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Stability Analysis of Prey-Predator Model Holling Type II with Infected Predator

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Abstract - The prey-predator model is a model of interaction between two species, predators and prey. The interaction affects the growth of the two species. The Holling type II prey-predator model with disease-infected predators was discussed. Furthermore, the stability of the model was analyzed the around the critical point determined from eigenvalues of the Jacobian matrix. Stability is also associated with threshold parameter. Finally, the analysis results were confirmed with numerical solutions for different parameter values. The results show that the stability of the system depends on the value of the capacity of the prey population.

Keyword - Prey-Predator, Holling Type II, Stability, Threshold Number, Numerical Solution.

I. Introduction

Study of the prey-predator model is important for controlling prey and predator populations in ecological theory. Interaction between prey and predator determines the specific form of the response function. The response function depends on the number of prey and predator population, the carrying capacity of the environment, and the level of saturation of predator. Ecological populations infected by diseases can affect population size. Determining a population is infected with a disease or free of disease based on a threshold parameters (R_0). The value of R_0 depends on the infectivity of disease and contact rate of a typical infective [4, 5]. The model prey-predator with predator infected has been developed [1, 2]. Those models are also reformulated by considering infection in both populations, prey and predator populations [3]. In this paper, the stability behavior related to threshold parameters in the model prey-predator Hollings type II with predator infected is discussed.

II. The Prey Predator Model

The mathematical model of prey predator was discussed [3]. The model was developed based on assumption:

- The susceptible prey follows logistical growth.
- The infected predator does not reproduce.
- The infected predator does not recover.
- The infected predator die more easily.

In this paper, the model describing interaction between prey and predator with infected predator lead to the following system equations

$$\dot{S} = rS\left(1-\frac{S}{K}\right)-\frac{\alpha_1 SP_1}{1+a_1 S},$$

$$\dot{P_1} = \frac{\beta SP_1}{1+a_1 S}-\gamma_1 P_1, \qquad (1)$$

$$\dot{P_2} = -\gamma_2 P_2,$$

Where S is susceptible prey population, P_1 is predator population, and P_2 is infected predator population.

Parameter r is prey growth rate, K is carrying capacity, α_1 are interaction between prey and predator, β is predator growth rate, μ is death rate of infected prey, γ_1 and γ_2 are predator mortality rate. All parameters are positive constant.

III. Stability Analysis

There are three critical points of the system (1)

A_1 = (0,0,0),
A_2 = (K,0,0),
$A_3 = \left(\frac{\gamma_1}{\beta-a_1\gamma_1}, \frac{r}{\alpha_1}\left(1-\frac{\gamma_1}{K(\beta-a_1\gamma_1)}\right)\left(1+\frac{a_1\gamma_1}{\beta-a_1\gamma_1}\right), 0\right)$.

Jacobian matrix of the system (1) is

$$J = \begin{bmatrix} r-\frac{2rs}{K}-\frac{\alpha_1 P_1}{(1+\alpha_1 S)^2} & -\frac{\alpha_1 S}{1+\alpha_1 S} & -\frac{\alpha_1 S}{1+\alpha_1 S} \\ \frac{\beta P_1}{(1+\alpha_1 S)^2} & \frac{\beta S}{1+\alpha_1 S}-\gamma_1 & 0 \\ 0 & 0 & -\gamma_2 \end{bmatrix}. \quad (2)$$

a. Critical point A_1 = (0,0,0)

There are three distinct real eigenvalues that is, r, $-\gamma_1$, and $-\gamma_2$. It means that the critical point A_1 is a saddle point and unstable.

b. Critical point A_2= (K,0,0)

There are three distinct real eigenvalues for critical point A_2, is $-r$, $\frac{\beta K}{1+a_1 K}-\gamma_1$, and $-\gamma_2$. It means that stability of critical points A_2 depend on value $\frac{\beta K}{1+a_1 K}-\gamma_1$. Let

$$R_0 = \frac{\beta K}{\gamma_1(1+\alpha_1 K)} \quad (3)$$

is a threshold parameters. If $R_0 < 1$, then the critical point A_2 is asymptotically stable. On the other hand, if $R_0 > 1$, then the critical point A_2 is unstable.

c. Critical point $A_3 = \left(\frac{\gamma_1}{\beta-a_1\gamma_1}, \frac{r}{\alpha_1}\left(1-\frac{\gamma_1}{K(\beta-a_1\gamma_1)}\right)\left(1+\frac{a_1\gamma_1}{\beta-a_1\gamma_1}\right), 0\right)$

Eigenvalues for critical points A_3 are

$$\lambda_{1,2} = \frac{\frac{r\gamma_1(a_1(K(\beta-a_1\gamma)-\gamma)-\beta)}{K\beta(\beta-\gamma a_1)} \pm \sqrt{\left(\frac{r\gamma_1(a_1(K(\beta-a_1\gamma)-\gamma)-\beta)}{K\beta(\beta-\gamma a_1)}\right)^2 - 4\frac{r(K(\beta-\gamma a_1)-\gamma)}{K a_1}}}{2},$$

$$\lambda_3 = -\gamma_2.$$

Stability of critical point A_3 depend on value $\frac{r\gamma_1(a_1(K(\beta-a_1\gamma)-\gamma)-\beta)}{K\beta(\beta-\gamma a_1)}$, so that $\frac{\beta K}{\gamma(1+a_1 K)} < 1 + \frac{\beta}{\gamma a_1(1+a_1 K)}$.

