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Generalized Equation of a Straight Line in the Interval Version

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Abstract – This article deals with the problem of the interval version of a straight line. Here are the definitions of a generalized straight line with an angular coefficient, the equation of a generalized straight line passing through two rectangles formed by the intersection of two pairs of given intervals, and an extended version of the generalized straight line containing n rectangles formed by the intersection of n pairwise given intervals. For each equation, special cases are considered and geometric images are illustrated. Examples are given where the introduced concepts can be applied as a tool for interval analysis, which justifies the consideration of various equations of the line in the interval version.

Keywords – Interval Analysis, Interval Arithmetic, Straight Line Equation, Generalized Straight Line Equation, Extended Generalized Line, Interval Midpoint, Interval Width, Borehole, Borehole, Band Width, Diagonal Length.

I. INTRODUCTION

When considering some technical problems, it is necessary to determine a solid strip with a certain width, where the elements of the technical systems under consideration are located.

You can give the following examples to confirm the above suggestions.

1. In the development of the mountain massif with the use of non-explosive destructive mixture is required to obtain a continuous line of cracks. In this case, a very important role is played by the location of wells along a solid line.

2. When conducting high-voltage lines of electric wires, it is taken into account the fact that in the process of ice formation in the wires, the supporting structures may collapse under the weight of iced wires. In this case, the location of the supports and supporting structures along a straight solid line is also important.

In the considered examples, their main parameters, for example, the diameter and depth of wells $d\epsilon[10;\ 30]$см in the first example, in the second example, the area $S = \pi r^2$or $S = ab$, and the depth of the support, are in a certain interval.

In this regard, these problems can be considered in the context of interval analysis.

In the problems under consideration, the location of wells and supports along a certain straight line located in a strip with a certain width leads us to introduce the definition of a generalized straight line.

First, we present some information from the interval analysis, which will be used in the reasoning of the following data.

Interval analysis and its specific methods are therefore of the highest value in problems where uncertainties and ambiguities arise from the very beginning and are an integral part of the problem statement.

We call an interval a closed segment of the real axis, and interval uncertainty is a state of incomplete (partial) knowledge about the quantity of interest, when only its belonging to a certain interval is known, i.e. when we can specify only the limits of possible values of this quantity (the limits of its change). Accordingly, interval analysis is a branch of mathematical knowledge that studies problems with interval uncertainties and methods for solving them [1]. According to [1, 2], we present the definition of the interval and some of its characteristics, which will be used in the future.

Definition 1: The interval [a, b] of the real axis R is the set of all numbers located between the given numbers a and b, including themselves, i.e. [a, b]:= $\{x \in R \mid a \le x \le b\}$. In this case, a and b are called the ends of the interval [a, b], left and right, respectively, and the set of all intervals is denoted by the symbol IR. In contrast to intervals and interval quantities, we will call point values those quantities whose values are individual points—a real axis, a plane, or, more generally, a space.

Symbolically, the intervals are denoted as

$$\mathbf{a} = [\,\underline{a},\ \overline{a}\,] = \{x \in R \mid \underline{a} \le x \le \overline{a}\}.$$

Next, we'll look at ways to define interval operations that serve problem statements with interval input data. Interval arithmetic is defined by the following fundamental principle:

$$\mathbf{a} \star \mathbf{b} = \{a \star b \mid a \in \mathbf{a}, b \in \mathbf{b}\}$$

For intervals a, b, such that performing a point operation

$$a \star b,\ \star \in \{+, -, \cdot, /\},$$

Makes sense for any $a \in \mathbf{a}$ and $b \in \mathbf{b}$. Thus, the result of the interval analog of the operation is assigned to the range of values of this operation over points from these intervals.

$$\mathbf{a} + \mathbf{b} = [\underline{a} + \underline{b}, \overline{a} + \overline{b}], \quad \mathbf{a} - \mathbf{b} = [\underline{a} - \overline{b}, \overline{a} - \underline{b}],$$

$$\mathbf{a} \cdot \mathbf{b} = [\min\{\underline{a}\,\underline{b}, \underline{a}\,\overline{b}, \overline{a}\underline{b}, \overline{a}\,\overline{b}\}, \max\{\underline{a}\,\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\}],$$

$$\mathbf{a}/\mathbf{b} = [\,\underline{a} \cdot 1/\overline{b}, \overline{a} \cdot 1/\underline{b}] \text{ для } 0 \notin \boldsymbol{b}.$$

Any interval is completely defined by two numbers — their ends, but in practice other characteristics of intervals are often used. The most important of them are the middle (center) of the interval

$$\text{mid}\mathbf{a} = \tfrac{1}{2}(\underline{a} + \overline{a}),$$

and its radius

$$\text{rad}\mathbf{a} = \tfrac{1}{2}(\overline{a} - \underline{a}).$$

Often the equivalent concept of the interval width is considered instead of the radius

$$\text{wid}\mathbf{a} = \overline{a} - \underline{a}.$$

Now we turn to the description of the definition of a generalized straight line and the various actions associated with it.

