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A Comparative Analysis of Multi-Criteria Decision Making Methods in Optimizing Water Loss Management Strategies used in Water Distribution System

A case of Moshi Urban Water Supply and Sanitation Authority, Tanzania

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Abstract - **This paper presents a comparative analysis of Multi-Criteria Decision Making (MCDM) methods integrated with numerical optimization technique to optimize water loss management strategies used in the water distribution system. The study was conducted at Moshi Urban Water Supply and Sanitation Authority (MUWSA), Tanzania. Two ranking methods, Complex Proportional Assessment (COPRAS) and Simple Additive Weighting (SAW) integrated with Integer Linear Programming (ILP) technique were compared. The aim is to help decision-makers to choose the best decision-making method that may help to optimize the water loss management strategies used in water distribution systems. The Multi-Criteria Decision Making methods were used in evaluating and prioritizing the strategies while the Integer Linear Programming technique was used to formulate a decision model and selecting the best strategies (alternatives) employed in water loss management. Results show that the methods rank the given alternatives differently while when integrated with integer linear programming technique, the formulated models select the same portfolios of alternatives. Both models select thirteen alternatives, which cost 97% (TZS 235.71 million) of the total budgets set by the water authority for WLM. Furthermore, the models showed robustness to their results; when the ranking of evaluation criteria changes, the portfolios of alternatives remained the same. In general, this paper elaborates how decision-making methods can be coupled with mathematics in operation research to solve issues in water management and planning.**

Keywords - Multi-Attribute Value Theory. Integer Linear Programming. Water Loss Management. SMARTER. COPRAS. SAW. MUWSA.

I. Background And Introduction

Water is very important for survival of both animals and plants, and it is critical to the economics of a country. Developing countries worldwide are facing big challenges in water management due to increasing demand for urban water supply caused by industrialization, urbanization, improvement of living standards, and impact of global warming in freshwater supply [1]. Water demand is increasing while resources are diminishing. In addition to the mentioned challenges, not all water produced in the distribution system reaches the consumers; some amount of water is lost on the way before reaching the consumers.

Water loss in a distribution system is a problem facing many water authorities in general, however, in developing countries; the problem is worse, due to poor infrastructures and poor sanitation.

Water loss is the difference between the water produced and water billed or consumed; (i.e. water loss = water produced – water billed or consumed). Water losses in the water distribution system (WDS) are grouped into two categories: Apparent (commercial) and real (physical) losses. Apparent losses include water theft such as illegal connection and meter tampering, meter inaccuracies, and unbilled water for firefighting and public use such as water used in public toilets. Physical losses, on the other hand, include water leakage from pipes, fittings, and joints, and water leakage from tank overflow, tank floor and walls [2]. Water leakage is the major contributing factor for water loss in WDS. About 70% of the total water loss in developing countries is caused by water leakage [3]. The leakage can be caused by aging of pipes, poor network design and construction, damage to exposed pipes, poorly sealed connections, and theft of pipes.

Total water loss is the total of real and apparent losses. It represents the non-revenue water (NRW), which is the unbilled water or water which does not bring revenue to the water companies (IWA). The NRW is the indicator of efficient operation of WDS in which high NRW indicates poor performance of the WDS. Water losses negatively affect the operation of water companies as there is no company can operate efficiently if it does not realize all its revenue. Moreover, water losses in WDS have economic, environmental, public health and social effects. Besides, water losses in WDS reduce revenue, interrupt the quality of water, and inflate operation and maintenance cost of the company [1],[2].

“Reference [1] shows that water loss in WDS worldwide ranges from 15% to 60% of the total water supply”. The situation is critical in developing countries where a combination of aging infrastructure, illegal connections, and theft of pipes worsen the problem. While the World Bank (WB) recommends that NRW should be less than 23% [1], the Southern Africa countries have set a standard of 20% as optimal for well-performing water company (Ministry of Water of Tanzania (MoW) report, 2014). In Tanzania, the average of NRW for the Urban Water Supply and Sanitation Authorities (UWSSAs) was recorded to be 46% in 2013 (MoW report, 2014), in which Dar es Salaam Water Supply and Sewage Authority (DAWASA) had the highest percent of NRW (55%), while other UWSSAs have an average of 37% of NRW. Despite the MoW setting a target for reducing NRW from 46% in 2013 to 25% in 2019, water loss in the WDS is still high. The MoW in its 2019/2020 budget report shows that DAWASA has 40% of NRW whereas other UWSSAs have an average of 32% of NRW.

There are various water loss control strategies, methodologies, and procedures set by the International Water Association (IWA). The IWA Water Loss Task Force (WLTF) developed and promote water management approach which can be applied worldwide to reduce water loss in the distribution system and increase the revenue generation of the water companies [4]. The developed methodologies broadly focus on accurate and comprehensive metering, water balance and managing apparent and real losses.

To manage real losses, the WLTF puts emphasis on the use of four strategies: active leakage control, pressure management, speed, and quality repair, and infrastructure management. All these strategies should be balanced to get the most cost-effective leakage management. As it is with real losses, apparent losses control, also use four basic methodologies: reduction of meter errors, reduction of human errors, reduction of a billing system error, and reduction of theft.

Studies on water loss problem in the WDS are of interest worldwide as the problem affects water authorities and consumers all over the world. Many studies in the aspect of technical and managerial have been done to reduce water loss in WDS, design, and operation of the WDS, and improving services. Most of the studies carried in this area are from Science and engineering, and mathematics. The focus of these studies has mostly been on water leakage control, design and operation of WDS.

This paper discusses water loss control or reduction in the aspect of management. It focuses on water loss management (WLM) in WDS. WLM is the management of the quantity of water which is the difference between supply and consumption. WLM is an important decision issue in meeting water company strategic goals outlined in the company Strategic Planning (SP). SP is a process of setting long-term goals for the future of the company based on predictions and analysis of strategies and key decisions. SP is a valuable tool for sustainable water management. Most of the problems facing water companies in developing countries are caused by a lack of strategic planning.

