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Analysis the Path of the Light in the Optical Waveguide with Quadratic Graded Index by Using Simple Approximation Method

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Abstract – Optical waveguides are photonic circuit elements where the light is guided inside. These photonic elements require a transparent medium that the refractive index of the layer is higher than that of the surrounding material. Optical waveguides usually consist of two parts. The inside part is called as core has high refractive index The second part is called as cladding with lower refractive index surrounded the core. Guiding property of optical waveguides are based on the principle of total reflection of light from planar dielectric interfaces. This property of optical waveguides is defined by some structure parameters. Since the refractive index of optical waveguides decreases from the core center to cladding gradually, the refractive index changing of optical waveguides with graded index should be taken into consideration. In this study, behavior of light is analyzed by the parameter changing of optical waveguides. Eikonal equation is used to determine the trajectory of the light. Determination of the trajectory of the light is useful for the explanation of guiding in these waveguides.

Keywords – Optical waveguides, refractive index, light trajectory, approximation technique, Eikonal equation.

I. INTRODUCTION

The optical signals are effectively used as carriers in communications since the innovation of the laser in 1960 [1]. The laser is suitable light source due to properties such as directionality, monochromaticity, brightness [2]. The light source problem has been eliminated by innovation of the laser. The convenient medium is necessary for the optical signals to propagate efficiently. This convenient medium is called optical waveguide.

Optical waveguide is the main element of modern integrated photonic circuits. The knowledge of optical propagation properties of optical waveguides is very important for engineering design of photonic devices. Therefore a considerable amount of work has been reported on the study of characteristics of optical waveguides used in integrated photonics [3], [4], [5], [6], [7], [8], [9], [10], [11]. This element has attracted much attention recently and is being extensively investigated because of potential applications in integrated photonics.

The investigation of electromagnetic fields in optical waveguide is fundamental to the analysis of integrated optical devices. Analysis is an important step in designing the optical waveguide. Also, the electromagnetic field propagation is an important requirement for optical waveguide. Many methods have been developed for the electromagnetic field propagation of these optical waveguides [12], [13], [14], [15]. The property of optical waveguide is defined by some structure parameters. The refractive index profile of an optical waveguide is one of these structure parameters.

In addition, various methods proposed to determine the refractive index profile of optical waveguides to predict the waveguide characteristics and to control waveguide production process. However, practical applications for integrated photonics are required to understand the propagation of light in optical waveguides having various geometries. Various analytical and numerical techniques have so far been used for the investigation of optical waveguides. This paper is aimed to give an analysis on optical waveguide, starting from modeling of the refractive index in optical waveguide.

II. Theory And Method

An optical waveguide is a dielectric structure which transports energy at optical wavelengths portion of the electromagnetic spectrum. Many types of optical waveguides can be used in an integrated photonics. One of these waveguide structures is a slab waveguide shown in Figure 1.

Slab waveguide structure has a core region with refractive index n_1 and second layer with the refractive index n_2. The refractive index n_3 of left side layer of the core equals n_2. Thus, the waveguide structure is generated such that the core region with index n_1 is surrounded on both sides by two dielectric materials provided that $n_1 > n_2 = n_3$. This optical waveguide is named symmetric waveguide. Optical waveguide is named asymmetric waveguide if the second layer and third layer is different that is $n_2 \neq n_3$. In this case, the outer layers can be called cladding. In order for the structure to guiding the light, the refractive index of the core must be bigger than the refractive indexes of the surrounding media.

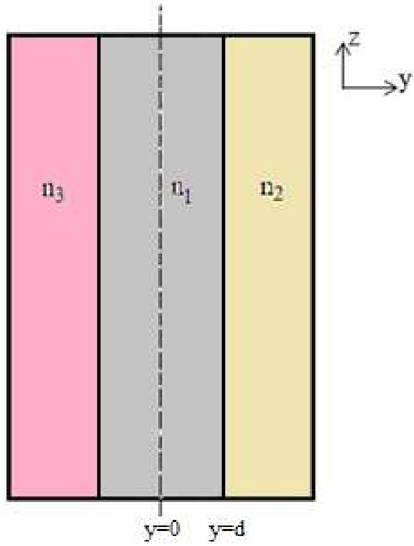

Fig. 1. A slab waveguide structure.

Optical slabs are simplified optical waveguides. The investigation of properties of optical slab waveguides is very useful in understanding of guiding properties of more complex optical waveguides. Refractive index is an important structure parameter in analysis of the electromagnetic field propagation inside slab optical waveguides.