I. Equation of a generalized straight line with an angular coefficient.

Definition 2. Expression specified by the equation

$$\boldsymbol{y} = k\boldsymbol{x} + \boldsymbol{b}, \tag{1}$$

it is called a generalized straight line if it is defined by the equality

$$\boldsymbol{y} = \begin{cases} [k\underline{x} + \underline{b},\ k\overline{x} + \overline{b}], & \text{if } k \ge 0, \\ [k\overline{x} + \overline{b},\ k\underline{x} + \underline{b}], & \text{if } k < 0, \end{cases} \tag{2}$$

$\boldsymbol{x} = [\underline{x},\ \overline{x}]$, $\boldsymbol{y} = [\underline{y},\ \overline{y}]$, $\boldsymbol{b} = [\underline{b},\ \overline{b}] \epsilon \boldsymbol{IR}$, i.e. intervals, $k \epsilon R$.

As in the real version, we can consider various special cases of equation (1). The following cases are possible:

1) $\boldsymbol{x} = [\underline{x},\ \overline{x}] \ge 0$ Or zero containing the interval;

2) $\boldsymbol{x} = [\underline{x},\ \overline{x}] < 0$.

Given that the coefficient k can also change its sign, we study these variants together with cases 1) and 2).

a) Let $\boldsymbol{x} = [\underline{x},\ \overline{x}] \ge 0$or zero containing the interval and $k \ge 0$.

Then the equation $\boldsymbol{y} = k\boldsymbol{x} + \boldsymbol{b}$ describes the following generalized line:

$$\begin{cases} \underline{y} = k\underline{x} + \underline{b} \\ \overline{y} = k\overline{x} + \overline{b}. \end{cases} \tag{3}$$

The lines defined by equation (3) are constructed as follows. A rectangle is constructed at the intersection of the x and y intervals, i.e. A rectangle with the base $[\underline{x},\ \overline{x}]$ and height $[\underline{y},\ \overline{y}]$. Based on the condition of point a), this rectangle is located to the right of the Oy axis. If we denote the length of the rectangle's diagonal by ρ, it will be equal to

$$\rho = \sqrt{\left((\overline{x} + \overline{b}) - (\underline{x} + \underline{b})\right)^2 + (\overline{y} - \underline{y})^2}. \tag{4}$$

Next, we construct straight lines defined by equation (3), perpendicular to the diagonal of the rectangle, which forms an obtuse angle with the axis ox, since k>0 (Fig.1). As a result, we get an inclined strip bounded below and above by the equations given by equality (3), respectively.

