International Journal of Progressive Sciences and Technologies (IJPSAT)
ISSN: 2509-0119.

http://ijpsat.ijsht-journals.org
Vol. 16 No. 2 September 2019, pp. 203-206

On Super Edge Magic Total Labeling of the Branched Prism Graph

Khairannisa Al Azizu, Lyra Yulianti, Syafrizal Sy, Narwen

Mathematics Department, Faculty Of Mathematics and Natural Science, Andalas University
Kampus UNAND Limau Manis Padang, Indonesia.

Abstract - Let $G=(V,E)$ be an arbitrary graph on p vertices and q edges. The edge magic total labeling of G is a bijection $f: V(G) \cup E(G) \rightarrow \{1,2,\ldots,p+q\}$ such that $f(u)+f(v)+f(uv)=k$ for every $uv \in E(G)$ and some positive integer k. The constant k is called the magic constant of the labeling. The labeling is called super if $f(V(G)) = \{1,2,\ldots,p\}$. This paper concerns about the super edge-magic total labeling of branched-prism graph, denoted by $(C_m \times P_2) \odot \overline{K_n}$, and shows that the magic constant $k = 5mn+5m+1/2(m+1)+1$, for odd m, $m \geq 3$ and $n \geq 1$.
.

Keywords - Super Edge Magic Total Labeling, Super, Bijection, The branched Prism Graphs, Magic constant.

I. Introduction

All graphs in this paper are considered simple, undirected and finite. A graph labeling is a mapping from the vertex set and/or edge set of graph to some positive integers called labels. If the domain of the labeling is the vertex set (or edge set) then the labeling is called vertex labeling (or edge labeling). The labeling is called total if the domain is the set of all vertices and edges [1].

Let $G=(V,E)$ be an arbitrary graph on p vertices and q edges. The bijection $f: V(G) \cup E(G) \rightarrow \{1,2,\ldots,p+q\}$ such that $f(u)+f(v)+f(uv)=k$ for every $uv \in E(G)$ and some positive integer k is called the edge-magic total labeling of G. The positive integer k is called the magic constant and $w(uv) = f(u)+f(v)+f(uv)$ is called the edge-weight of uv. If $f(V(G)) = \{1,2,\ldots,p\}$ then the labeling is called super [2]. Other notations and definitions in this paper refer to [3].

There are many interesting results regarding the super edge magic labeling of various types of graphs. Recent survey regarding the graph labeling can be seen in an updated survey by Gallian [5]. In [6], Sugeng has shown that the generalized prism graph, denoted by $C_m \times P_r$ has a super edge-magic total labeling if m in odd, for $m \geq 3$ and $r \geq 2$. In this paper, it will be shown that the branched-prism graph, denoted by $(C_m \times P_2) \odot \overline{K_n}$ has a super edge-magic total labeling for odd m, $m \geq 3$ and $n \geq 1$.

II. Materials And Methods

2.1. Cartesian Product *(cross)*

Cartesian product (cross) of graph $G_1 = (V_1, E_1)$ and $G_2 = (V_1, E_1)$ is a graph $G = G_1 \times G_2$. $V = V_1 \times V_2$ and two vertices (u_1, u_2) and (v_1, v_2) in graph G are incidence if and only if one of this condition is happened, there are $u_1 = v_1$ and $u_2 v_2 \in E_2$ or $u_2 = v_2$ and $u_1 v_1 \in E_1$[4].

2.2. Corona Product

Let two graphs G and H with $V(G) \rightarrow \{x_1, x_2, \cdots, x_n\}$ and $V(H) \rightarrow \{y_1, y_2, \cdots, y_m\}$.

Definition a graph with the vertex $V(H_i) \rightarrow \{y_{i1}, y_{i2}, \cdots, y_{im}\}$ where H_i is i duplicate from graph H. Graph $G \odot H$ have the set of vertex

$$V(G \odot H) = V(G) \cup \bigcup_{i=1}^{n} V(H_i),$$

and the set of edge $E(G \odot H)$

$$= E(G) \cup \bigcup_{i=1}^{n} E(H_i) \cup \{x_i y_{ij} | 1 \leq j \leq m, y_i \in V(H_i)\},$$

with H_i is duplicate of graph H. It can be find $|V(G \odot H)| = |V(G)| + |V(G)| \times |V(H)|$ and $|E(G \odot H)| = |E(G)| + |V(G)| \times |E(H)| + |V(G)| \times |V(H)|$.

2.3. Super Edge Total Labeling Graph

Graph G denote $G=(V,E)$ on p vertices and q edges is edge total labeling if it can make one of bijection of $f : V(G) \cup E(G) \rightarrow \{1, 2, \cdots, p + q\}$. Furthermore, to any edge $uv \in E(G)$, edge weight $w(uv) = f(u) + f(uv) + f(v) = k$ is constant. f is super edge total labeling in graph G if $f: V(G) \rightarrow \{1,2, \dots, p\}$.

