

Simulations of Flow of Fluid Through Porous Media

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Abstract — ‘Fluid flowing through porous media’ have application in practically almost every. Owing to its huge applicability, this paper intends to simulate this flow by constructing a self-explanatory model which at the end is validated by Darcy’s law. The theory preceding the model and this law is explained in detail. The results compliance with this law proves the credibility of this model for industrial application.

Keywords — *Darcy’s Law; Percolation; Nodes; Pores; Resistance.*

I. INTRODUCTION

The basic law governing the flow of fluids through porous media is Darcy’s law, which was formulated by the French civil engineer Henry Darcy in 1856 on the basis of his experiments on vertical water filtration through sand beds. Darcy found that his data could be described by:

$$Q = CA \Delta(P - \rho gz)$$

Where:

P = pressure [Pa]

ρ = density [kg/m³]

g = gravitational acceleration [m/s²]

z = vertical coordinate (measured downwards) [m]

L = length of sample [m]

Q = volumetric flowrate [m³/s]

C = constant of proportionality [m²/Pa s]

A = cross-sectional area of sample [m²]

The term ‘porous media’ encompasses a wide variety of contacting devices such as packed towers, sand beds and substances like limestone rock, filter paper and catalytic particles. It is desirable to classify the porous media, according to the types of pore spaces they contain. A proposed classification was by dividing the pore spaces into

voids, capillaries and force spaces (Manegold, 1937). This work concentrates on the pores of the size of the capillaries.

To describe the flow through a porous medium, we need to specify two parameters, applied pressure across the network and the flow-rate (i.e. the net amount of fluid passing through the network per unit time).

II. USEFUL CONCEPTS TO UNDERSTAND THE FLOW

A. Percolation Theory

According to the nature of the problem, the random mechanism can be attributed to the fluid or the medium. In percolation processes (such as a fluid soaking into a porous medium), there is a distinction between the fluid particles and the scattering medium. This medium, although it varies in random fashion from point to point, is invariant in time.

The medium is described as a set of points placed at equal intervals along a straight line and the particles of fluid can move in steps of unit length in either direction with equal probability. In the percolation process, the points of the medium are assigned a direction to the left or right with equal probability. A particle entering the medium moves in accordance with the arrows at each point and so the medium plays the active role.

Consider a 2-dimensional matrix through which a path has to be found (Fig. 2). If all elements are open to fluid flow, then they form part of a single 'percolation path' for the entire medium. As elements are blocked off, the size of the percolation path is reduced, but it still connects both ends of the network.

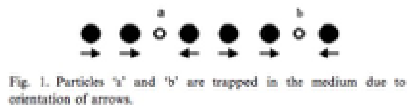


Fig. 1. Particles 'a' and 'b' are trapped in the medium due to orientation of arrows.

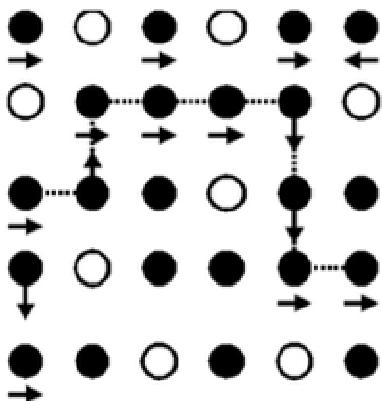


Fig. 2. Percolation pathway found in the network. Pores part of pathway have been shown connected with a dashed line. Black filled circles are inter-connected pores. Open circles are blocked pores. Arrows indicate direction of motion of a particle at a point in the medium.

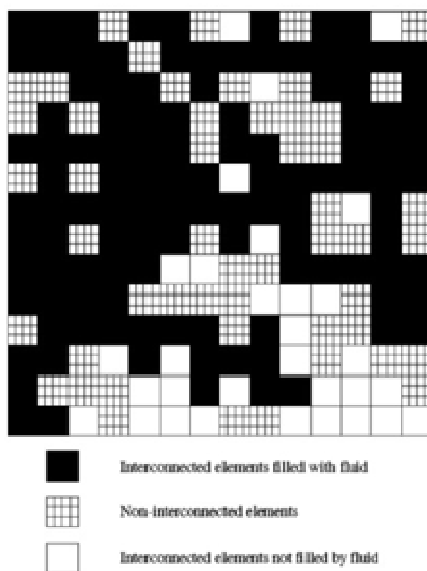


Fig. 3. Intermediate stage of percolation in 2-dimensional network.