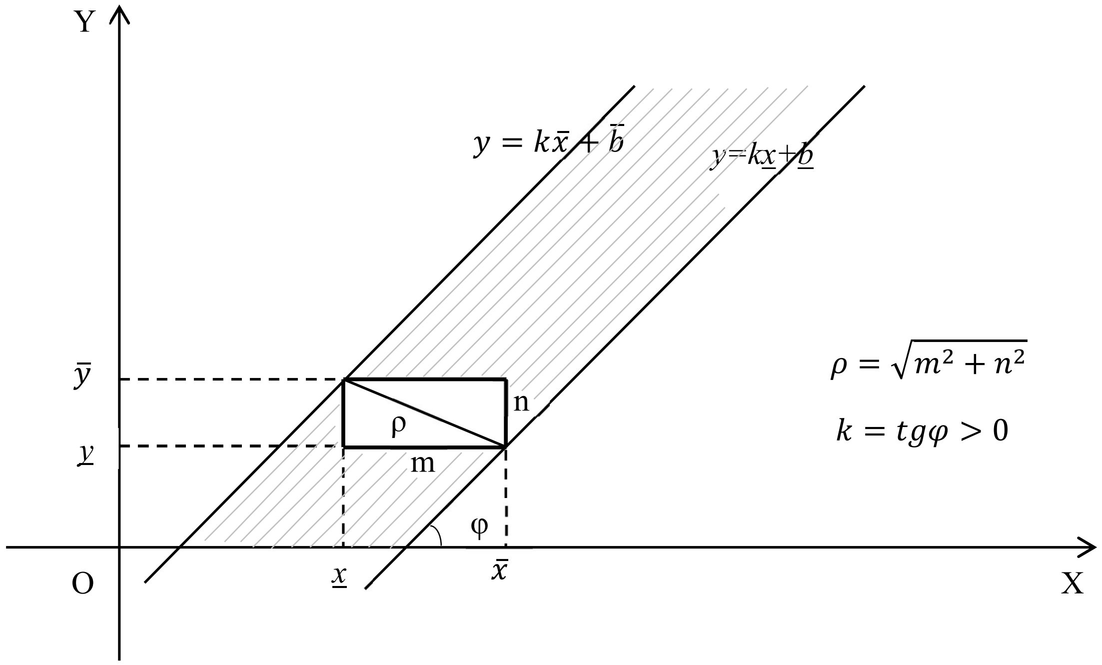

Figure 1

b) Let $\boldsymbol{x} = [\underline{x}, \overline{x}] \geq 0$ and $k < 0$. The difference between the construction of this line and the case a) is that they are built perpendicular to the diagonal of the rectangle, which forms an acute angle with the axis ox, since k<0 (Fig.2). As a result, we get an inclined strip bounded below and above by the equations given by (3).

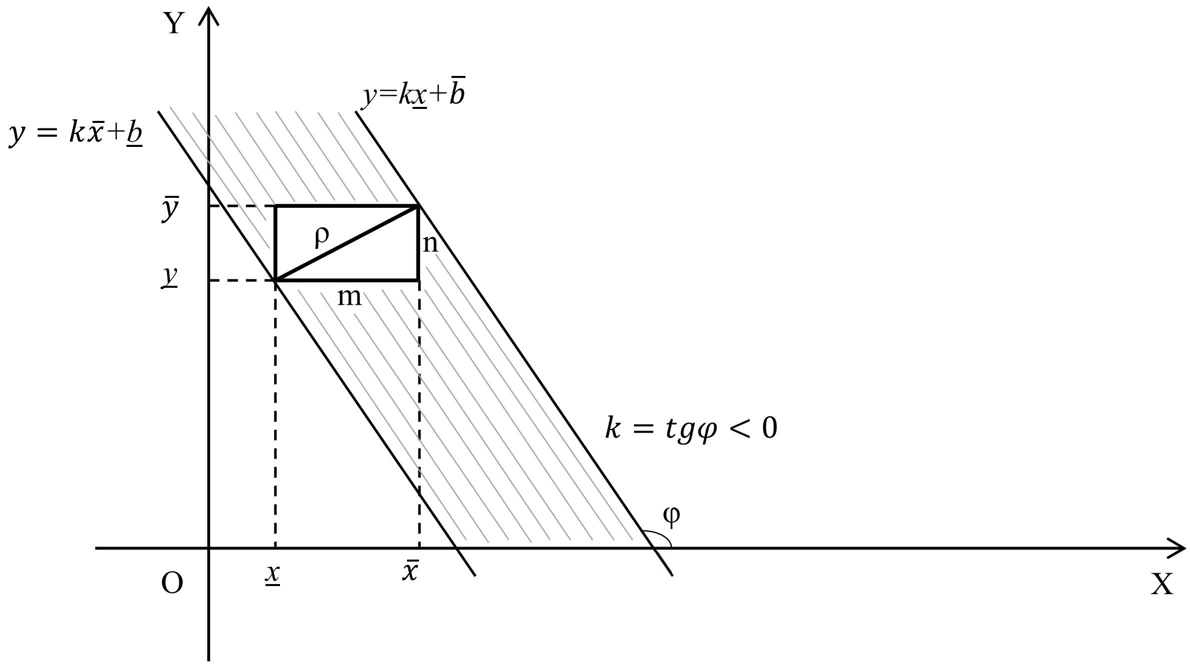

Figure 2

In the case of $\boldsymbol{x} = [\underline{x},\ \overline{x}] < 0$, the construction of a generalized line is discussed in the same way as in the case of 1), with the difference that the equation

$$\boldsymbol{y} = k\boldsymbol{x} + \boldsymbol{b}$$

Describes the following generalized line:

$$\begin{cases} \underline{y} = k\overline{x} + \underline{b} \\ \overline{y} = k\underline{x} + \overline{b}. \end{cases} \qquad (5)$$

In both cases, adding the corresponding boundaries of the interval $\boldsymbol{b} = [\underline{b},\ \overline{b}]$, to the right-hand sides of equations (3) and (5) means shifting the boundaries of the interval $\boldsymbol{y} = [\underline{y},\ \overline{y}]$ along the Oy axis by $\underline{b}$ и $\overline{b}$ values, respectively.

