}{\partial t^2} + \alpha_1 S(x) \frac{\partial^3 W}{\partial x \partial t^2} + \frac{\partial}{\partial x} \left[\left(\alpha_7 + \sigma_1 \alpha_3 \frac{\partial U}{\partial x} \right) P(\sigma_{xx}) - \sigma_1 \alpha_4 \frac{\partial^2 W}{\partial x^2} M(\sigma_{xx}) \right] + F_2(x, t) = 0 \end{aligned} \right\} \quad (8)$$

$x, t \in Q_T$

$$\left. \begin{aligned} & \left\{ -\alpha_1 S(x) \frac{\partial^2 u}{\partial t^2} + \alpha_2 J(x) \frac{\partial^3 W}{\partial x \partial t^2} + \alpha_3 \frac{\partial W}{\partial x} P(\sigma_{xx}) + \right. \\ & \left. + \frac{\partial}{\partial x} \left[\left(\alpha_3 + \sigma_1 \alpha_4 \frac{\partial u}{\partial x} \right) M(\sigma_{xx}) - \sigma_1 \alpha_3 \frac{\partial^2 W}{\partial x^2} B(\sigma_{xx}) \right] + F_3(x, t) \right\} \delta(W) \Big|_x = 0 \\ & \left\{ \left(\alpha_7 + \sigma_1 \alpha_3 \frac{\partial u}{\partial x} \right) P(\sigma_{xx}) - \sigma_1 \alpha_4 \frac{\partial^2 W}{\partial x^2} M(\sigma_{xx}) - \alpha_8 P_{\bar{N}_1} \right\} \delta(u) \Big|_x = 0 \\ & \left\{ \left(\alpha_3 + \sigma_1 \alpha_4 \frac{\partial u}{\partial x} \right) M(\sigma_{xx}) - \sigma_1 \alpha_3 \frac{\partial^2 W}{\partial x^2} B(\sigma_{xx}) - \alpha_3 M_{\bar{N}_1} \right\} \delta \left(\frac{\partial W}{\partial x} \right) \Big|_x = 0 \\ & t > 0 \end{aligned} \right\}; (9)$$

$$\left. \begin{aligned} & \left\{ F(x) \frac{\partial W}{\partial t} + \alpha_1 S(x) \frac{\partial^2 u}{\partial x \partial t} - \alpha_2 J(x) \frac{\partial^3 W}{\partial x^2 \partial t} \right\} \delta W \Big|_t = 0 \\ & \left\{ F(x) \frac{\partial u}{\partial t} - \alpha_1 S(x) \frac{\partial^2 W}{\partial x \partial t} \right\} \delta u \Big|_t = 0 \quad x \in (0, 1) \\ & \left\{ -\alpha_1 S(x) \frac{\partial u}{\partial t} + \alpha_2 J(x) \frac{\partial^2 W}{\partial x \partial t} \right\} \delta W \Big|_{x_t} = 0 \end{aligned} \right\}; (10)$$

Here $Q_T = \{0 < x < 1, 0 < t \leq T\}$;

$$\bar{F}(x) = F_0 F(x), \quad \bar{S}(x) = S_0 S(x), \quad \bar{J}(x) = J_0 J(x),$$

$$\bar{S1}(x) = S_{10} S1(x), \quad \bar{B1}(x) = B_{10} B1(x).$$

$$\text{Here } \bar{F}(x) = \int_y \int_z dz dy, \quad \bar{S}(x) = \int_y \int_z z dz dy, \quad \bar{J}(x) = \int_y \int_z z^2 dz dy,$$

$$\bar{S1}(x) = \int_y \int_z z^3 dz dy, \quad \bar{B1}(x) = \int_y \int_z z^4 dz dy, \quad \bar{F}_0(x) = \max_x |\bar{F}(x)| \dots;$$

$$P(\sigma_{xx}) = \int_y \int_z \sigma_{xx} dz dy = F(x) \left[\left(\beta_1 + \sigma_1 \beta_2 \frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial x} + \beta_2 \left(\frac{\partial W}{\partial x} \right)^2 \right] - \left[S(x) \left(\beta_3 + \sigma_1 \beta_4 \frac{\partial u}{\partial x} \right) - \frac{1}{2} \sigma_1 J(x) \frac{\partial^2 W}{\partial x^2} \right] \frac{\partial^2 W}{\partial x^2};$$