B. Theory of capillary pressure

In porous media, capillary pressure is the force necessary to squeeze a Fluid droplet through a pore throat and is higher for smaller pore diameter.

In a single capillary, the curvature R_c of the interface gives rise to the pressure differential equal to:

$$P_c = 2(\sigma)/R_c.$$

The radius of curvature of the meniscus is equal to:

$$R_c = r/\cos(\theta).$$

So that for a single circular capillary:

$$P_c = 2(\sigma)\cos(\theta)/r.$$

As can be seen from the above expression, the surface tension force is inversely related to capillary radius. Hence, capillary pressure can be regarded as the resistance offered by a capillary to the flow of fluid through it, the larger the capillary radius, the lower the resistance. In this simulation, the resistance of an element is dimensionless and so surface tension values are irrelevant, i.e. the analysis is system non-specific.

C. Pore structure models

This simulation uses a simplified model of the porous medium. The model consists of a network of elements that represent cylindrical capillary tubes of different diameters and equal lengths. A single pore, then, is a series of elements placed one after another and so incorporates the effect of varying diameter along its length. Our model has all pores of the same length. The effect of 'arrows' is obtained by the random generation and assignment of resistances (i.e. through assigning radii values) to elements of the medium. The concept of least resistance is used to determine the percolation path of the fluid.

If pressure is applied to a fluid-filled porous medium, or to the fluid at the entrance to a capillary system, the fluid will penetrate those capillaries whose capillary pressure are lower than the applied pressure. In other words, the largest-diameter capillaries would be filled first and at increasing pressures, the smaller capillaries would get filled. This is referred to as the concept of 'least resistance'.

III. 3. SIMULATION

A. Initialization

The simulation requires a random generation of resistances of the elements that mimic the porous medium. The parameters of relevance are the size of the network or lattice and the desired phase fraction of blocked elements.

B. Least resistant pathway

The simulation employs Dijkstra's algorithm to determine the least resistant pathway.

Dijkstra's algorithm:

Dijkstra's algorithm is an algorithm for finding the shortest paths between nodes in a graph. The algorithm exists in many variants, but a more common variant fixes a single node as the "source" node and finds shortest paths from the source to all other nodes in the graph, producing a shortest-path tree.

By definition, the resistances of all pores are positive, implying positive edge weights for the graph. The concept of edge weight is equivalent to the resistance of a pore and is stored in the pore itself.

After all the pores have been assigned the minimum possible source distance value, the source distance of all pores on the exit face is compared and the one having the least value is selected. This is the value of resistance of the 'least resistant pathway' or the 'breakthrough' pressure required across the network.

For a pathway to exist between the entry and the exit face of the network, the value of 'breakthrough' pressure must be finite. If a pathway has been found, the elements are stored in order of their appearance in the percolation path and consequently, the order in which they will be filled up by the fluid.

C. Opening of a node

The next step is to carry out a depth-first search across all pores having finite resistance which have opened up. A pore is deemed to have 'opened-up' when its capillary pressure is smaller than the effective pressure available at that pore. The effective pressure at a pore is the difference of the cumulative resistance of all pores that precede the current pore from the applied pressure across the network. The recursion for a pore ends when the capillary resistance is greater than the effective pressure or all adjacent pores having finite resistance have been explored.

The pressure is incremented in steps until all the pores that have finite resistance and are reachable have 'opened-up'. Although the pressure is incremented slowly, at a given increment there can be multiple pores that 'open-up'. This is similar to the morphological approach used to study fractal dimensions (Hilpert & Miller, 2001).

D. Flowrate

In the case of an actual porous medium, the physical quantity 'flow-rate' is defined only when the fluid actually exits from the pores at the end opposite to the one at which it entered (assuming 1-dimensional percolation). The simulated network, in reality, has several exit points, each of which has its own flowrate. The flowrate through the network is defined as the cumulative flowrate from all the pores on the exit face.