If k=0, then the generalized line given by the equation

$$\boldsymbol{y} = \boldsymbol{b} = [\underline{b}, \overline{b}]$$

Describes a horizontal bar with a width equal to

$$\omega id(\boldsymbol{b}) = \overline{b} - \underline{b}\,.$$

4) if the interval $\boldsymbol{y} = [\underline{y},\ \overline{y}]$ is a degenerate interval, i.e. $y = \underline{y} = \overline{y}$, then the generalized line is given by the equation

$$\boldsymbol{x} = \frac{1}{k}(y - \boldsymbol{b}) = \frac{1}{k}[y - \underline{b}, y - \overline{b}]$$

and it describes a vertical bar.

Now, using the characteristics of the interval, we consider a geometric image, i.e., a method for constructing a generalized line given by equation (1). Based on the given intervals $\boldsymbol{x} = [\underline{x},\ \overline{x}]$ and $\boldsymbol{y} = [\underline{y},\ \overline{y}]$, considering them as a rectangle with sides $m = \omega id(\boldsymbol{x})$ and $n = \omega id(\boldsymbol{y})$, we construct a rectangle with a base equal to m and a height equal to n. Let's determine the length of the diagonal of this rectangle. Next, draw parallel lines perpendicular to the diagonal that forms an obtuse angle with the Ox axis if k>0 (Fig. 1) and to the diagonal that forms an acute angle with the Ox axis if k<0 (Fig.2). As a result, we get a band that is the desired generalized straight line.

a) If k=0, we get a generalized straight line, given by the equation y=b, parallel to the ox axis, the width of which is equal to $n = \omega id(\boldsymbol{y})$, i.e. it is a strip with a width equal to $\rho = n$ (horizontal strip).

b) If y turns out to be a degenerate interval, we get a generalized line given by the equation $\boldsymbol{x} = -\frac{1}{k}\boldsymbol{b} + y$, parallel to the axis Oy, whose width is equal to $m = \omega id(\boldsymbol{x})$, i.e., $\rho = m$ (vertical band).

I. The equation of a generalized straight line passing through two rectangles formed by the intersection of two pairs of given intervals.

The real version of the equation of a straight line passing through the data of two points [3] can be extended in the interval version.

Real version:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1},$$

$M_1(x_1;\ y_1)$ и $M_2(x_2;\ y_2)$ given point.

The interval option: Let two intervals be given: $\boldsymbol{x}_1$ и $\boldsymbol{x}_2$.

Then these intervals correspond to two intervals

$\boldsymbol{y}_1 = k_1\boldsymbol{x}_1 + \boldsymbol{b}_1$ and $\boldsymbol{y}_2 = k_2\boldsymbol{x}_2 + \boldsymbol{b}_2$,

in accordance with the introduced equation of the generalized line (1) or (2).

As the point x_1 we take the middle of the interval $\boldsymbol{x}_1$, i.e. $x_1 = \mathrm{mid}(\boldsymbol{x}_1)$, and as the point x_2 we take the middle of the interval $\boldsymbol{x}_2$, i.e. $x_2 = \mathrm{mid}(\boldsymbol{x}_2)$. Similarly, we define the corresponding points for the intervals $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$.

Now we introduce the definition of a generalized line passing through rectangles formed by the intersection of two pairs of given intervals.

Definition 3: A Generalized straight line passing through rectangles formed by the intersection of two pairs of given intervals is an expression defined by the equality

$$\frac{\boldsymbol{x} - \mathrm{mid}(\boldsymbol{x}_1)}{\mathrm{mid}(\boldsymbol{x}_2) - \mathrm{mid}(\boldsymbol{x}_1)} = \frac{\boldsymbol{y} - \mathrm{mid}(\boldsymbol{y}_1)}{\mathrm{mid}(\boldsymbol{y}_2) - \mathrm{mi}\ \ (\boldsymbol{y}_1)}, \qquad (5)$$

$\mathrm{mid}(\boldsymbol{x}_i)$ and $\mathrm{mid}(\boldsymbol{y}_i)$ $(i = 1,2)$, the midpoint of the $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ intervals, respectively.

Since the midpoint of the interval, according to [2], is a real number, equation (5) is easily reduced to equation (1). Indeed, by performing an elementary transformation in equation (5) we obtain

$$\boldsymbol{y} - \mathrm{mid}(\boldsymbol{y}_1) = \frac{\mathrm{mid}(\boldsymbol{y}_2) - \mathrm{mid}(\boldsymbol{y}_1)}{\mathrm{mid}(\boldsymbol{x}_2) - \mathrm{mid}(\boldsymbol{x}_1)}(\boldsymbol{x} - \mathrm{mid}(\boldsymbol{x}_1)).$$