1.1 Multi-Criteria Decision Making (MCDM) Methods

Multi-Criteria Decision Making (MCDM) is the study of methods and procedures by which multiple and conflicting criteria can be incorporated in the decision process. The main objective of MCDM is to provide a tool to DMs to enable them to advance in solving multi-criteria decision problems, where several conflicting criteria are taken into account [5]. Several names can be used to refer to MCDM: Multi-Criteria Decision Analysis (MCDA), Multi-Objective Decision Making (MODM), Multi-Attribute Decision Making (MADM), and Multi-Dimensions Decision Making (MDDM).

The MCDM weighting methods are classified into two main categories as the first category includes compensatory weighting methods such as Multi-Attribute Utility Theory (MAUT), Multi-Attribute Value Theory (MAVT), Analytic Hierarchy Process (AHP), (Simple Multi-Attribute Rating Technique Swings (SMARTS), Simple Multi-Attribute Rating Technique Extended to Ranking (SMARTER), Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) which are used in Multi-Attribute Utility (MAU) methods. The second category includes and non-compensatory weighting methods like Elimination Et Choix Traduisant la Realite (ELECTRE), Preference Ranking Organization Method for Enrichment Evaluation

(PROMETHEE) and their families which are used in outranking methods [6], [5]. In a compensatory approach, the evaluation of alternatives considers trade-offs between criteria while in non-compensatory methods, a loss of an alternative on a criterion cannot be compensated by other criteria.

Furthermore, "references [7], and [5] put various MCDM methods into the following three categories based on their similarities":

1. Multi-Attribute Theory

This group of methods aggregates the different point of view into a unique function for optimization. The methods use numbers to represent the preference of the considered action. The methods in this category are MAUT (Multi-Attribute Utility Theory), MAVT (Multi-Attribute Value Theory), SMARTS (Simple Multi-Attribute Rating Technique Swings), AHP (Analytical Hierarchy Process) and TOPSIS (Technique for Order by Similarity to Ideal Solution).

2. Outranking methods

These methods are based on pair-wise comparison of action. They develop a relationship which is called outranking relationship which represents DMs' preferences to solve their problems. The methods in this category are ELECTRE (Elimination Et Choix Traduisant la Realite (French) or Elimination and Choice Translating Reality (english)), PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluation).

3. Elementary methods

These are the simplest MCDM methods which use simple preference models. They include weighted sum, weighted product, conjunctive, disjunctive, and linear assignment.

Weighting to the evaluation criteria in MCDM methods is very important as the final result of decision making largely depend on weights. "Reference [5] classifies weighting methods into three categories".

1. Subjective weighting methods

These determine criteria weights basing on the preferences of the DMs. Elicitation process in subjective methods is explained clearly and mostly used for MCDM in water resource management. Popular subjective weighting methods are direct rating, ranking method, SMART/SMARTER, point allocation, swing method, ratio method, pair-wise comparison, SIMOS method, graphical weighting, and Delphi method.

2. Objective weighting methods

In these methods, criteria weights are obtained through mathematical methods in which DMs have no role in the determination of relative importance of criteria. Popular objective weighting methods include: mean weight, standard deviation, Statistical variance procedure, Criteria Importance Through Inter-criteria Correlation (CRITIC), entropy method, TOPSIS, least mean square, Multi-Objective Optimization.

3. Combination or optimal weighting methods

These are hybrid methods that include multiplication and additive synthesis. The weights are derived from both subjective and objective information on criteria weights. The methods in this category are swing weights, ranking, rating, pair-wise comparison, trade-off analysis, and qualitative translation.

1.2 Studies in MCDM methods and their application in water loss and resource management

Studies in decision making are guided by MCDM methods. These are tools developed in the field of decision theory for resolving operation research problems, the methods form a finite number of decision alternatives in which decision-makers (DMs) have to evaluate and rank based on the weights of the finite set of evaluation criteria [7]. Many researchers have done researches in MCDM methods and their application in various fields, particularly in water loss and resource management, and planning. "References [8], [9], and [10] show studies conducted in MCDM methods": "reference [8] found that when COPRAS and ARAS (Additive Ration Assessment) methods are used in selecting a suitable air conditioner, the two methods ranked the same the performance of alternatives". "Reference [9] compared COPRAS and SAW (Simple Additive Weighting) and found that the two methods are closely related in the ranking of alternatives". "Reference [10] reviewed the common MCDM methods and came up with advantage and disadvantage as well as the area of application for each method". On the other hand, "references [11], [12], and [13] show studies in application of MCDM methods in water loss and resource management": "Reference [12] formulated a model based on a problem structuring method (PSM) and PROMETHEE method to assess strategies used to balance water supply – demand in WDS. Results show that the model was robust for decision making". "Reference [13] used SAW, CP (compromise programming) and TOPSIS methods to formulate a model for water resource management. Results show that the best alternative does not depend on the MCDM method used but it is slightly sensitive to the weights of the criteria and data used in the analysis". "Reference [11] did a comparison of compensatory and non-compensatory MCDM methods for water resources strategic management. SAW and AHP (analytical hierarchy process) represented compensatory methods and ELECTRE III represented non-compensatory methods. Results show that the ELECTRE III method has lower sensitivity than SAW and AHP methods to change in the weights. Also, the ranking obtained from the ELECTRE III method is more reliable for decision making".

This study compares two integrated decision models of MCDM Methods and Integer Linear Programming (ILP) a numerical optimization technique to evaluate, prioritize and select the best strategies (alternatives) used in WLM. The compared models are formulated from Complex Proportional Assessment, (COPRAS) and Simple Additive Weighting (SAW) methods; these are MCDM ranking methods which use SMARTER to assign weights to evaluation criteria [10]. COPRAS and SAW are the method of Multi-Attribute Utility Theory (MAUT) [14]. MAUT and MAVT are two related methods in compensatory weighting methods, which allow compensation of weak performance of one criterion by a good performance of another criterion. They use additive weighted value function which combines the multiple evaluation criteria into a single measure of the overall value of each alternative. They all use SMARTS or SMARTER to assign weights to the evaluation criteria [10]. COPRAS method uses both benefit (maximization) and cost (minimization) criteria values for evaluation of MCDM alternatives while the SAW method requires conversion of cost criteria to benefit criteria [15].

II. Materials And Methods

This section presents the case study, model framework, data collection methods, and instrument used to analyze the data.