$$\begin{aligned} M(\sigma_{xx}) &= \int_y \int_z z \sigma_{xx} dz dy = S(x) \left[\left(\beta_5 + \sigma_1 \beta_6 \frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial x} + \beta_6 \left(\frac{\partial W}{\partial x} \right)^2 \right] - \\ &- \left[J(x) \left(\beta_7 + \sigma_1 \beta_8 \frac{\partial u}{\partial x} \right) - \sigma_1 \beta_9 S_1(x) \frac{\partial^2 W}{\partial x^2} \right] \frac{\partial^2 W}{\partial x^2}; \end{aligned}$$

$$\begin{aligned} B(\sigma_{xx}) &= \int_y \int_z z^2 \sigma_{xx} dz dy = J(x) \left[\left(\beta_{10} + \frac{1}{2} \sigma_1 \beta_7 \frac{\partial u}{\partial x} \right) \frac{\partial u}{\partial x} + \frac{1}{2} \beta_7 \left(\frac{\partial W}{\partial x} \right)^2 \right] - \\ &- \left[S_1(x) \left(\beta_{11} + \sigma_1 \beta_{12} \frac{\partial u}{\partial x} \right) - \sigma_1 \beta_{13} B_1(x) \frac{\partial^2 W}{\partial x^2} \right] \frac{\partial^2 W}{\partial x^2}; \end{aligned}$$

$$F_1(x, t) = \alpha_6 \left(\frac{\partial M_{q_1^-}}{\partial x} + Q_3 + \frac{\partial M_{q_1^+}}{\partial x} - \frac{\partial M_{q_1^-}}{\partial x} + q_3^+ + q_3^- \right);$$

$$F_2(x, t) = \alpha_6 (Q_1 + q_1^+ + q_1^-);$$

$$F_3(x, t) = \alpha_8 (P_{\bar{N}_3} - M_{\bar{Q}_1} - M_{q_1^+} + M_{q_1^-});$$

$$\alpha_1 = \frac{S_0}{F_0 l}, \quad \alpha_2 = \frac{J_0}{F_0 l^2}, \quad \alpha_3 = \frac{1}{l^2}, \quad \alpha_4 = \frac{1}{l^3}, \quad \alpha_5 = \frac{1}{l^4}, \quad \alpha_6 = \frac{l^4}{E J_0}, \quad \alpha_7 = \frac{1}{l};$$

$$\alpha_8 = \alpha_6 / l, \quad \alpha_9 = \alpha_6 / l^2;$$

$$\beta_1 = \frac{F_0}{J_0} l^3, \quad \beta_2 = \frac{1}{2} \frac{F_0}{J_0} l^2, \quad \beta_3 = \frac{S_0}{J_0} l^2, \quad \beta_4 = \frac{S_0}{J_0} l, \quad \beta_5 = \frac{S_0}{J_0} l^3, \quad \beta_6 = \frac{1}{2} \frac{S_0}{J_0} l^2, \quad \beta_7 = l^2,$$

$$\beta_8 = l, \quad \beta_9 = \frac{1}{2} \frac{S_0}{J_0}; \quad \beta_{10} = l^3, \quad \beta_{11} = \frac{S_{10}}{J_0} l^2, \quad \beta_{12} = \frac{S_{10}}{J_0} l, \quad \beta_{13} = \frac{1}{2} \frac{B_{10}}{J_0};$$

l – the length of the beam

We enter the following operators

$$L_1^{(0)}(\mathcal{U}, W) = \frac{\partial^2}{\partial x^2} [\alpha_3 M^{(0)}(\sigma_{xx})];$$