Then, opening the bracket, we have

$$\boldsymbol{y} - \mathrm{mid}(\boldsymbol{y}_1) = \frac{\mathrm{mid}(\boldsymbol{y}_2) - \mathrm{mid}(\boldsymbol{y}_1)}{\mathrm{mid}(\boldsymbol{x}_2) - \mathrm{mid}(\boldsymbol{x}_1)}\boldsymbol{x} - \frac{\mathrm{mid}(\boldsymbol{y}_2) - \mathrm{mid}(\boldsymbol{y}_1)}{\mathrm{mid}(\boldsymbol{x}_2) - \mathrm{mid}(\boldsymbol{x}_1)}\mathrm{mid}(\boldsymbol{x}_1).$$

Enter the following notation:

$$k = \frac{\mathrm{mid}(\boldsymbol{y}_2) - \mathrm{m}\quad(\boldsymbol{y}_1)}{\mathrm{mid}(\boldsymbol{x}_2) - \mathrm{mid}(\boldsymbol{x}_1)},$$

$$b = -kmid(\boldsymbol{x}_1) + mid(\boldsymbol{y}_1).$$

Substituting these expressions in the last equality, we get

$$\boldsymbol{y} = k\boldsymbol{x} + b, \qquad (6)$$

where k and b are real numbers.

Equation (5) describes a generalized straight line (band) that contains rectangles formed by the intersection of two pairs of intervals $\boldsymbol{x}_1$ and $\boldsymbol{y}_1$, as well as $\boldsymbol{x}_2$ and $\boldsymbol{y}_2$. To construct a generalized straight line given by equation (5), we proceed as follows. Let's build two rectangles at the intersection of the intervals ($\boldsymbol{x}_1$,$\boldsymbol{y}_1$), and ($\boldsymbol{x}_2, \boldsymbol{y}_2$). Then, depending on the relative position of the intervals, we draw diagonals to the rectangles. We construct diagonals that form an obtuse angle with the ox axis if $\boldsymbol{x}_1 > 0$, $\boldsymbol{x}_2 > 0$ and $\boldsymbol{x}_2 > \boldsymbol{x}_1$, i.e. the interval $\boldsymbol{x}_2$ is located to the right of the interval $\boldsymbol{x}_1 > 0$, $\boldsymbol{x}_2 > 0$ and $\boldsymbol{x}_2 < \boldsymbol{x}_1$, i.e. the$\boldsymbol{x}_2$interval is located to the left of the $\boldsymbol{x}_1$, interval. Let's construct diagonals that form an acute angle with the Ox axis. Next, we draw two parallel lines from the ends of the diagonals. This will be the desired generalized line containing two rectangles formed by the intersection of the intervals ($\boldsymbol{x}_1$,$\boldsymbol{y}_1$), and ($\boldsymbol{x}_2, \boldsymbol{y}_2$) (Fig.3).