1.3 Case Study

The study was carried out at Moshi Urban Water Supply and Sanitation Authority (MUWSA), the town of Mt. Kilimanjaro in the northern part of Tanzania. The water distribution system of MUWSA covers 590.3 km and based on 30th April 2019 information, serves 36625 customers. The WDS is divided into 12 zones and has 19 sources in which 12 are springs and 7 are boreholes. The Utility experience water loss of about 2.6 million m^3 per year in the WDS which is equivalent to 22% of the total water produced per year (Research data, 2019). The utility aims to reduce this loss to 15% within five coming years.

1.4 Data Collection and Analysis

Both primary and secondary data was collected. Primary data was collected from knowledgeable and experienced DM by using questionnaire and interview tools. While secondary data was collected through the documentary review which includes reading the reports, brochures, flyers, and posters. The collected data were; the strategies/alternatives used by the company in WLM, evaluation criteria, budget set for implementation, the quantity of water produced and loss, etc.

Data were analyzed through the integrated models of MCDM family methods and ILP the numerical optimization technique. The models aimed to aid the DMs to prioritize and select a portfolio of strategies used in WLM. The ILP equations were solved by using LINDO 6.1 software package.

1.5 Model Framework

The proposed model framework is divided into two main phases: (1) MCDM family methods approach, and (2) ILP numerical optimization technique.

III. RESULTS

3.1 Description of Problem and Objectives

The first step in decision making is to define explicitly the problem and objectives of what is needed in the study. In doing so, the researcher needs to identify the root causes of the problem and understand the context of the company under study. Also at this step, alternatives which contribute to the achievement of the objectives and their evaluation criteria are identified. Expert DMs should be engaged in the evaluation of alternatives.

Thus, based on the case study, the problems of the study were: To identify, prioritize and select the portfolio of alternatives used at MUWSA in water loss management. The goals are derived from the quality policy statement of the company which states that: *MUWSA aims to achieve the provision of adequate, sustainable and competitive clean and safe water supply and sanitation service which consistently satisfy the needs and expectations of its customers.*

The derived goals are:

1. To maximize the revenue
2. To minimize the operational cost
3. To maximize water saving
4. To maximize water supply reliability
5. To maximize water quality

3.2 Identification of Alternatives

The surveyed alternatives for WLM in this study were:

1. **Alternative (A_1): Education** on the effective use of water which may facilitate saving water inside and outside the home, and encourage people to reports visible leaks and faults which may speed up the maintenance process and avoid excessive water loss,

   A_{11} – Advertising campaigns

   A_{12} – Educational campaigns in schools

   A_{13} - Ward meetings with the society

   A_{14} – Meeting with local leaders

2. **Alternative (A_{21}): Illegal use control** - A measure aimed at losses that occur with the illegal use of water from end-users (illegal connection, a reversal of the meter and violation of the infrastructure).

3. **Alternative (A_{31}): Network zoning (Establishment of District Metering Areas -DMA)**

4. **Alternative (A_4): Using indicators to quantify the losses** - important information for the planning of action needs to be taken to control losses.

A_{41} –24 hours Zone Measuring (HZM)

5. **Alternative (A_5): Strategies used to control inaccuracy meter**.

   A_{51} - Calibration of the meter

   A_{52} - Replacement of the defect meters

6. **Alternative (A_6): Detection of apparent/physical losses**

   A_{61} – Visual inspection of the WDS

   A_{62} – comparison between the bulk water meter and customer water meter readings

   A_{63} – Report from the community on the detected leak through a toll-free telephone

7. **Alternative (A_{71}):** Replacement of dilapidated pipes

8. **Alternative (A_{81}):** Installation of quality pipes

9. **Alternative (A_{91}):** Timely repair of pipe leaks (active leakage control)