Theorem 1 [2] *A (p,q)graph G admits a edge-magic vertex labeling if and only if there exists a bijection f:V(G)→{1,2,...,p} such that the set S={f(u)+f(v)|uv∈E(G)} consists of q consecutive integers. In such a case, f* can *be extends to a super edge-magic total labeling of G with constant k=p+q+s, where s= min{S}, where S = {k-(p+1),k-(p+2),...,k-(p+q)}.*

III. Results And Discussions

Let C_m, P_2 and $\overline{K_n}$ be a cycle on m vertices, a path on 2 vertices and a complement of complete graph on n vertices, for odd m, $m \geq 3$ and $n \geq 1$, respectively. The branched prism graph, denoted by $H = (C_m \times P_2) \odot \overline{K_n}$, is coming from the corona operation between the prism graph $C_m \times P_2$ and $\overline{K_n}$ for odd m, $m \geq 3$ and $n \geq 1$ (see Figure 3.1). The vertex set and edge set of H are as follows.

$$V(H) = \{v_{i,j}, v_{i,j,k} | 1 \leq i \leq 2, 1 \leq j \leq m, 1 \leq k \leq n\},$$
$$E(H) = \{v_{i,j}v_{i,j+1}, v_{i,m}v_{i,1}, v_{1,j}v_{2,j}, v_{i,j}v_{i,j,k}, v_{i,m}v_{i,m,k} | 1 \leq i \leq 2, 1 \leq j \leq m-1, 1 \leq k \leq n\}.$$

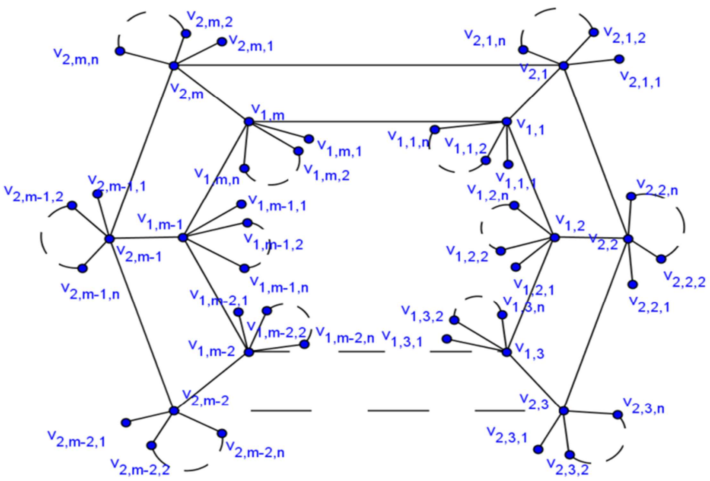

Figure 3.1 The Branched Prism Graph

In the following theorem, it is determined that the branched prism graph $H = (C_m \times P_2) \odot \overline{K_n}$ admits the super edge-magic total labeling for odd m, $m \geq 3$ and $n \geq 1$.

Theorem 2.1 *There exists some super edge-magic total labeling in the* branched prism graph $H = (C_m \times P_2) \odot \overline{K_n}$ *for odd m, m ≥ 3 and n ≥ 1. The magic constant of the labeling is k = 5mn+5m+1/2(m+1) +1.*

Proof.

From the definition of branched prism graph H, it clear that $|V(H)|$ = $2mn+2m$ and $|E(H)|=2mn+3m$.

Define the bijection $f: V(H) \cup E(H) \rightarrow \{1, 2, \ldots, 4mn+5m\}$ as follows.

$$f(v_{1,i,j}) = \begin{cases} \dfrac{i+1}{2} + m(j-1) & \text{for odd } i, 1 \le i \le m, \text{and } 1 \le j \le n \\ \dfrac{m+i+1}{2} + m(j-1) & \text{for even } i, 2 \le i \le m-1, \text{and } 1 \le j \le n \end{cases}$$

$$f(v_{1,i}) = \begin{cases} mn + \dfrac{m+i}{2} & \text{for odd } i, 1 \le i \le m, \\ mn + \dfrac{i}{2} & \text{for even } i, 2 \le i \le m-1, \end{cases}$$

$$f(v_{2,i}) = \begin{cases} mn + 2m & \text{for } i = 1 \\ mn + m + \dfrac{m+i-1}{2} & \text{for even } i, 2 \le i \le m-1, \\ mn + m + \dfrac{i-1}{2} & \text{for odd } i, 3 \le i \le m \end{cases}$$

$$f(v_{1,i,j}) = \begin{cases} mn + 2m + \dfrac{m-1}{2} + m(j-1) & \text{for } i = 1 \text{ and } 1 \le j \le n, \\ mn + 3m + m(j-1) & \text{for } i = 2 \text{ and } 1 \le j \le n, \\ mn + 2m + \dfrac{m+i-2}{2} + m(j-1) & \text{for odd } i, 3 \le i \le m, \text{and } 1 \le j \le n, \\ mn + 2m + \dfrac{i-2}{2} + m(j-1) & \text{for even } i, 4 \le i \le m-1, \text{and } 1 \le j \le n. \end{cases}$$

The vertex labeling of H is given in Figure 3.2.