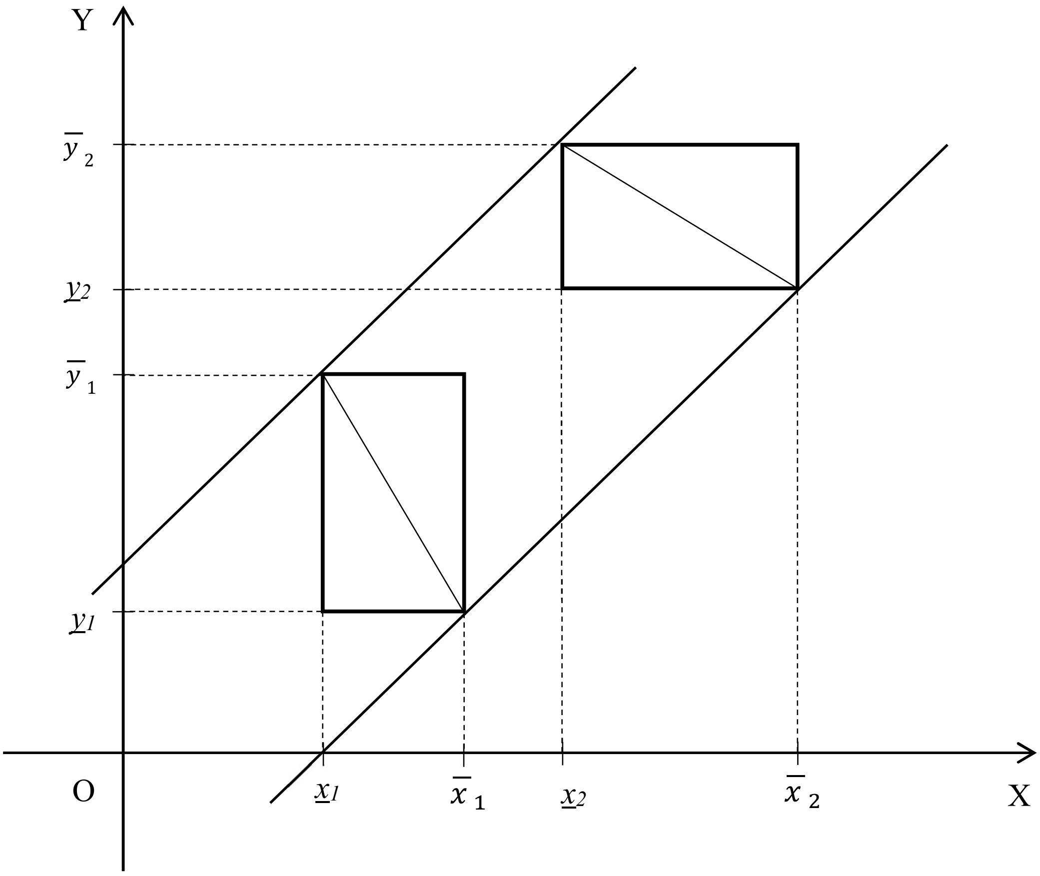

Figure 3

I. an Extended version of a generalized straight-line containing n rectangles formed by the intersection of n pairwise given intervals

Let us solve the following problem: how to determine the width of a generalized straight line containing n rectangles formed by the intersection of n pairwise given intervals?

To solve this problem, we need to introduce the concept of a natural extension of a generalized straight line.

Definition 4: A natural extension of a generalized line is an expression defined by the equation

$$Y = kX + b, \qquad (7)$$

$\boldsymbol{Y}, \boldsymbol{X}$ и $\boldsymbol{b} \in IR$, $k \in R$.

In equality (7), the interval X defines the set of intervals $\boldsymbol{x}_1 = [x_1, \overline{x_1}], \boldsymbol{x}_2 = [x_2, \overline{x_2}], \dots, \boldsymbol{x}_n = [x_n, \overline{x_n}]$, a the y interval defines a set of intervals $\boldsymbol{y}_1 = [y_1, \overline{y}], \boldsymbol{y}_2 = [y_2, \overline{y_2}], \dots, \boldsymbol{y}_n = [y_n, \overline{y_n}]$.

Let's answer the question, how is the width of a natural extended generalized line determined?

Width of a natural extended generalized line is defined as follows. Find the largest width of the intervals $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$, $(i = \overline{1, n})$.

After max max $\omega id(\boldsymbol{x}_i)$ and max $\omega id(\boldsymbol{y}_i)$, $(i = \overline{1, n})$, we take the first of them as the base, and the second as the height of the rectangle obtained as a result of the intersection of the intervals $\boldsymbol{x}_i$and $\boldsymbol{y}_i$. On the largest base max $\omega id(\boldsymbol{x}_i)$ completing the rectangle with the highest heightmax $\omega id(\boldsymbol{y}_i)$, and at the highest heightmax $\omega id(\boldsymbol{y}_i)$, let's complete the rectangle with the largest base max $\omega id(\boldsymbol{x}_i)$. As a result, we get two rectangles with equal widths and heights, i.e. equal areas. Then, along the vertices of the rectangle whose diagonals form an obtuse angle with the Ox axis, if k>0, and to the diagonal, which form an acute angle with the Ox axis, if k<0, we draw two parallel lines perpendicular to the diagonals of both rectangles.

The desired width of the natural extended generalized line, denoted by ρ,will be equal to the length of the diagonal of this rectangle, i.e.

$$\rho = \sqrt{(\max \omega id(\boldsymbol{x}_i))^2 + (\max \omega id(\boldsymbol{y}_i))^2}.$$

Then the resulting natural extended generalized straight line contains all n rectangles formed by the intersection of the given n intervals, as the band with the largest width.

Now let's start discussing the geometric representation of a natural extended generalized line containing the given n intervals and plotting its graph, i.e., plotting a band with the width specified by the equation

$$\boldsymbol{Y} = k\boldsymbol{X} + \boldsymbol{b}, \qquad (\boldsymbol{Y}, \boldsymbol{X}\ \boldsymbol{b} \in \boldsymbol{IR}, k \in R),$$

где $\boldsymbol{X} \ni \{\boldsymbol{x}_1, \ \boldsymbol{x}_2, \dots, \ \boldsymbol{x}_n\}$, $\boldsymbol{Y} \ni \{\boldsymbol{y}_1, \ \boldsymbol{y}_2, \dots, \ \boldsymbol{y}_n\}$.