10. **Alternative (A_{101}):** Pressure management

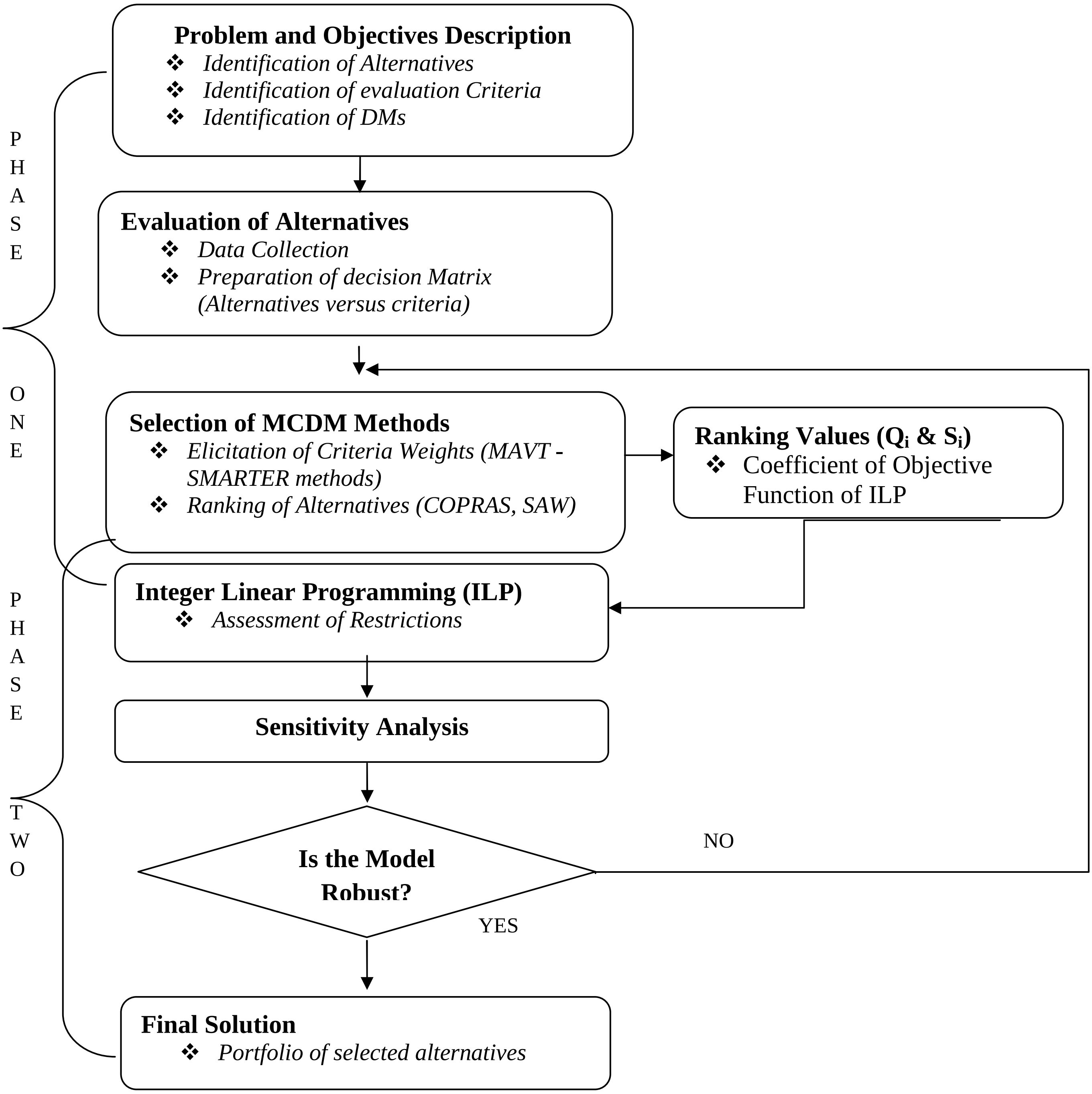

Fig.1. Framework of the Proposed Model

3.2.1 Evaluation Criteria

The identified evaluation criteria to this study were;

1. **Criteria (C_1): Revenue generation** – The ability of alternative to improve revenue. The higher the score value, the most preferable the alternative is.

2. **Criteria (C_2)**: **Investment cost**- cost needed to implement the alternative. The lower the score value (cost) the most preferable the alternative is.

3. **Criteria (C_3)**: **Operation &Maintenance cost** - The cost related to the implementation of the alternative. The lower the score value (cost) the most preferable the alternative is.

4. **Criteria (C_4): Saving of Water** – The ability of alternative to reduce water loss. The higher the score value, the most preferable the alternative is.

5. **Criteria (C_5): Quality of Water –** The ability of the alternative to retaining water quality. The higher the score value, the most preferable the alternative is.

6. **Criteria (C_6): Water Supply reliability –** The ability of alternative to reduce supply disruptions. The fewer the frequency of disruptions (burst, leaks, and illegal uses) the most preferable the alternative is.

7. **Criteria** (C_7): **Efficiency** of the alternative to reduce water losses. The higher the score value, the most preferable the alternative is.

Note: The evaluation score values for criteria are scaled as: ***Very high – 5; High – 4; Fair – 3; Low – 2; Very low – 1***

3.2.2 Identification of the DM

This study considered only one DM who is knowledgeable about both technical and managerial issues. The DM filled a questionnaire, evaluated the Alternatives against the Criteria, and responded to the interview questions.

3.3 Evaluation of Alternatives

The table below shows the scores of Alternatives given by the DM against each Criterion.

Table 1: Score evaluation matrix: Alternatives versus Criteria

Alternative	**Criteria**						
	c_1	c_2	c_3	c_4	c_5	c_6	c_7
A_{11}	4	3	2	4	3	3	4
A_{12}	3	1	1	3	3	3	3
A_{13}	5	2	2	5	3	3	4
A_{14}	4	2	2	4	3	3	4
A_{21}	4	3	2	4	2	3	3
A_{31}	4	3	2	4	2	2	3
A_{41}	4	2	1	4	2	3	3
A_{51}	5	3	2	5	1	2	4
A_{52}	5	4	2	5	1	2	4
A_{61}	4	2	1	4	3	3	4
A_{62}	4	1	1	4	3	3	4
A_{63}	4	2	1	4	3	3	4
A_{71}	5	4	2	5	4	4	4
A_{81}	4	4	2	4	4	4	4
A_{91}	4	3	3	4	4	4	4
A_{101}	4	3	1	4	3	3	3
Max/Min	Max	Min	Min	Max	Max	Max	Max

3.3.1 Normalization of data

The normalization process is done to converts the data to the commensurable unit. The value of alternatives ranges between 0 and 1, where 0 is the worst alternative value and 1 the best alternative value in each attribute if its goal is to maximize or minimize [6]. "Reference [15] explained normalization methods for COPRAS and SAW methods" as follows:

- *COPRAS method*

Normalization of data for COPRAS method is done without transforming the cost to benefit type criteria by using linear transformation – sum method.

$$\boldsymbol{a_{ij}} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad (1)$$

- *SAW method*

SAW method uses various normalization techniques; Linear scale transformation - max method, linear scale transformation – sum method, linear scale transformation – MaxMin method, and vector normalization. In this study normalization was done by using linear scale transformation- sum method which separate benefit (maximum) and cost (minimum) criteria.

Benefit criteria

$$\boldsymbol{a_{ij}} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad (2)$$

Cost criteria

$$\boldsymbol{a_{ij}} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}}$$

Where $\boldsymbol{x_{ij}}$ is the score of ***i -th*** alternative with respect to ***j -th*** criterion before normalization, and $\boldsymbol{a_{ij}}$ is the normalized value.

3.4 Selection of MCDM Methods

This stage describes the elicitation of weights to criteria and ranking of Alternatives by using MCDM methods. This study compares COPRAS and SAW methods used in the ranking of alternatives. The SMARTER method was used to assign weights to criteria basing on their importance as put by the DMs for both methods.

3.4.1 Elicitation of weights

SMARTER was chosen because it uses a swing procedure to obtain a constant scale and use linear function values for intra- criterion evaluation. Furthermore, SMARTER is more precise in generating weights to the criteria than the weights of criteria given by the DMs.

"References [16], [17] identified various methods in which SMARTER uses in generating the weights to the evaluation criteria"; the common methods are: rank-sum (RS), rank reciprocal (RR), rank exponent (RE) and rank order centroid (ROC). In this study, the ROC weights method was selected to calculate the weights of criteria because weights of this method reflect the centroid (center of mass) of the simplex defined by the ranking of the criteria. Besides with more criteria, the method has much less error for ranked criteria and has clear statistic basis [18].