$$L_1^{(1)}(\mathcal{U}, W) = \frac{\partial^2}{\partial x^2} \left[\alpha_3 M^{(1)}(\sigma_{xx}) + \sigma_1 \left(\alpha_4 \frac{\partial \mathcal{U}}{\partial x} M(\sigma_{xx}) - \alpha_5 \frac{\partial^2 \mathcal{U}}{\partial x^2} B(\sigma_{xx}) \right) \right] + \alpha_3 \frac{\partial}{\partial x} \left[\frac{\partial W}{\partial x} P(\sigma_{xx}) \right];$$

$$L_2^{(0)}(\mathcal{U}, W) = \frac{\partial}{\partial x} [\alpha_7 P^{(0)}(\sigma_{xx})];$$

$$L_2^{(1)}(\mathcal{U}, W) = \frac{\partial}{\partial x} \left[\alpha_7 P^{(1)}(\sigma_{xx}) + \sigma_1 \left(\alpha_3 \frac{\partial \mathcal{U}}{\partial x} P(\sigma_{xx}) - \alpha_4 \frac{\partial^2 \mathcal{U}}{\partial x^2} M(\sigma_{xx}) \right) \right]; \setminus$$

$$l_1^{(0)}(\mathcal{U}, W) = \frac{\partial}{\partial x} [\alpha_3 M^{(0)}(\sigma_{xx})];$$

$$l_1^{(1)}(\mathcal{U}, W) = \frac{\partial}{\partial x} \left[\alpha_3 M^{(1)}(\sigma_{xx}) + \sigma_1 \left(\alpha_4 \frac{\partial \mathcal{U}}{\partial x} M(\sigma_{xx}) - \alpha_5 \frac{\partial^2 \mathcal{U}}{\partial x^2} B(\sigma_{xx}) \right) \right] + \alpha_3 \frac{\partial W}{\partial x} P(\sigma_{xx});$$

$$l_2^{(0)}(\mathcal{U}, W) = \alpha_7 P^{(0)}(\sigma_{xx});$$

$$l_2^{(1)}(\mathcal{U}, W) = \alpha_7 P^{(1)}(\sigma_{xx}) + \sigma_1 \left[\alpha_3 \frac{\partial \mathcal{U}}{\partial x} P(\sigma_{xx}) - \alpha_4 \frac{\partial^2 \mathcal{U}}{\partial x^2} M(\sigma_{xx}) \right];$$

$$l_3^{(0)}(\mathcal{U}, W) = \alpha_3 M^{(0)}(\sigma_{xx});$$

$$l_3^{(1)}(\mathcal{U}, W) = \alpha_3 M^{(1)}(\sigma_{xx}) + \delta_1 \left[\alpha_4 \frac{\partial \mathcal{U}}{\partial x} M(\sigma_{xx}) - \alpha_5 \frac{\partial^2 \mathcal{U}}{\partial x^2} B(\sigma_{xx}) \right];$$

$$P^{(0)}(\sigma_{xx}) = \beta_1 F(x) \frac{\partial \mathcal{U}}{\partial x} - \beta_3 S(x) \frac{\partial^2 W}{\partial x^2};$$

$$P^{(0)}(\sigma_{xx}) + P^{(1)}(\sigma_{xx}) = P(\sigma_{xx});$$

$$M^{(0)}(\sigma_{xx}) = \beta_5 S(x) \frac{\partial u}{\partial x} - \beta_7 J(x) \frac{\partial^2 w}{\partial x^2};$$

$$M^{(0)}(\sigma_{xx}) + M^{(1)}(\sigma_{xx}) = M(\sigma_{xx}),$$

Thus we write questions (8) - (10) as follows:

$$\left. \begin{aligned} \frac{\partial^2}{\partial x^2} \left[F(x)W + \alpha_1 S(x) \frac{\partial u}{\partial x} - \alpha_2 J(x) \frac{\partial^2 w}{\partial x^2} \right] - [L_1^{(0)}(u, W) + L_1^{(1)}(u, W) + F_1(x, t)] &= 0 \\ \frac{\partial^2}{\partial x^2} \left[F(x)u - \alpha_1 S(x) \frac{\partial w}{\partial x} \right] - [L_2^{(0)}(u, W) + L_2^{(1)}(u, W) + F_2(x, t)] &= 0 \end{aligned} \right\}; \quad (11)$$