The light is treated as particles and trajectory of these particles follows along paths that called rays in geometrical optics. We will determine the trajectory of light with a simple method using geometric or ray optics in this slab waveguide structure. Actually geometrical optics is a special case of wave or physical optics.

In optical slab waveguides with the graded refractive index profile, the trajectory of a ray is determined by the ray path, or Eikonal equation, in terms of the profile n(y). This equation can be regarded as a generalization of Snell's law, and can be obtained by various methods [16], [17], [18]. The graded index profile can be viewed as the limit of many thin uniform layers. So, we can be modeled the refractive index gradient as a series of thin homogeneous layers, each with an index n(y). Where parameter y is the distance from the axis (y=0) of the core layer.

On a slab waveguide, the refractive-index profile n(y) depends only on y, so that each position (y, z) on the ray path is determined by in the y- and z-directions. Snell's law can be defined in terms of □ angle at the interface between two layers in slab waveguide shown Figure 2.

$$n(y)\cos\theta=n(y+\Delta y)\cos(\theta+\Delta\theta) \qquad (1)$$

The term $n(y+\Delta y)$ that is one of the products on the right side of Eq.1 can be written by Taylor series expansion of n(y) around y

$$n(y+\Delta y) \cong n(y)+\frac{dn}{dy}\Delta y \qquad (2)$$

if $\Delta\theta$ is very small.

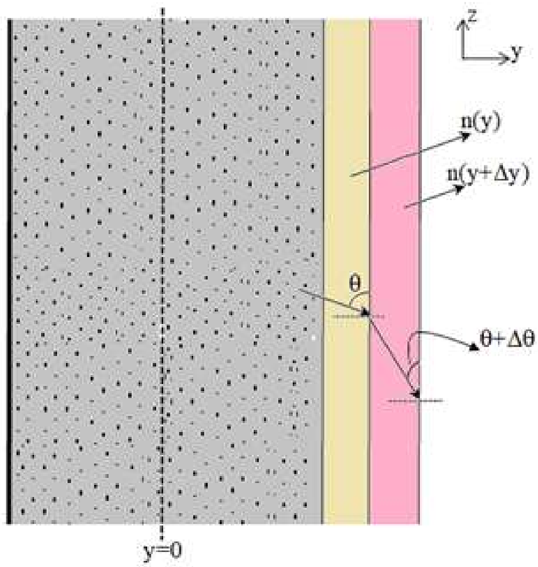

Fig. 2. Modeling of ray refraction in optical slab waveguides with the graded refractive index profile.

The term $\cos(\theta+\Delta\theta)$ that is other one of the products on the right side of Eq.1 can be expanded using the trigonometric identity

$$\cos(\theta+\Delta\theta)=\cos\theta\cos\Delta\theta-\sin\theta\sin\Delta\theta\cong\cos\theta-\sin\theta\Delta\theta \qquad (3)$$

where we have assumed that $\cos\theta$~1 and $\sin\Delta\theta$~$\Delta\theta$ for small θ angles Using these equations we get

$$n(y)\sin\theta\Delta\theta=\frac{dn}{dy}\cos\theta\Delta y-\frac{dn}{dy}\Delta y\sin\theta\Delta\theta \qquad (4)$$

The last term is negligible compared to the other terms, so can be written as

$$\frac{dn}{dy}\cong n(y)\tan\theta\frac{\Delta\theta}{\Delta y} \qquad (5)$$

The angle θ for the most waveguide situations is going to be small (on the order of 10°), so small angle approximations can be used to simplify the expression:

$$\tan\theta\cong\theta\cong\frac{\Delta y}{\Delta z} \qquad (6)$$

Substituting this into Eq.5 so that $\Delta y=\Delta z\tan\theta$ we obtain:

$$\frac{dn}{dy}\cong n(y)\frac{\Delta\theta}{\Delta z} \qquad (7)$$

Finally, we carried out

$$\frac{d^2y}{dz^2}=\frac{1}{n(y)}\frac{dn(y)}{dy} \qquad (8)$$

This equation is called the Eikonal equation. The function y(z) describes the exact ray path, and can be determined once n(y) is known.

III. Modelling And Simulations

In this study, an optical slab waveguide is modeled. The structure, as shown in Figure 1, has three layers: core layer with refractive index of 1,520. Second and third layer have refractive indexes of 1,545. We have chosen this structure as a symmetric slab waveguide. The refractive indexes are equal at the same distance on both sides of the y-axis. The thickness of the core layer is d=10 μm and the cladding layers have 15 μm thicknesses.