Rank Order Centroid (ROC) weights: The ROC weights are given by;

$$W_i(ROC) = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{j} \qquad i = 1, 2, 3, \ldots, n \qquad (3)$$

Where $\boldsymbol{n}$ is number of criteria; $\boldsymbol{i}$ represent the rank

The criteria are assigned with weights with vector $W = [W_1, W_2, \ldots\ldots, W_n]$, in which $W_1 \succ W_2 \succ W_3 \succ W_4 \succ \ldots\ldots \succ W_n$, which satisfies,

$$\sum_{j=1}^{n} W_j = 1 \qquad (4)$$

Thus, the weights generated by SMARTER – ROC weights method for seven criteria are;

$$W_1 = 0.3704, W_2 = 0.2276, W_3 = 0.1561, W_4 = 0.1085, W_5 = 0.0728, W_6 = 0.0442,\ W_7 = 0.0204$$

The weights were assigned according to the DM's rank of the evaluation criteria as $C_1 \succ C_4 \succ C_7 \succ C_3 \succ C_2 \succ C_6 \succ C_5$.

This means that; $C_1 \rightarrow W_1$, $C_4 \rightarrow W_2$, $C_7 \rightarrow W_3$, $C_3 \rightarrow W_4$, $C_2 \rightarrow W_5$, $C_6 \rightarrow W_6$, $C_5 \rightarrow W_7$.

The weighted normalized values ($V_{ij}(a)$) for alternatives are calculated by the MAVT weighted normalized value function which is given by:

$$V_{ij}(a) = w_j a_{ij} \qquad i = 1,2,\ldots,n \qquad (5)$$

Where $\boldsymbol{w_j}$ is the weight of $\boldsymbol{j}$ criteria and a_{ij} is the normalized value of alternative $\boldsymbol{i}$ with respect to criteria $\boldsymbol{j}$.

The weighted normalized matrix for COPRAS and SAW methods are given in tables 2 and 3 respectively.

Table 2: Weighted normalized matrix: Alternatives versus Criteria – COPRAS method

Alternative	**Criteria**						
	c_1 **W$_1$ =0.3704**	c_2 **W$_5$=0.0728**	c_3 **W$_4$=0.1085**	c_4 **W$_2$=0.2276**	c_5 **W$_7$=0.0204**	c_6 **W$_6$=0.0442**	c_7 **W$_3$=0.1561**
A_{11}	0.02211	0.00520	0.00804	0.01359	0.00139	0.00276	0.01058
A_{12}	0.01659	0.00173	0.00402	0.01019	0.00139	0.00276	0.00794
A_{13}	0.02764	0.00347	0.00804	0.01699	0.00139	0.00276	0.01058
A_{14}	0.02211	0.00347	0.00804	0.01359	0.00139	0.00276	0.01058
A_{21}	0.02211	0.00520	0.00804	0.01359	0.00093	0.00276	0.00794
A_{31}	0.02211	0.00520	0.00804	0.01359	0.00093	0.00184	0.00794
A_{41}	0.02211	0.00347	0.00402	0.01359	0.00093	0.00276	0.00794
A_{51}	0.02764	0.00520	0.00804	0.01699	0.00046	0.00184	0.01058
A_{52}	0.02764	0.00693	0.00804	0.01699	0.00046	0.00184	0.01058
A_{61}	0.02211	0.00347	0.00402	0.01359	0.00139	0.00276	0.01058
A_{62}	0.02211	0.00173	0.00402	0.01359	0.00139	0.00276	0.01058
A_{63}	0.02211	0.00347	0.00402	0.01359	0.00139	0.00276	0.01058
A_{71}	0.02764	0.00693	0.00804	0.01699	0.00185	0.00368	0.01058
A_{81}	0.02211	0.00693	0.00804	0.01359	0.00185	0.00368	0.01058
A_{91}	0.02211	0.00520	0.01206	0.01359	0.00185	0.00368	0.01058
A_{101}	0.02211	0.00520	0.00402	0.01359	0.00139	0.00276	0.00794
Max/Min	Max	Min	Min	Max	Max	Max	Max

Table 3: Weighted normalized matrix: Alternatives versus Criteria – SAW method

Alternative	**Criteria**						
	c_1 **W$_1$ =0.3704**	c_2 **W$_5$=0.0728**	c_3 **W$_4$=0.1085**	c_4 **W$_2$=0.2276**	c_5 **W$_7$=0.0204**	c_6 **W$_6$=0.0442**	c_7 **W$_3$=0.1561**
A_{11}	0.02211	0.00335	0.00501	0.01359	0.00139	0.00276	0.01058
A_{12}	0.01659	0.01004	0.01002	0.01019	0.00139	0.00276	0.00794
A_{13}	0.02764	0.00502	0.00501	0.01699	0.00139	0.00276	0.01058
A_{14}	0.02211	0.00502	0.00501	0.01359	0.00139	0.00276	0.01058
A_{21}	0.02211	0.00335	0.00501	0.01359	0.00093	0.00276	0.00794
A_{31}	0.02211	0.00335	0.00501	0.01359	0.00093	0.00184	0.00794
A_{41}	0.02211	0.00502	0.01002	0.01359	0.00093	0.00276	0.00794
A_{51}	0.02764	0.00335	0.00501	0.01699	0.00046	0.00184	0.01058
A_{52}	0.02764	0.00251	0.00501	0.01699	0.00046	0.00184	0.01058
A_{61}	0.02211	0.00502	0.01002	0.01359	0.00139	0.00276	0.01058
A_{62}	0.02211	0.01004	0.01002	0.01359	0.00139	0.00276	0.01058
A_{63}	0.02211	0.00502	0.01002	0.01359	0.00139	0.00276	0.01058
A_{71}	0.02764	0.00251	0.00501	0.01699	0.00185	0.00368	0.01058
A_{81}	0.02211	0.00251	0.00501	0.01359	0.00185	0.00368	0.01058

A_{91}	0.02211	0.00335	0.00334	0.01359	0.00185	0.00368	0.01058
A_{101}	0.02211	0.00335	0.01002	0.01359	0.00139	0.00276	0.00794
Max/Min	Max	Min	Min	Max	Max	Max	Max

3.4.2 Ranking of Alternatives

Ranking of alternatives was done through COPRAS and SAW methods. These methods are among of MAUT methods used for ranking the alternatives as explained by [14]. The COPRAS method uses both the maximizing (benefits) and minimizing (costs) criteria values in evaluation, while SAW method use only benefit criteria after transforming cost to benefit criteria [9], [10],[14], [15], [19]

The ranking process passes through the following steps:

- *COPRAS Method*

1. Firstly, we find the sum of weighted normalized values for both the maximization (Beneficial) and minimization (cost/non-beneficial) attributes.