To construct a natural extended generalized line containing the given n intervals, we proceed as follows. Let's define the largest of the interval widths $\boldsymbol{x_i}$ and $\boldsymbol{y_i}$ $(i = \overline{1, n})$. Let's build two rectangles based on them. These rectangles, with equal sides respectively $m = \max\limits_i(\omega id(\boldsymbol{x}_i))$ and $n = \max\limits_i(\omega id(\boldsymbol{y}_i))$ there will be rectangles with the largest area equal to S= mn. The diagonal of this rectangle is equal to $d = \sqrt{m^2 + n^2}$, we take as the width p of the strip that you want to build. In this case, we get a natural extended generalized line with the largest width, which contains all the rectangles with sides $m_i = \omega id(\boldsymbol{x}_i)$ and $n_i = \omega id(\boldsymbol{y}_i)$, if the following conditions are met: the distance between the midpoints of the intervals $\boldsymbol{x_i}$ and mid-intervals $\boldsymbol{y_i}$ must be constant and equal, i.e.

$$mid(\boldsymbol{x}_2) - mid(\boldsymbol{x}_1) = mid(\boldsymbol{x}_3) - mid(\boldsymbol{x_2}) = \cdots = mid(\boldsymbol{x}_n) - mid(\boldsymbol{x}_{n-1}) = a, \qquad (8)$$

$$mid(\boldsymbol{y}_2) - mid(\boldsymbol{y}_1) = mid(\boldsymbol{y}_3) - mid(\boldsymbol{y_2}) = \cdots = mid(\boldsymbol{y_n}) - mid(\boldsymbol{y}_{n-1}) = \mathrm{b}, \qquad (9)$$

If conditions (8) and (9) are met, the distance between the intersection points of the diagonals of the rectangles obtained as a result of $\boldsymbol{x_i} \cap \boldsymbol{y}_i (i = \overline{1, n})$ they will be equal (Fig. 4). If we denote this distance by l, that $l = \sqrt{a^2 + b^2}$.

If conditions (8) and (9) are not met, i.e., the equality of distances between points that determine the midpoint of the intervals along the Ox and Oy axes, then the generalized line does not contain all the rectangles obtained as a result of intersections of the intervals $\boldsymbol{x_i} \cap \boldsymbol{y}_i$ $(i = \overline{1, n})$. To demonstrate the above, figure 5 shows a generalized straight line that does not contain all three rectangles. The image shows that the distances between the centers of the rectangles are unequal, since the distance between the midpoints of the intervals is different, i.e. $a_1 \neq a_2$ and $b_1 \neq b_2$.