The refractive index variation is given by power law relationship for symmetric optical waveguide [19], [20]:

$$n(y) = \begin{cases} n_1\left[1-2\Delta(\frac{y}{d})^g\right]^{1/2} & \text{for } y<|d| \\ n_2 = n_1[1-2\Delta]^{1/2} & \text{for } y>|d| \end{cases} \quad (9)$$

Where parameter g defines the shape of the index profiles. For g=∞, Eq.(9) reduces to step index profile. Parameter Δ is called refractive index difference and given by

$$\Delta= \frac{n_1^2-n_2^2}{2n_1^2} \quad (10)$$

The change of refractive index profile versus y distance is plotted in Figure 3 for step index optical waveguide by means of Eq. (9). The refractive index is constant in the core region. At the interface of the two regions, this value abruptly decreases as shown in Figure 3.

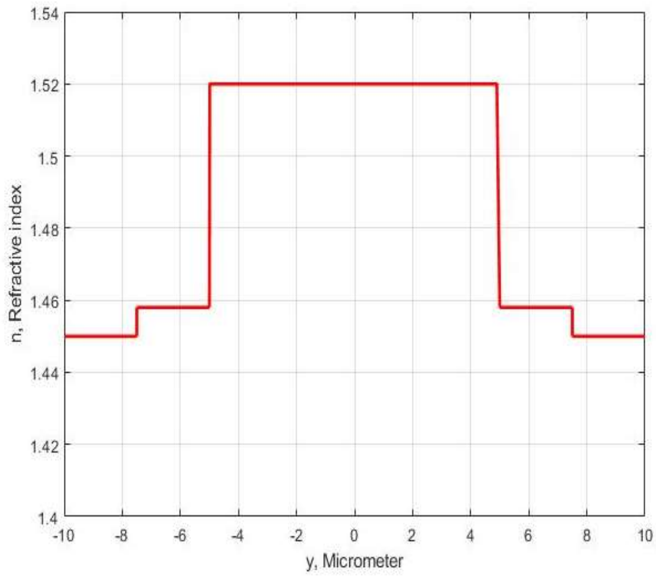

Fig. 3. Change of the refractive index profile versus y distance for step index symmetric optical waveguide.

The change of refractive index profile versus y distance is plotted in Figure 4 for graded index optical waveguide. The refractive index has maximum value at y=0. The refractive index value decreases continuously with distance from y axis in core region. The refractive index has a constant value when the distance from the y-axis reaches the interface of the core-cladding. This refractive index value is constant in the cladding region.

As a practical matter, n(y) in Eq. (8) is often replaced with n_1, the index of refraction at y = 0. Thus, Eikonal equation written as

$$\frac{d^2y}{dz^2} = \frac{1}{n(0)}\frac{dn(y)}{dy} \quad (11)$$

Eikonal equation involves determining the trajectory of a ray path depending on refractive index of graded index slab waveguide. This equation can be interpreted so that the ray always bends toward the higher-index material (toward core material).

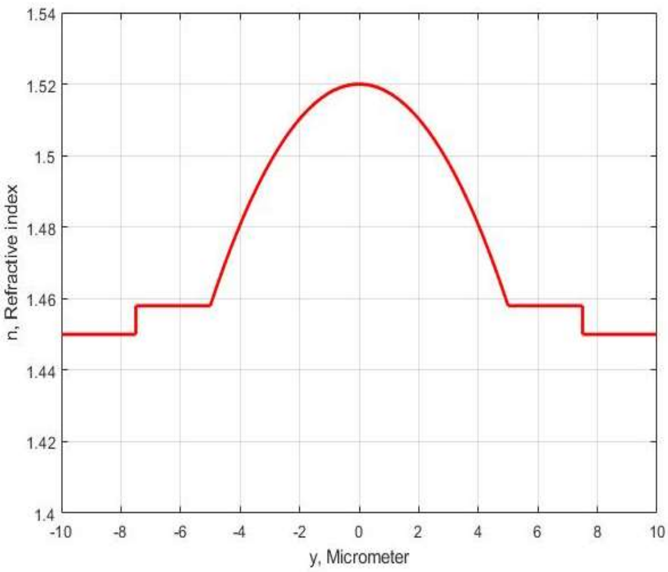

Fig. 4. Change of the refractive index profile versus y distance for graded index symmetric optical waveguide.