The sums of weighted normalized values are calculated by using the following formulas:

$$S_{+i} = \sum_{j=1}^{n} W_{+j}\, a_{+ij} = \sum_{j=1}^{n} V_{+ij}\,(a) \quad (6)$$

$$S_{-i} = \sum_{j=1}^{n} W_{-j}\, a_{-ij} = \sum_{j=1}^{n} V_{-ij}\,(a)$$

Where S_{+i} and S_{-i} are sums of maximizing and minimizing weighted normalized criteria respectively.

The greater the value of $\boldsymbol{S_{+i}}$, the better is the alternative; and the lower the value of $\boldsymbol{S_{-i}}$, the better is the alternative. The $\boldsymbol{S_{+i}}$ and $\boldsymbol{S_{-i}}$ values express the degree of goals attained by each alternative. The significances of the alternatives on the basis of defining the positive alternatives $\boldsymbol{S_{+i}}$ and negative alternatives $\boldsymbol{S_{-i}}$ characteristics is given in the table 4 below.

Table 4: Sums of the weighted normalized values

Alternative	S_{+i}	*Value*	S_{-i}	*Value*
A_{11}	S_{+11}	0.05044	S_{-11}	0.01324
A_{12}	S_{+12}	0.03887	S_{-12}	0.00575
A_{13}	S_{+13}	0.05936	S_{-13}	0.01150
A_{14}	S_{+14}	0.05044	S_{-14}	0.01150
A_{21}	S_{+21}	0.04733	S_{-21}	0.01324
A_{31}	S_{+31}	0.04641	S_{-31}	0.01324
A_{41}	S_{+41}	0.04733	S_{-41}	0.00749
A_{51}	S_{+51}	0.05752	S_{-51}	0.01324
A_{52}	S_{+52}	0.05752	S_{-52}	0.01497
A_{61}	S_{+61}	0.05044	S_{-61}	0.00749
A_{62}	S_{+62}	0.05044	S_{-62}	0.00575
A_{63}	S_{+63}	0.05044	S_{-63}	0.00749
A_{71}	S_{+71}	0.06075	S_{-71}	0.01497
A_{81}	S_{+81}	0.05182	S_{-81}	0.01497
A_{91}	S_{+91}	0.05182	S_{-91}	0.01726
A_{101}	S_{+101}	0.04779	S_{-101}	0.00922

2. Secondly, we determine the relative significances or priorities $\boldsymbol{(Q_i)}$ of the alternatives.

The relative significances or priorities of the alternatives are found by the relationship given below:

$$Q_i = S_{+i} + \frac{S_{-min}\sum_{i=1}^{m} S_{-i}}{S_{-i}\sum_{i=1}^{m}\frac{S_{-min}}{S_{-i}}}$$

Where S_{-min} is the minimum value of S_{-i}, m is the number of alternatives. (7)

The equation can be simplified to

$$Q_i = S_{+i} + \frac{\sum_{i=1}^{m} S_{-i}}{S_{-i}\sum_{i=1}^{m}\frac{1}{S_{-i}}} \quad (8)$$

The greater the value of Q_i, the higher the priority of the alternative. The relative significance value of an alternative shows the degree of satisfaction attained by that alternative. The alternative with the highest relative significance value (Q_{max}) is the best choice among the evaluated alternatives.

For example, i = 1

$$Q_1 = S_{+1} + \frac{\sum_{i=1}^{16} S_{-i}}{S_{-1}\sum_{i=1}^{16}\frac{1}{S_{-i}}}$$

$$= 0.050438 + \frac{0.1813}{0.013237(1591.3707626)}$$

$Q_1 = 0.059045$

Note: $Q_1 = Q_{11}$, $S_{+1} = S_{+11}$, $S_{-1} = S_{-11}$

As shown above, the values of Q_i of other alternatives are calculated and shown in table 5.

3. Thirdly, we calculate the quantitative Utility (U_i)

The quantitative utility (U_i) was determined by comparing the priorities of all alternatives with the one with maximum relative significance.

$$U_i = (\frac{Q_i}{Q_{max}}) \text{ X } 100\ \% \quad (11)$$

The utility values of the alternatives range from 0 % to 100 %.

Table 5 shows the values of relative significance *(Q_i)* and quantitative utility *(U_i)* of each alternative.

Table 5: Values of preference (Q_i), quantity utility (U_i) and ranking of alternative

Rank	*Alter*	*Q_i*	*U_i (%)*	*Rank*	*Alter*	*Q_i*	*U_i (%)*
1.	A_{71}	0.073308	100.00000	9.	A_{41}	0.062150	84.77214
2.	A_{62}	0.070372	95.99498	10.	A_{91}	0.061041	83.26649
3.	A_{63}	0.070125	95.65805	11.	A_{14}	0.060883	83.05096
4.	A_{13}	0.069877	95.31974	12.	A_{11}	0.059045	80.54373
5.	A_{51}	0.066524	90.74589	13.	A_{12}	0.058143	79.31331
6.	A_{52}	0.065131	88.84569	14.	A_{101}	0.057011	77.76914
7.	A_{61}	0.064344	87.77214	15.	A_{21}	0.056338	76.85109
8.	A_{81}	0.063692	86.88274	16.	A_{31}	0.055125	75.19643

Basing on the results, the complete ranking of alternatives for WLM is obtained as:

$\mathbf{A_{71} \succ A_{62} \succ A_{63} \succ A_{13} \succ A_{51} \succ A_{52} \succ A_{61} \succ A_{81} \succ A_{41} \succ A_{91} \succ A_{14} \succ A_{11} \succ A_{12} \succ A_{101} \succ A_{21} \succ A_{31}}$ with $\mathbf{A_{71}}$ indicating the best alternative with 100% utility degree and $\mathbf{A_{31}}$ indicating is the worst alternative with 75.19643% utility degree.