(i) If $1 < R_0 < 1 + \frac{\beta}{\gamma a_1(1+a_1 K)}$, then critical point E_3 is asymptotically stable.
(ii) If $R_0 > 1 + \frac{\beta}{\gamma a_1(1+a_1 K)}$, then critical point E_3 is unstable.

IV. Numerical Solutions

Analytical result of the prey predator system (1) is confirmed by some numerical solutions that shows phase portrait on phase plane and graphic of solutions with fix parameter [3]: $r = 2$, $a_1 = 0.01$, $\gamma = 1$, $\beta = 0.015$, $\mu = 0.9$, $\alpha_1 = 0.1$, $\gamma_2 = 0.5$. Let initial condition $(S(0), P(0)) = (200, 40)$ for K = *600*, and $(S(0), P(0)) = (200, 150)$ for K = *490*. Figure 1 and 2 show solutions and phase portrait of the system (1), respectively.

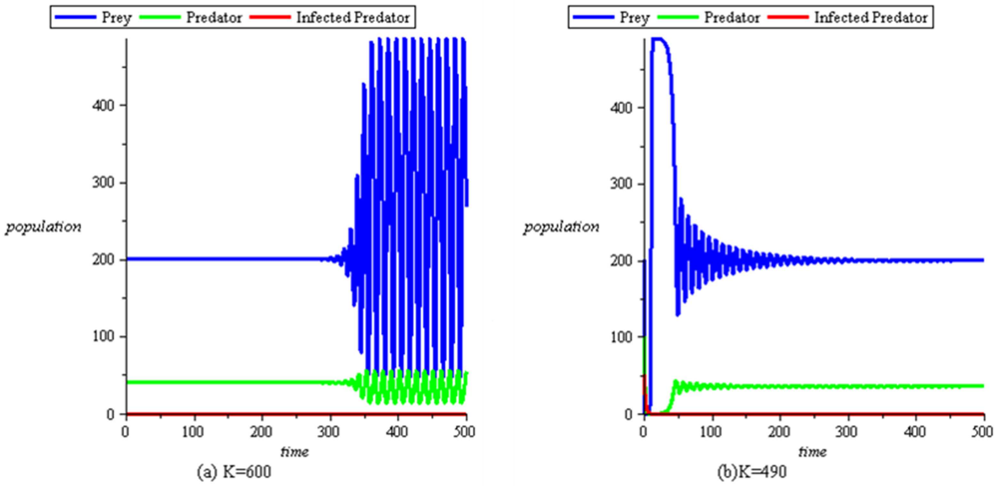

Figure 1. Solutions of the system (1)

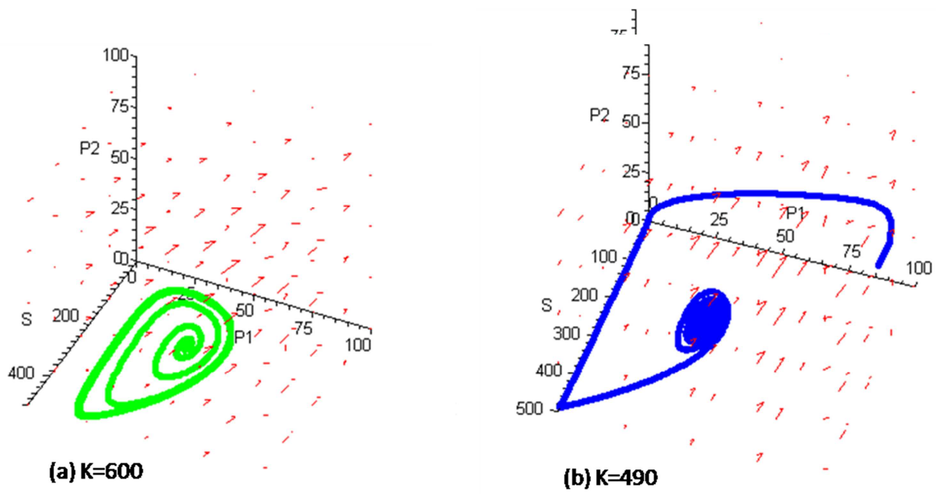

Figure 2. Phaseportrait of the system (1)

V. Conclusion

Stability of prey predator system with free disease and infected predator has been analyzed. The result shows that the stability of the system depends on the value *K*, carrying capacity of prey population.

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