Let us examine a slab waveguide that has an index profile in the y direction described by

$$n(y) = n_1 \left[1 - 2\Delta(\frac{y}{d})^g\right]^{1/2} \quad (12)$$

If we take derivative of Eq. (12), we get following relation:

$$\frac{dn(y)}{dy} = \left[-\frac{n_1 g\, \Delta}{d^g}\right][y]^{(g-1)}\left[1 - 2\Delta(\frac{y}{d})^g\right]^{-(\frac{1}{2})} \quad (13)$$

The derivative of refractive index according to y-coordinate is shown in Figure 5 for optical waveguide with quadratic graded index profile i.e. for g=2.

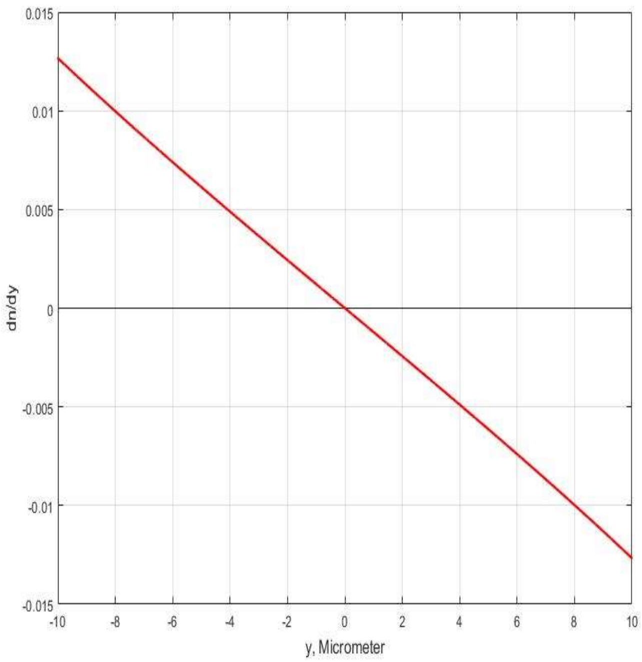

Fig. 5. The derivative of refractive index versus y-coordinate.

The refractive index of the core decreases gradually as it approaches the cladding layer as seen in Figure 5.

Ray path in the core region of the slab optical waveguide with quadratic graded index profile are plotted in Figure 6 from solution of second-order differential equation in Eq.11.

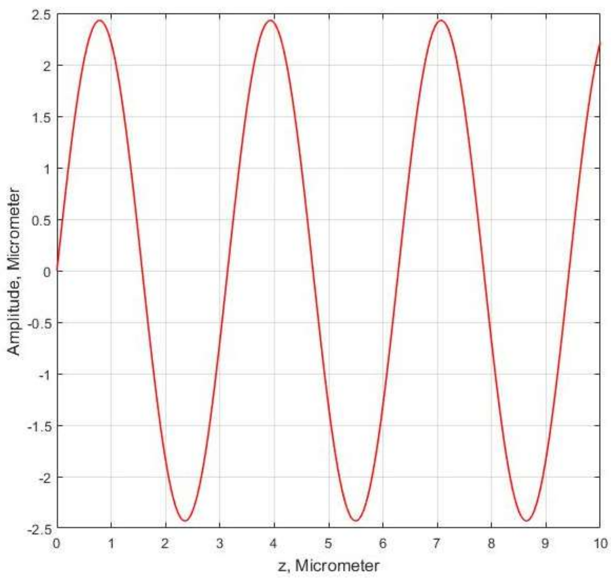

Fig. 6. The Ray path in the medium with quadratic graded index profile.

The trajectory of the ray always bends toward the high index material (toward core material) as seen in Figure 5. The behavior of the ray inside core medium has periodic characteristic as shown in the Figure 6.

IV. CONCLUSION

In this paper we we have introduced the concept of Eikonal equation in the analysis of the optical slab waveguide. The propagating rays in the optical slab waveguide always bend toward the higher refractive index region of the material. When the ray crosses the axis of optical waveguide that has highest refractive index, the curvature of its path changes sign in such a way as to return the ray toward the axis. The reason for this is the graded decreasing of the index of refraction as it moves away from the axis of the waveguide toward the cladding media. The refractive index changing plays a important role optical waveguide design. Thus, optical slab waveguides are useful in integrated photonic circuits.

REFERENCES

[1] Lifante, *Integrated Photonics: Fundamentals,* John Wiley & Sons, West Sussex, 2003.

[2] Peter W. Milonni, Joseph H. Eberly, *Laser Physics,* John Wiley & Sons, 2010.