- *SAW Method*

The sum S_j of the weighted values of all criteria is calculated as follows:

$$S_i = \sum_{j=1}^{n} W_j\, a_{ij} = \sum_{j=1}^{n} V_{ij}\,(a)$$

Where $\boldsymbol{w_j}$ is the weight of ***j*** criteria and a_{ij} is the normalized value of alternative ***i*** with respect to criteria ***j.***

The highest value of S_i is the best ranked or most preferable alternative

Table 6 below shows the value of S_i and ranking of alternatives.

Table 6: Value of S_i and ranking of alternative

Rank	*Alternative*	*Value (S_i)*	*Rank*	*Alternative*	*Value (S_i)*
1.	A_{62}	0.07049	8.	A_{101}	0.06115
2.	A_{13}	0.06939	9.	A_{14}	0.06047
3.	A_{71}	0.06827	10.	A_{81}	0.05934
4.	A_{51}	0.06587	11.	A_{12}	0.05892
5.	A_{61}	0.06547	12.	A_{11}	0.05879
5.	A_{63}	0.06547	13.	A_{91}	0.05851
6.	A_{52}	0.06503	14.	A_{21}	0.05568
8.	A_{41}	0.06236	15.	A_{31}	0.05476

Basing on the results, the complete ranking of alternatives for WLM is obtained as:
$A_{62} \succ A_{13} \succ A_{71} \succ A_{51} \succ A_{61}, A_{63} \succ A_{52} \succ A_{41} \succ A_{101} \succ A_{14} \succ A_{81} \succ A_{12} \succ A_{11} \succ A_{91} \succ A_{21} \succ A_{31}$ with $\mathbf{A_{62}}$ indicating the best alternative and $\mathbf{A_{31}}$ indicating is the worst alternative.

3.5 Integer Linear Programming (ILP)

ILP is a mathematical optimization technique in which all variables are restricted to integers. In this study, the ILP was used to assess the operational restrictions of the company and to select the portfolio of alternatives. A special form of ILP, the Binary Integer Programming (BIP) was used. In BIP, all decision variables are restricted to take on binary values either 0 or 1. The BIP gives the model more meanings in decision making. Numbers in this programming represent the selection choice of alternatives instead of their arbitrary values.

The formulation of ILP models for COPRAS and SAW methods is the same. The constraints of the two models are the same; the difference is on the objective functions in which they differ on their coefficients.

The developed ILP models are as follows;

- *COPRAS Method*

Objective function:

Max $Z = 0.059045A_{11} + 0.058143A_{12} + 0.069877A_{13} + 0.060883A_{14} + 0.056338A_{21} + 0.055125A_{31} + 0.062150A_{41} + 0.066524A_{51} + 0.065131A_{52} + 0.064344A_{61} + 0.070372A_{62} + 0.070125A_{63} + 0.073308A_{71} + 0.063692A_{81} + 0.061041A_{91} +$ (9)

Subject to constraints

$14.58A_{11} + 14.58A_{12} + 14.58A_{13} + 14.58A_{14} + 31.59A_{21} + 29.16A_{31} + 2.43A_{41} + 75.33A_{51} + 75.33A_{52} + 4.86A_{61} + 4.86A_{62} + 4.86A_{63} + 29.16A_{71} + 51.03A_{91} + 4.86A_{101} \leq 243$ (10)

$A_{11} + A_{12} + A_{13} + A_{14} \geq 1$ (11)
$A_{51} + A_{52} \geq 1$ (12)
$A_{61} + A_{62} + A_{63} \geq 1$ (13)
$A_{11} + A_{12} + A_{13} + A_{14} + A_{21} + A_{31} + A_{41} + A_{51} + A_{52} + A_{61} + A_{62} + A_{63} + A_{71} + A_{81} + A_{91} + A_{101} \leq 16$ (14)

$A_{11}, A_{12}, A_{13}, A_{14}, A_{21}, A_{31}, A_{41}, A_{51}, A_{52}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, A_{101}$ = [0 or 1] (15)

- *SAW method*

Objective function:

Max $Z = 0.05879A_{11} + 0.05892A_{12} + 0.06939A_{13} + 0.06047A_{14} + 0.05568A_{21} + 0.05476A_{31} + 0.06236A_{41} + 0.06587A_{51} + 0.06503A_{52} + 0.06547A_{61} + 0.07049A_{62} + 0.06547A_{63} + 0.06827A_{71} + 0.05934A_{81} + 0.05851A_{91} + 0.06115A_{101}$ (16)

Subject to constraints

$14.58A_{11} + 14.58A_{12} + 14.58A_{13} + 14.58A_{14} + 31.59A_{21} + 29.16A_{31} + 2.43A_{41} + 75.33A_{51} + 75.33A_{52} + 4.86A_{61} + 4.86A_{62} + 4.86A_{63} + 29.16A_{71} + 51.03A_{91} + 4.86A_{101} \leq 243$ (17)
$A_{11} + A_{12} + A_{13} + A_{14} \geq 1$ (18)
$A_{51} + A_{52} \geq 1$ (19)
$A_{61} + A_{62} + A_{63} \geq 1$ (20)
$A_{11} + A_{12} + A_{13} + A_{14} + A_{21} + A_{31} + A_{41} + A_{51} + A_{52} + A_{61} + A_{62} + A_{63} + A_{71} + A_{81} + A_{91} + A_{101} \leq 16$ (21)
$A_{11}, A_{12}, A_{13}, A_{14}, A_{21}, A_{31}, A_{41}, A_{51}, A_{52}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, A_{101}$ = [0 or 1] (22)

Where: Eqs. (9 &16) are the objective functions in which the coefficients are the relative preferences (Q_i) and sum of weighted values (S_i) of the MCDM methods. Eqs. (10 &17) are the budget restriction constraints for implementation of preventive actions which are in 1,000.000/= Tanzanian Shillings (TZS). Eqs. (11 – 13 & 18 – 20) represent the multiple-choice alternatives found in one category of strategies. The constraints ensure that at least one sub- alternative can be adopted in WLM. Eqs. (14 &21) are the constraints that represent the optimal number of alternatives to be selected in a portfolio. Eqs. (15&22) represent binary (decision variables, A_i) constraints; the values are integers 0 or 1, where 1 means that A_i is the selected alternative and 0 otherwise.