$$\begin{aligned} \left\{ \frac{\partial^2}{\partial x^2} \left[\alpha_1 S(x)u - \alpha_2 J(x) \frac{\partial w}{\partial x} \right] - [l_1^{(0)}(u, W) + l_1^{(1)}(u, W) + F_3(x, t)] \right\} \delta w \Big|_x &= 0; \\ \left\{ l_2^{(0)}(u, W) + l_2^{(1)}(u, W) - \alpha_8 P_{\bar{N}_1} \right\} \delta u \Big|_x &= 0; \\ \left\{ l_3^{(0)}(u, W) + l_3^{(1)}(u, W) - \alpha_9 P_{\bar{N}_1} \right\} \delta \left(\frac{\partial w}{\partial x} \right) \Big|_x &= 0; \end{aligned}$$

We consider the following specific issue, for example the moving beam. The initial and boundary conditions for this are as follows:

$$u(0, t) = \varphi_1(t), \quad w(0, t) = \varphi_2(t), \quad \frac{\partial w(0, t)}{\partial x} = \varphi_3(t); \quad (12)$$

$$\left. \begin{aligned} \frac{\partial^2}{\partial x^2} \left[\alpha_1 S(x)u - \alpha_2 J(x) \frac{\partial w}{\partial x} \right] - [l_1^{(0)}(u, W) + l_1^{(1)}(u, W) + F_3(x, t)] \Big|_{x=1} &= 0 \\ l_2^{(0)}(u, W) + l_2^{(1)}(u, W) - \alpha_8 P_{\bar{N}_1} \Big|_{x=1} &= 0 \\ l_3^{(0)}(u, W) + l_3^{(1)}(u, W) - \alpha_9 P_{\bar{N}_1} \Big|_{x=1} &= 0 \\ t &> 0 \end{aligned} \right\}; \quad (13)$$

$$u(x, 0) = \bar{u}(x), \quad w(x, 0) = \bar{w}(x), \quad x \in [0, 1]; \quad (14)$$

$$\frac{\partial}{\partial t} \left[F(x)u - \alpha_1 S(x) \frac{\partial w}{\partial x} \right] \Big|_{t=0} = 0; \quad x \in [0, 1]; \quad (15)$$

$$\frac{\partial}{\partial t} \left[F(x)W + \alpha_1 S(x) \frac{\partial u}{\partial x} - \alpha_2 J(x) \frac{\partial^2 w}{\partial x^2} \right] \Big|_{t=0} = 0; \quad x \in (0, 1) \quad (16)$$

$$\frac{\partial}{\partial t} \left[\alpha_1 S(x)u - \alpha_2 J(x) \frac{\partial w}{\partial x} \right] \Big|_{x=1} \Big|_{t=0} = 0, \quad (17)$$

Here $\bar{u}(x)$ i $\bar{w}(x)$ – is the solution to the problem.

$$\left. \begin{aligned} L_1^{(0)}(u, W) + L_1^{(1)}(u, W) + F_1(x, 0) &= 0 \\ L_2^{(0)}(u, W) + L_2^{(1)}(u, W) + F_2(x, 0) &= 0 \\ u(0) = \varphi_1(0); \quad w(0) = \varphi_2(0), \quad \frac{\partial w(0)}{\partial x} &= \varphi_3(0) \\ l_1^{(0)}(u, W) + l_1^{(1)}(u, W) + F_3(x, 0) \Big|_{x=1} &= 0 \\ l_2^{(0)}(u, W) + l_2^{(1)}(u, W) - \alpha_8 P_{\bar{N}_1} \Big|_{x=1} &= 0 \\ l_3^{(0)}(u, W) + l_3^{(1)}(u, W) - \alpha_9 P_{\bar{N}_1} \Big|_{x=1} &= 0 \end{aligned} \right\}; \quad (18)$$

IV. CONCLUSION

From the above formulas it can be seen that if the cross section of the beam is constant and symmetrical, i.e. $S(x)=0$, $\sigma_1=0$ and (11) comes to the nonlinear problem given in formula [3].

Thus, the mathematical model of the general problem of the oscillating beam without a geometric line is described in the form of formulas (11), (18) based on the principle of variation.

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