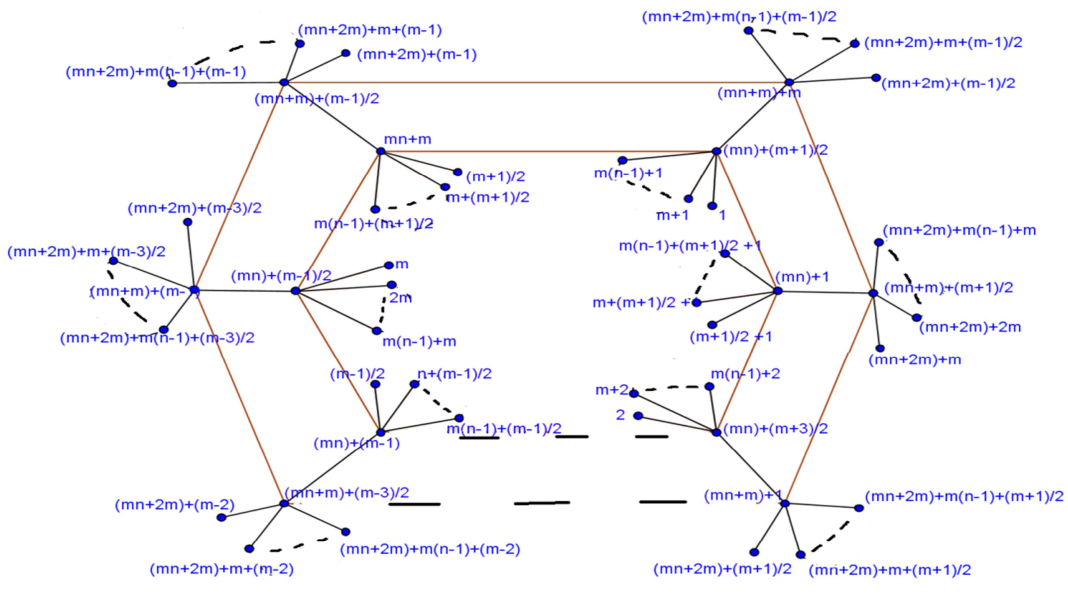

***Figure 3.2** The vertex labeling of H*

From this vertex labeling, it is obtained that $S = \{ f(x)+f(y) \mid xy \in E(H)\} = \{mn+1+1/2(m+1), mn+2+1/2(m+1), \ldots, 3mn+3m-1+1/2(m+1), 3mn+3m+1/2(m+1)\}$. It can be easily seen that the members of S form an arithmetic progression with difference 1, and therefore, S consists of consecutive integers. From Theorem 1, we can construct the edge labeling of H such that H admits super edge-magic total labeling as follows.

Define $f(E(H)) = \{2mn+2m+1, 2mn+2m+2, \ldots, 4mn+5m\}$, where the smallest label $2mn+2m+1$ is given to the edge with the largest member of S, $3mn+3m+1/2(m+1)$, the second smallest label $2mn+2m+2$ is given to the edge with the second largest member of S, $3mn+3m-1+1/2(m+1)$. The process continues until the largest label $4mn+5m$ is given to the edge with the smallest member of S, $mn+1+1/2(m+1)$. Moreover, it can be shown that the constant $k = 2mn+2m+1 + 3mn+3m+1/2(m+1) = 5mn+5m+1+1/2(m+1)$. Therefore, the branched prism graph $H = (C_m \times P_2) \odot \overline{K_n}$ admits the super edge-magic total labeling for odd m, $m \geq 3$ and $n \geq 1$.

IV. CONCLUSIONS

In this paper, it has been shown that the branched prism graph $H = (C_m \times P_2) \odot \overline{K_n}$ admits the super edge-magic total labeling for odd m, $m \geq 3$ and $n \geq 1$.

REFERENCES

[1] Kotzig, A., and Rosa, A., 1970, Magic valuation of finite graphs, *Canad. Math. Bull.* **13** : 451 – 461

[2] Bondy, J.A dan Murty, U.S.R. 1976. Graph Theory with Application.The Macmillan Press LTD, London.

[3] Chartrand, G and Zhang,P. 2005. Introduction to Graph Theory. Mc GrawHill Press, Boston.

[4] Chartrand, G and Lesniak L.1996. Graphs and Digraphs Third Edition. Chapman and Hall/CRC, America.

[5] Gallian, J. A. 2018. A Dynamic Survey of Graph Labeling. *The Electronic Journal of Combinatorics*, #DS6

[6] Sugeng.K.A. 2006. Super edge antimagic total labelings.Utilitas Mathematica.**71**:131-141.