After solving the ILP equations for each model by LINDO 6.1 software package, the variables, A_{11}, A_{12}, A_{13}, A_{14}, A_{41}, A_{51}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, and A_{101} yielded value 1, while variables,A_{21}, A_{31},and A_{52} yielded 0 value. This means variables whose value is 1 are alternatives selected to form a portfolio of best alternatives and those whose value is 0 are eliminated. The eliminated alternatives are of less importance and their roles can be performed by the remaining alternatives.

On both models, the selected alternatives represent a total cost of TZS 235.71 million which is 97 % of the total cost budgeted by the water authority. This means the authority will save TZS 7.29 million if the alternatives of this portfolio are implemented. The maximum sum of values of the selected alternatives are Z = 0.836515 and Z = 0.82456 for COPRAS and SAW methods respectively.

3.6 Sensitivity Analysis

“Reference [20] defines Sensitivity analysis as a fundamental concept in the effective use and implementation of quantitative decision models, whose purpose is to assess the stability (robustness) of an optimal solution under changes in the parameters”.

For that, the ranking of the criteria was changed after doing mathematical calculations as explained by [20] for evaluating the critical of each criterion.

In the analysis, we find that the new rankings of decision criteria are:

- COPRAS method: $C_3 \succ C_1 \succ C_4 \succ C_2 \succ C_6 \succ C_5 \succ C_7$
- SAW method: $C_1 \succ C_4 \succ C_3 \succ C_2 \succ C_7 \succ C_6 \succ C_5$

Considering the weights of criteria generated by SMARTER- ROC method in section (3.3.1); weighted normalization and ranking process of alternatives were done as per section (3.3.2) procedures.

The new ranking of alternatives becomes;

- COPRAS method: $\mathbf{A_{63} \succ A_{62} \succ A_{12} \succ A_{71} \succ A_{41} \succ A_{61} \succ A_{81} \succ A_{13} \succ A_{14} \succ A_{51} \succ A_{91} \succ A_{11} \succ A_{52} \succ A_{21} \succ A_{101} \succ A_{31}}$

- SAW method: $\mathbf{A_{62} \succ A_{13} \succ A_{61}\,,\, A_{63} \succ A_{71} \succ A_{41} \succ A_{51} \succ A_{12} \succ A_{101} \succ A_{52} \succ A_{14} \succ A_{81} \succ A_{11} \succ A_{91} \succ A_{21} \succ A_{31}}$

Furthermore, by using the values of new relative significance (Q_i) and the sum of weighted normalized (S_i) of the alternatives, new ILP models were formulated; and when solved the same portfolio of alternatives, A_{11}, A_{12}, A_{13}, A_{14}, A_{41}, A_{51}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, and A_{101} was selected. Three alternatives,$A_{21}, A_{31}, and\ A_{52}$ were eliminated from the list as well. This implies that the ILP model for selecting the portfolio of the best alternative is robust regardless of the ranking of alternatives and the change of weights of evaluation criteria.

IV. Overall Discussion

When considering table 5 the values of utility (U_i) show that no alternatives which are below 50%, and when comparing the values of alternatives versus the value of the most preferable alternative in table 6 we find that, no alternatives which are below 50% too. This implies that all alternatives are important in WLM. The relative significance values (Q_i) in table 5 ranks alternative A_{71} (Replacement of dilapidated pipes) as the best alternative and A_{31} (Network zoning) as the least alternative in water loss management. Table 6 shows that the most preferable alternative is A_{62} (comparison between the readings of bulk meter and customers’ meter) and again the least preferable alternative is A_{31} (Network zoning). Moreover, in both ranking of alternatives, we found that pressure management (A_{101}) is not a critical strategy for water loss management at MUWSA like in many WDS. The reason is that most of the water sources at MUWSA are springs hence water flow is by gravity. The water authority’s 5 years corporate strategic plan is to avoid the use of boreholes sources which will lead to zero use of pumps and according to research data

(MUWSA 2019) the water authority will reduce the operation costs to about 2%, the budget set for pressure management in the WDS.

However, after solving the ILP problems, the selected portfolio of alternatives include the following alternatives: A_{11}, A_{12}, A_{13}, A_{14}, A_{41}, A_{51}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, and A_{101}.

Three alternatives,$A_{21}, A_{31}, and\ A_{52}$ were eliminated from the list. The selected portfolio of alternatives cost 97% (TZS 235.71 million) of the total budget set by the water authority for WLM. Thus, the authority will save 3% (TZS 7.29 million) if this portfolio of alternatives is implemented. These alternatives have maximum sum values of Z = 0.836515 and 0.82456 for the models formulated from COPRAS and SAW methods respectively. The similarities of the two results of the models indicate that the two methods COPRAS and SAW are efficient and appropriate for decision-making and in solving issues in water management and planning.

When the ranking of decision criteria was changed to evaluate the sensitivity analysis, the ILP models showed robustness in selecting the portfolio of alternatives. The selected alternatives before and after sensitivity analysis were the same for both models.

V. Conclusion

This paper presents a comparative analysis of two MCDM methods, COPRAS and SAW integrated with ILP technique to solve issues related to water management in a WDS. The main objectives of using these combined models were to compare their efficiency in evaluating the identified alternatives, and in selecting the group of best alternatives which are to be used in WLM. Moreover, the comparison aimed to check their appropriateness in optimizing the WLM strategies used in the WDS. The results showed that the two methods rank differently the alternatives while selecting the same portfolio of the best alternatives. Both models select thirteen alternatives, A_{11}, A_{12}, A_{13}, A_{14}, A_{41}, A_{51}, A_{61}, A_{62}, A_{63}, A_{71}, A_{81}, A_{91}, and A_{101} which cost 97% (TZS 235.71 million) of the total budgets set by the water authority for WLM. For this case, the authority will save 3% (TZS 7.29 million) if this portfolio of alternatives will be implemented. The results prove that the two methods are appropriate and efficient for decision-making and in solving issues in water management and planning.

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