[3] A. Çetin and Kiliçkaya M. S. "Investigation of the Transverse Electromagnetic Modes in an Embedded Optical Waveguide," *Advanced Studies in Theoretical Physics,* vol. 9, no.15, pp. 723-735, 2015.

[4] A. K. Thander and B. Sucharita, "Study of Optical Waveguide Using HOC Scheme," *Applied Mathematical Sciences, vol. 8, no.* 79, pp. 3931-3938, 2014.

[5] A. K. Thander and B. Sucharita, "Optical Confinement Study of Different Semiconductor Rib Waveguides Using Higher Order Compact Finite Difference Method," *Optik,* vol. 127, no. 4, pp. 2116-2120, 2016.

[6] A. Nerukh, H. Semenova, and N. Sakhnenko. "Alternative Calculations of Initial Value Problem for Electromagnetic Field in Dielectric Waveguide," *Optical and Quantum Electronics, vol.* 40, no. 11-12, pp. 943-956, 2008.

[7] P. Roy Chaudhuri and S. Roy, "Analysis of Arbitrary Index Profile Planar Optical Waveguides and Multilayer Nonlinear Structures: A Simple Finite Difference Algorithm," *Optical and Quantum Electronics*, vol. 39, no. 3, pp. 221-237, 2007.

[8] Y. Ding, Z. Q. Cao and Q. S. Shen. "Determination of Optical Waveguide Refractive-Index Profiles With the Inverse Analytic Transfer Matrix Method," *Optical and Quantum Electronics, vol.* 36, no. 6, pp. 489-497, 2004.

[9] W. Yan, Y. Tang, Q. Ji, X. Yan and H. Zhanget, "The Transmission Characteristics of a Subwavelength Metal Slab Waveguide," *Optik*, vol. 185, pp. 151-155, 2019.

[10] M. Jelínek, "Progress in Optical Waveguiding Thin Films," *Czechoslovak Journal of Physics,* vol. 53, no. 5, pp. 365-377, 2003.

[11] M, Najafi-Hajivar and M. Hosseini-Farzad, "Broadband Polarization-Insensitive Amplitude and Phase Modulators Based on Graphene-Covered Buried and Ridge Silicon Waveguides," *Optics Communications, vol.* 125860, 2020.

[12] C. S. Hsiao, Y. J. Chiang, Likarn Wang and L. Wen-Wei, "A Semi-Analytic Method for Solving Rib-Type Waveguide Problems," *Journal of Modern Optics*, vol. 55, no. 8, pp. 1315-1329, 2008.

[13] D. W. Nkemzi and N. D. Nguti, "A Uniform Approximation for the Eigenmodes of a Planar Dielectric Waveguide," *Optics communications*, vol. 281, no. 9, pp. 2485-2490, 2008.

[14] A. Çetin Ali, E. Uçgun and M. S. Kilickaya, "Determining the Effective Refractive Index of Algaas-GaAs Slab Waveguide Based on Analytical and Finite Difference Method, " *Journal of Physical Science and Application, vol.* 2, no. 9, pp. 381, 2012.

[15] S. P. Mohanty, S. K. Sahoo, A. Panda and G. Palai, "FDTD Method to Photonic Waveguides for Application of Optical Demultiplexer at 3-Communication Windows, " *Optik, vol.* 185, pp. 146-150, 2019.

[16] L. Vázquez, S, Jiménez and A. B. Shvartsburg, "The Wave Equation: From Eikonal to Anti-Eikonal Approximation," *Modern Electronic Materials, vol.* 2, no. 2, pp. 51-53, 2016.

[17] M. D. Malykh, N. E. Nikolaev and L. A. Sevastianov, "The Geometrical Description of Electromagnetic Radiation," *Journal of Electromagnetic Waves and Applications*, vol. 30, no.15, pp. 2055-2066, 2016.

[18] J. H. Stosch, T. Kühler and E. Griese, "Analysis and Optimisation of Bidirectional Optical Couplers in PCBs*," Optical and Quantum Electronics, vol.* 50, no. 2, p. 109, 2018.

[19]] Z. Cao, J. Jiang, Q. Shen, X. Dou and Y. Chen, "Exact Analytical Method for Planar Optical Waveguides with Arbitrary Index Profile," *JOSA A, vol.* 16, no. 9, pp. 2209-2212, 1999.. Tichi, *Electronic Hearth: Creating an American Television Culture*, Oxfool.rd University Press, 1991.

[20] A. W. Snyder and J. Love, *Optical Waveguide Theory,* Springer Science & B.