In this simulation, mass and volume conservation have been assumed to hold. The pores do not rupture in the pressure range being studied. Since fluid enters from one face and leaves from the opposite face, the cumulative flowrate could be alternatively defined as the sum of the flowrate entering the network through the pores on the entry face.

When the first flowpath is obtained (using Dijkstra's algorithm), the net flowrate through it (and in this case, through the network) is zero. As the inlet pressure is increased, a new sequence of pores starts getting filled by the fluid, which may result in another flowpath joining one of the existing flowpath or flowpath emerging from the exit face. All flowpath have the same inlet and outlet pressures at any given time, and vary only in their individual resistances. Thus, the net pressure driving force across a particular path, rather than the inlet pressure is taken for calculation of the flowrate.

$$P_i = P_a - R_i$$

The flowrate through a path is then computed by dividing the driving force by the path resistance. The total flowrate is then the sum of individual flowrates and can be expressed as Q.

$$Q = P / R$$

The flowrate is calculated each time a pressure increment is made and also a new pore 'opens up'. The pressure after which no more pores open up is called 'maximum pressure'. Once 'maximum pressure' is attained, the matrix is regenerated and the process repeated a few dozen times over.

IV. RESULTS

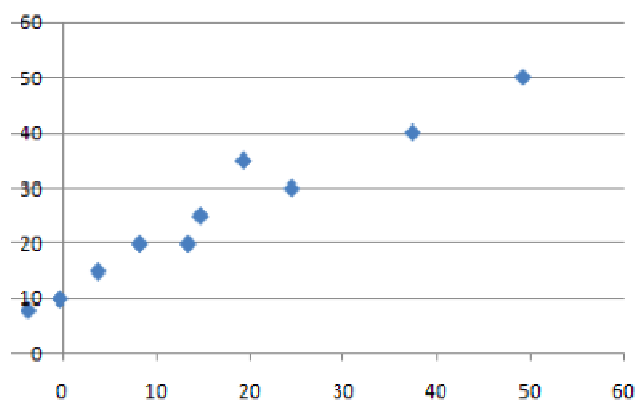


Fig. 4. Plot of flow rate vs applied pressure when size of the grid is 10 $\square$ 10

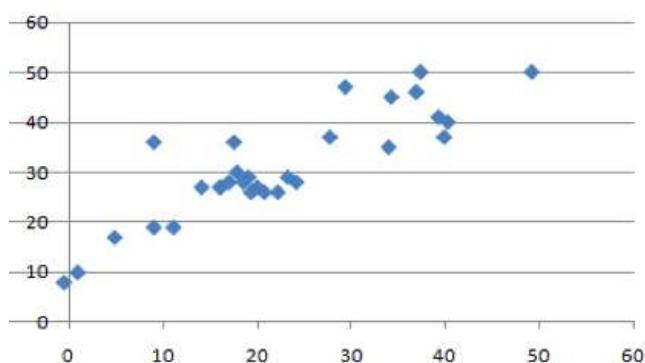


Fig. 5. Plot of flow rate vs applied pressure When the grid size is 30 $\square$ 30

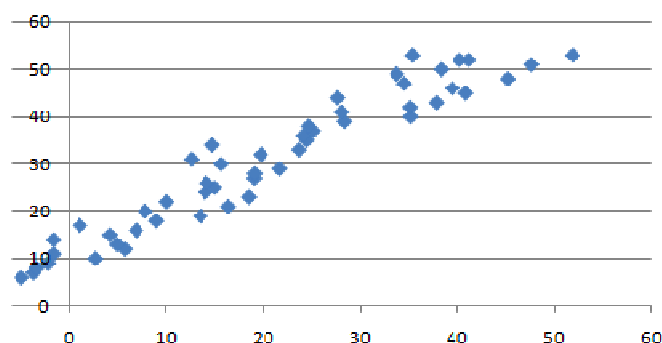


Fig. 6. Plot of flow rate vs applied pressure When the grid size is 50 $\square$ 50

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