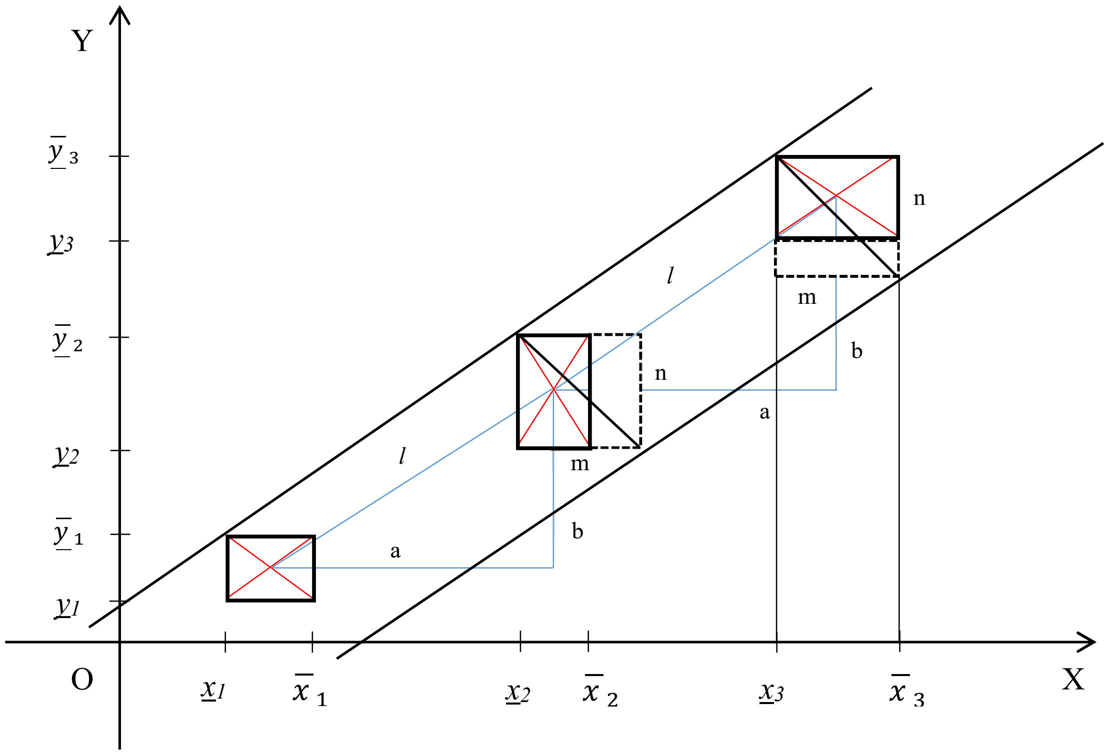

Figure 4

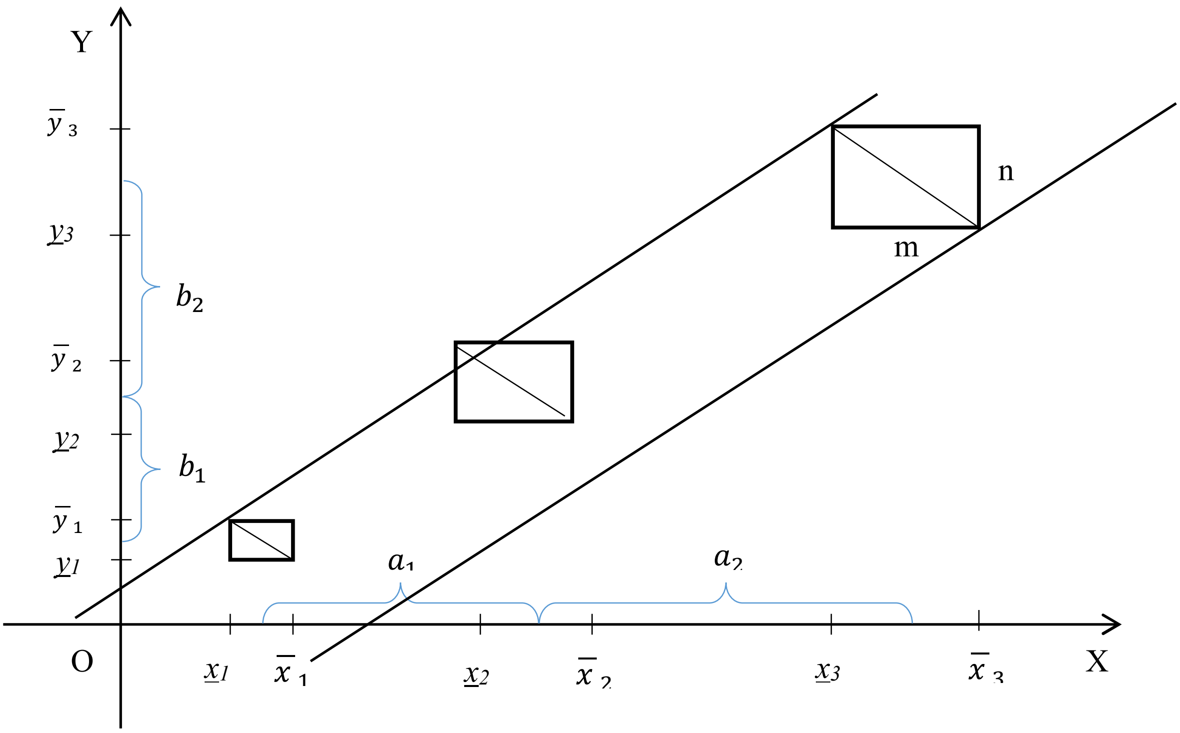

Figure 5

II. CONCLUSION

Thus, we come to the following conclusion. This article provides definitions of a generalized straight line with an angular coefficient, an equation of a generalized straight line passing through two rectangles formed by the intersection of two pairs of given intervals, and an extended version of a generalized straight line containing n rectangles formed by the intersection of n pairwise given intervals. Each equation is analyzed in detail, as well as considered in special cases and illustrated with geometric images.

REFERENCES

[1] S. p. Shary, finite-Dimensional interval analysis. XYZ publishing house, Novosibirsk-2019, 630p.

[2] S. A. Kalmykov, Yu. I. Shokin, Z. H. Yuldashev, methods of interval analysis. Nauka publishing house, Novosibirsk-1986, 220p.

[3] D. T. Written synopsis of the lectures on higher mathematics. Izdatelstvo iris-Press, Moscow, 2008, 608p.

[4] Maxsudov, D. (2019). THE DEVELOPMENT OF TRANSMITTED AND RATIONAL TAFSIRS. *The Light of Islam*, *2019*(1), 9.

[5] Махсудов, Д. (2020). Развитие исламоведения (корановедения) в Узбекистане: история и современность. *Вестник КазНУ. Серия религиоведение*, *21*(1), 60-67.